TREATISE ON PRACTICAL LIGHT

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TREATISE ON PRACTICAL LIGHT

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INTRODUCTION

ALTHOUGH Optics is one of the oldest of the sciences, it has been pursued mainly for its theoretical interest, and until comparatively recently spectacles, the telescope and the microscope may be said to have been its only outcome of commercial importance. But in the last fifty years the development of photography, the improvement in the microscope following upon the work of Abbé and his successors, the invention of three-colour reproduction, and the introduction of the coal-tar products into textile dyeing, have created problems of great and increasing importance. Those problems have, of course, arisen in the factories, and although manufacturers have frequently appointed distinguished mathematicians and men of science as experts to deal with them, these have—partly perhaps through trade jealousy—in the main worked independently, and only occasionally published any of their results.

Meanwhile the treatment of Optics in most of our colleges, so far as it goes beyond the elementary notions, deals with it either as a framework upon which to hang pretty mathematical problems, or else deals with the fascinating, but comparatively non-practical questions of Physical Optics, such as interference, polarisation anomalous dispersion, etc. For these reasons I have found the chapters to which I attach the greatest importance upon the Compound Lens, the Microscope, and upon Colour, the most difficult to write, and I realise that they are still far from complete.

There can be no doubt that it is to the advantage of the optical industry both that the difficulties which manufacturers find should

be known to those whom scientific training may assist in their solution, and also that the experimental methods which have been adopted by individual manufacturers should be known to each other, as well as to those whose interest may be chiefly theoretical. I feel, therefore, no need to apologise for having treated the Compound Lens and Colour Measurement more fully than usual, but on the contrary, in order that the book may be made as useful as possible, I should be deeply grateful to any who have practical methods of dealing with the many problems in these connections, if they would give me an opportunity of learning them.

I shall also be glad if any one finding mistakes will send me a note of them for future correction.

The greater portion of this book was already written some ten years ago, when I published the more elementary portions as *Practical Exercises in Light.*

I am hopeful that this larger book may serve as something more than a mere text-book, and that it may help to interest some of the younger mathematicians in the experimental and practical side of a subject in which there is a large field for useful work.

For those who have to deal with Optics commercially and wish to read the theory, I would recommend Edser's *Light* (Macmillan) as an elementary text-book. This may be followed by Southall's *Principles and Methods of Geometrical Optics* (Macmillan), a book giving to English readers a very clear exposition of the problems connected with the Compound Lens and their treatment by the modern German school. Dennis Taylor's *System of Optics* is a valuable book for a lens computator, but is difficult to read, owing to his involved system of signs.

I desire to thank Mr. Edser for permission to reproduce Fig. 280, the only illustration not specially drawn for the book.

R. S. CLAY.

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TREATISE ON PRACTICAL LIGHT

CHAPTER I

PIN EXPERIMENTS

Laws of Reflection of Light.

Apparatus.—Drawing-board and paper, plane mirror, that can be supported in a vertical plane, about 3 inches broad by 1 inch high, which must be of good glass—preferably "patent plate" (the great thickness of ordinary plate-glass is a disadvantage).

1. **Equality of Angles of Incidence and Reflection.**—Draw a straight line, AB, on the paper. Stand the mirror on the paper so that its silvered surface is vertically over this line. Stick a pin, Q, in the paper close to the front surface of the mirror, and another, P, as far as the paper will allow distant from it in an oblique direction. Look into the mirror along the line PQ joining these two pins so that the one, Q, nearest the mirror is hidden by the pin P. Without moving the head insert two more pins, R and S, one close to the surface of the mirror and one a long way off, in such positions that their images appear in the mirror along the continuation of the line PQ.

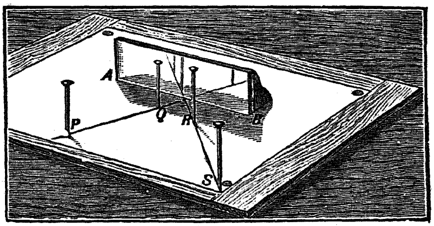


FIG. 1.—Reflection in a Plane Mirror.

If the head be moved so as to look along the line SR, the images of Q and P will be found to appear on the continuation of the line SR.

Remove the mirror, join PQ and SR, produce them to meet the line which coincided with the silvered surface of the mirror. They should intersect approximately at O on this line. With a set square draw a line, ON, through O perpendicular to AB. Then the angles PON, NOS should be equal.

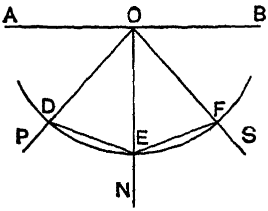


FIG. 2.—Law of Reflection.

With O as centre, and a radius of about 12 cms., describe an arc of a circle cutting PO, ON, OS, in D, E, F. Measure the distances DE, EF with a millimetre scale. Copy this figure on a reduced scale in your notebook, and enter these distances. The equality of these distances will measure the accuracy with which the experiment has been performed.

2. Equality of Distance of Image and Object from the Reflecting Surface.—(*a*) *Sighting Method.*—Draw a line, AB, upon the paper, and place the mirror so that this line may coincide with its silvered surface. About 6 cms. in front of the mirror place a pin, Q. To find the position of the image, q, of this pin, insert pins, P_1, P_2, in such positions that they seem in a line with this image, so that looking along the line P_1P_2, the pin P_1 appears to hide both P_2 and q, the image of Q. In the same way, insert a pair of pins, P_3, P_4, in some other direction. Remove the mirror, join P_1P_2 and P_3P_4, and produce these lines till they meet at q. Join qQ. The line qQ should be at right angles to AB and be bisected by it. Actually measure the distances of q and Q from this line and insert them in your notebook.

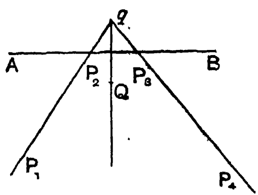


FIG. 3.—Position of Image.

(*b*) *Parallax Method.*—Place the mirror and pin Q as before. Looking in the mirror nearly normally, with the eye at such a height that only the lower part of the image of Q is visible in the

mirror, place a pin, Q′, in the paper at the back of the mirror so that the upper part of this pin may seem to be a continuation of the image q of Q. Q′ is evidently then on the line joining the eye and the image q; it may be in front of or behind this image.

To determine this, move the eye to the right as far as the mirror will allow. If Q′ still appears to be the continuation of the image of Q it has been so placed as to coincide with q exactly; but generally it will be found to separate from q. If it appears

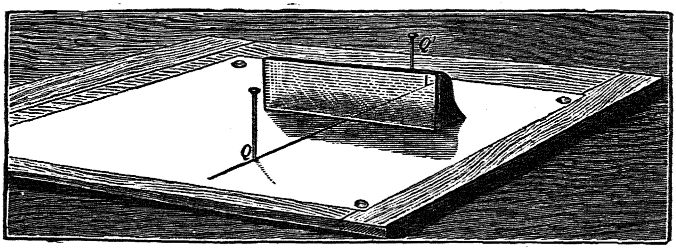


FIG. 4.—Position of Image by Parallax. The image of Q appears to the right of Q′, and is therefore behind Q′. Q′ is to be moved farther from the mirror.

to have moved to the right of q (*i.e.* to have moved in the same direction as the eye has been moved), it is too far away from the mirror and must be brought a little nearer. By repeated trials a position for Q′ will be found such that, wherever the eye be placed, the image of Q and pin Q′ will appear to be continuous. If the distances of Q and Q′ from the line AB be now measured, they should be found equal. Their actual lengths must be taken and entered in the notebook, in which a reduced drawing should be made.

3. **Note on the Parallax Method.**—We are accustomed to judge of the position of ordinary objects by two methods: firstly *the apparent size*, and secondly what we may call *the stereoscopic method.* In the first, knowing the actual size of the object, we are able by accumulated experience, very accurately to estimate the actual distance by the angle the object subtends at the eye; *i.e.* by its apparent size. In the second method, if we can see the object with both eyes we can estimate its distance by the convergence of the axes of the eyes.

In addition to these two methods, we can to some extent judge the distance by the change in the *accommodation* of the eye, and by observing its *position* relative to that of objects at a known distance.

When, however, an image is formed by a lens or mirror, most of these methods fail us. As the lens or mirror is usually not large enough to allow the image to be seen simultaneously by both eyes, we are unable to judge of its position by the convergence of the axes, and for near objects this is probably the one upon which we mostly rely in our everyday experience. The size of the image being as a rule quite different from that of the object, it is obvious that we shall be unable to judge its distance by its apparent size, since we have no means of estimating it. Indeed, as it is very difficult to dissociate the image from the object, the size of which we really know, the apparent size makes the distance of the image appear very different from its true distance and produces a powerful optical illusion; an object seen through a concave lens, for instance, appears to be at a very much greater distance than it really is, since its apparent size is less than the size of the original; and students frequently find it difficult to realize the fact that the image is really closer to them than the object. This illusion is so powerful that it more than neutralizes the estimate of the distance that should be furnished us by the accommodation of the eye. We are therefore left with only one method by which we can, without further apparatus, judge of the distance of the image—namely, its position relatively to other objects. It is this method which is referred to as *the method of parallax.*

If an observer will stand in front of a window, and between him and the window there is some object such as a retort stand or hanging gas lamp, if he moves sideways, say to the left, he will find that the retort stand appears to move across the window in the opposite direction—namely, to the right. On the other hand, if he looks through the window at some object outside, and moves to the left, he will find that the object moves across the window in the same direction—namely, to the left. So that an object which is further from him than the window (outside it) will appear to move across the window in the same direction as the observer moves, to the left when he moves to the left, to the right when he moves to the right. Exactly the opposite occurs when the object is within the room, *i.e.* nearer to him than the window; it moves across the window to the right when he moves to the left, and *vice versa.*

To apply this, let us suppose that a lens is placed in front of some object—a convex lens, for instance, at a distance of 2 or 3 feet from a sheet of newspaper, or better still, a poster. Look into the lens from some little distance. An image of the poster will probably be visible in the lens. Sometimes it will be erect and at others inverted, depending upon the focal length and the distance of the object. Thinking of this lens as a window, it is obvious that if the image is on the far side

of the lens, when the observer moves to the left, the image of the poster will move across the lens to the left also. Whilst if the image is on the observer's side of the lens, a movement of the observer to the left should cause it to appear to move across the lens to the right. By observing whether the movement of the image of the poster across the lens is in the same direction as the observer's own movement, or whether it is in the opposite direction, he can therefore ascertain if the image is formed on the far side of the lens or on his side of the lens.

Again, suppose there to be two objects at different distances outside the window, practically in a line with one another. If the observer

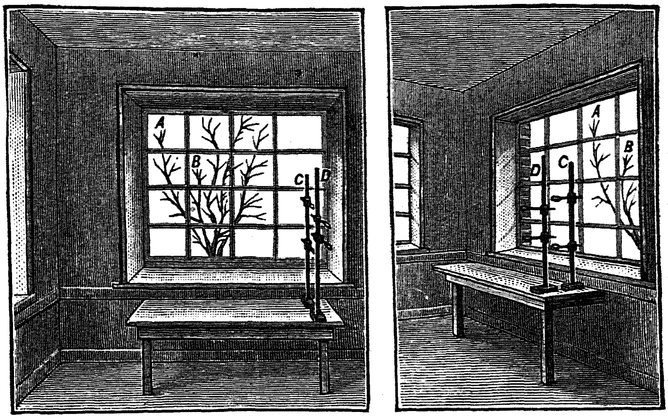


FIG. 5.—*Effect of Parallax.* When the observer has moved to the right, C and D, which are within the room, have moved across the window to the left; A and B have moved to the right. C, which is further from the observer, has moved to the right of D (although both have moved to the left relatively to the window).

moves to the left both of these will move across the window to the left, but it will be found that they no longer remain in line with one another—the further one moves to the left across the window by a greater amount than the nearer one, so that the further one moves to the left of the nearer one.

If he tries this with objects within the room, say two retort stands, placed in a line between him and the window, when he moves to the left both retort stands will move across the window to the right, but they will not appear by equal amounts—the retort stand which is further from him moving less across the window to the right than the one nearer to him ; so that, considering the retort stands alone, the further retort stand is now apparently to the left of the nearer one.

Therefore, when we have two objects either without or within the room at different distances from the observer (and whether, therefore the objects appear to move across the window in the same direction as the observer or the opposite one), in all cases **the further object of the two will appear to move, relatively to the nearer object, in the same direction as the observer.**

This rule should be firmly impressed on the memory.

Laws of Refraction.

Apparatus.—A half plate cutting-shape,[1] drawing-board, paper pins, and millimetre scale.

4. **The Ratio of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, for any one Medium, is a Constant, and is called the Refractive Index.**—(*a*) Draw a line on the paper and place the

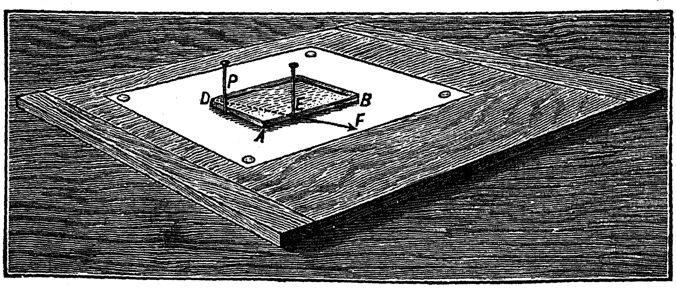


FIG. 6.—Refraction from a Glass Plate into Air.

edge, AB, of the plate upon this line. Looking through the surface AB, in a line with a pin, E, observe the corner D, and place a pin, P, in such a position close to the edge AD, that the corner D, as seen through the block, may seem to be continued above the block by the pin P. Then the ray DE on emergence traverses along a line EF which is a continuation of the line PE. The refractive index is the ratio $\frac{\text{ED}}{\text{EP}}$: for by drawing the normal at E it is easy to see that the angles EPA and EDA are equal

[1] The glass plate used by a photographer to trim his prints. It must have *rectangular* edges, one with bevelled edges will be useless.

to the angles of incidence and refraction, NEF and N'ED, respectively, and therefore

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \text{EPA}}{\sin \text{EDA}} = \frac{\frac{\text{AE}}{\text{EP}}}{\frac{\text{AE}}{\text{ED}}} = \frac{\text{ED}}{\text{EP}}.$$

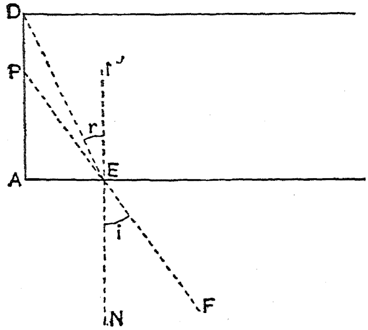


FIG. 7.—Refractive Index of a Glass Plate.

Repeat the experiment with the pin E in a different position, and again find the ratio $\frac{\text{ED}}{\text{EP}}$; see that the two values of μ are the same, or nearly so.

Copy the figures in your notebook on a reduced scale, and fill in the distances you measure.

(*b*) Draw a line on the paper and place the edge AB of the block over it. Close to the edges CD and AB insert pins P and Q so that the line PQ may be oblique to the face AB. Holding the head down so that P is seen through the block and

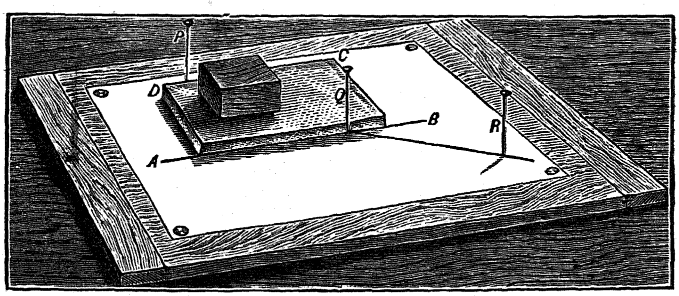


FIG. 8.—Refraction through a Glass Plate.

closing one eye, place yourself in such a position that the pin Q is in a line with (and therefore covers) the image of P as seen through the glass. (To make sure that beginners are really viewing the pin P through the block, it is helpful to put a piece of wood on the upper surface of the glass, so that the pin P can only be seen through the glass itself. To make sure that

the student is looking at P, and not at the image of one of the corners, C or D, the pin may be rocked slightly and the student told to observe that its image, at which he is looking, moves also.) Now insert at a distance of 10 or 12 cms. a third pin, R, so that it may appear in a line with Q and this image of P.

Remove the block, draw the normal MM'. With Q as centre, describe a circle of about 8 cms. radius, cutting QP, QR in K and L. Drop perpendiculars KM', LM, and measure their lengths.

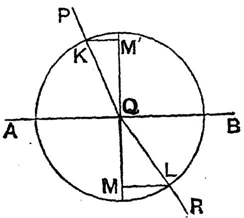


FIG. 9.—Refractive Index.

The refractive index, μ, is given by $\frac{LM}{KM'}$.

Replace the block, and the pin Q, and repeat the experiment for a new position of R. See that the value of μ now obtained agrees with the former one.

Copy the figure about one-third size in your notebook, and insert the actual lengths found for KM', LM, in each case.

5. **When a Ray of Light passes obliquely through a Block of Glass with Parallel Sides, the Incident and Emergent Rays will be Parallel.**—Place the block of glass on paper, and run a fine pencil line along the surfaces AB, CD. Insert the pins P, Q, R, as directed in the last experiment. Then looking in through the other side of the block, insert another pin in S, so that S, P, Q seem in a line. Remove the block, produce the line SP, and drop perpendiculars on it from Q and R.

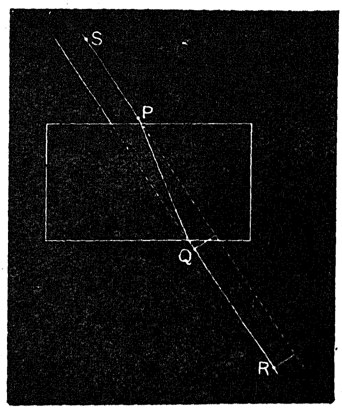


FIG. 10.—Path of a Ray through a Parallel Glass Plate.

Measure these perpendiculars: they should be equal.

Make a reduced copy of the figure in your notebook, and insert the actual lengths of the lines.

Refraction Experiments with a Prism.

Apparatus.—Glass prism,[1] drawing board, paper, pins, scale, and protractor.

6. **Symmetrical Refraction.**—Place the prism on the paper, and with a fine pencil draw a line along the edges AB and AC.

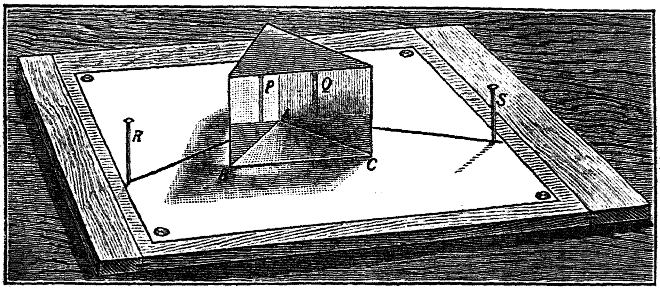


FIG. 11.—Refraction through a Prism.

Remove the prism for a moment, and insert pins at P and Q, so that the triangle APQ may be roughly isosceles. Replace the prism, seeing that its edges exactly coincide once more with the lines AB, AC. Looking in the prism in the direction RP with one eye closed, place yourself so that P is in line with

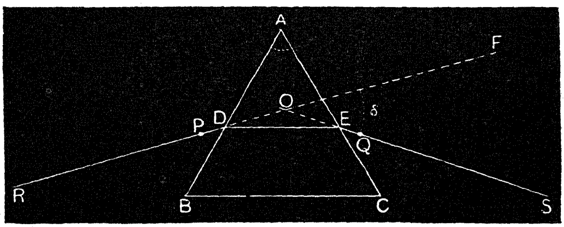


FIG. 12.—Refraction through a Prism.

the image of Q, and insert a pin, R, as far from P as the paper will allow. Then, looking in the direction SQ, insert another pin to appear in a line with Q and the image of P as seen through the prism. Remove the prism. Join the pins R, P, by

[1] Large prisms of about 3-inch face can be obtained at a low price for this experiment from Messrs. Pye and Co., Mill Lane, Cambridge.

a line, and produce this line to F, cutting the line AB at D. In the same way join S, Q, by a line, and produce it to meet the other one at O, cutting the surface AC at E.

Measure the angle FOS.

If we imagine the ray to be incident along the line RP, it passes through the prism in the direction DE and emerges along ES. If the prism had not been there, this same ray would have proceeded along OF; thus, the prism has bent the ray from the direction OF to the direction OS, and the angle FOS measures the *deviation* which the prism has produced.

Find this angle with the protractor, and also the angle BAC. Copy the figure into your notebook, and enter the magnitudes of these angles.

Observe that, when looking along SQ to insert the pin S, the image of R, seen in the distance, appears somewhat indistinct and coloured on emergence, blue on one side and red on the other. This is because the blue and red rays are unequally bent by the prism. It is on this account that you are advised to insert S looking along the line SQ, instead of along the line RP; for it is difficult to say exactly where S should be placed if it is observed through the prism.

7. **The Path through a Prism of a Ray which has been internally reflected.**—Begin by drawing the lines AB, AC, and BC, and

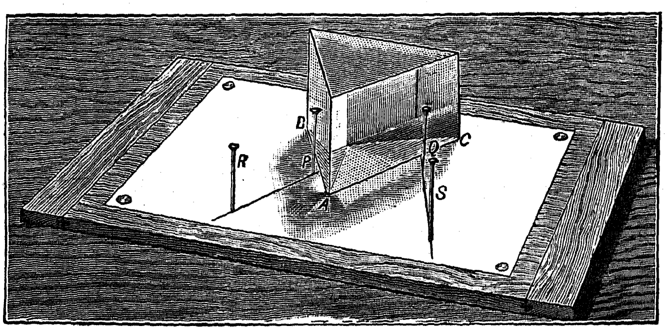


FIG. 13.—Reflection in a Prism.

insert the pins P and Q as in the last experiment. Looking in the direction RP, an image of Q will be seen which has been

formed by reflection at the surface BC. That it is the image of Q can be verified by slightly rocking the pin, when the image will be seen to move also. Insert the pin R as far from P as possible to appear in a line with P and this image. In the same way, the pin S may be inserted by looking along the line SQ. But as it will be found that S is not coloured this time it may be equally well inserted by looking along RP.

[If two students are working together on this experiment, one may look in the direction RP and the other in the direction SQ; thus they may check each other.]

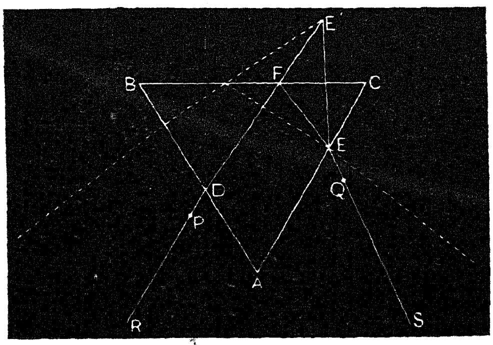


FIG. 14.—Reflection in a Prism.

Remove the block, join RP and produce it to meet AB in D, and SQ to meet AC in E. To obtain the direction of the ray DFE in the block, we must remember that it has been regularly reflected at F, and therefore that the image of any point such as E will be formed at E′ at an equal distance on the other side of BC. Therefore drop a perpendicular EM and produce it, making ME′ = ME. Join DE′ cutting BC at F. Join FE. Then EFD is the path of the ray in the prism.

8. **Determination of the Angle of a Prism.**—The following experiment, although not giving good results, is useful as illustrating the method which is used for determining the angle with the spectrometer.

Place the prism on the paper and draw the lines AB and AC Insert the pin P as far away from A as possible (on a separate

block at a distance) so that the line PA is approximately equally inclined to AB and AC; and as far away as possible insert a pin, Q, in such a position that looking along the line QA the image of P in the surface BA may be formed as close as possible to the angle A; that is, so that the pin Q and the image P′ may be on

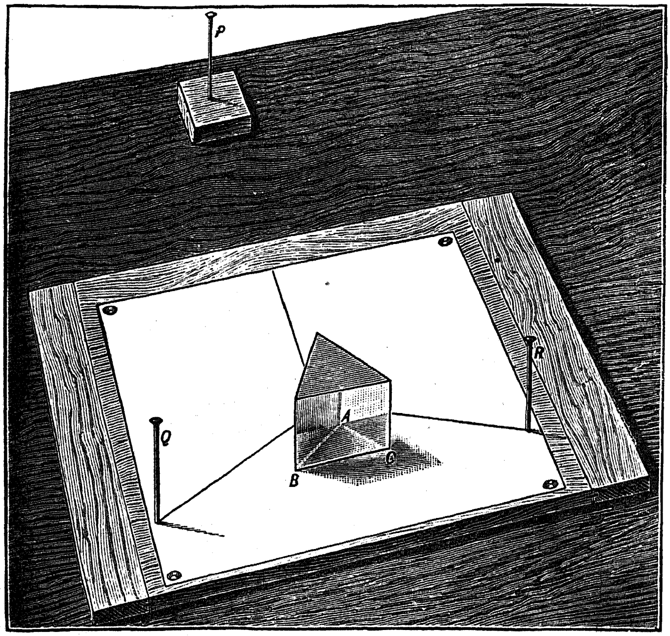


FIG. 15.—Angle of a Prism.

the line passing through the corner A. In the same way insert the pin R so that the image of P formed in the surface AC may be as near the corner A as possible.

Remove the prism. Join PA, QA, RA. The angle QAR will be double the angle of the prism BAC.

To prove this, produce the line PA to L. Then, as the lines AP and AQ are equally inclined to the surface AB,

$$\angle LAB = \angle BAQ,$$

and therefore $\angle LAQ = \text{twice} \angle LAB.$

In the same way ∠ LAR = twice ∠ LAC, so that ∠ QAR = twice ∠ BAC.

Copy the figure in your notebook on a reduced scale, and insert the actual values as found by the protractor of the angles QAR and BAC.

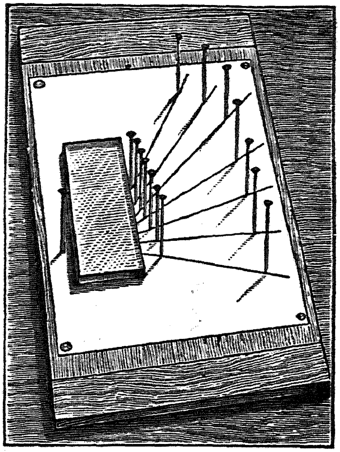


FIG. 16.—Caustic by Refraction at a Plane Surface.

Caustic by Refraction at a Plane Surface.

Apparatus.—A half plate cutting-shape [1] with rectangular edges, a drawing-board, pins, and set square.

9. Lay the plate down flat on a piece of drawing paper and draw a line along one edge of the glass. Place a pin, P, against the opposite long edge near one corner. Looking through the block at this pin, place two pins to appear in a line with it, one, R_1, close to the glass, and the other, S_1, at a distance of about 10 cms. In the same way place a series of pins, R_2, R_3, R_4, R_5, ... along the front edge of the block, with another series of pins, S_2, S_3, S_4, S_5, ... each at about a distance of 10 cms., so that the pins appear in pairs to be in a line with the one seen through the block.

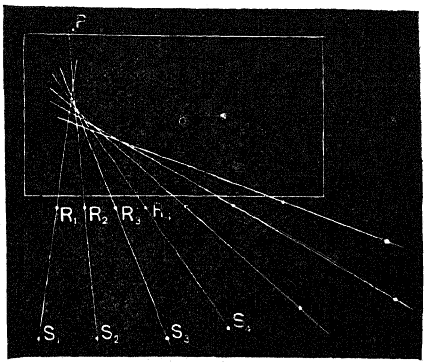


FIG. 17.—Caustic by Refraction at a Plane Surface.

[1] The glass shapes used by photographers to trim their prints (they sometimes have bevelled edges, but these would be useless for this experiment).

Remove the block and join each pair by a line, producing each line until it meets its neighbouring one. Draw a curve to touch all these lines and having its apex at the apparent position of the first pin when seen normally through the glass. This curve is the *caustic*, or rather is one half of it, the other half being on the other side of the normal line OP, and being similar.

Reflection at a Curved Surface.

Apparatus.—A cylindrical mirror (the makers of the glass shades used to cover wax flowers, etc., will cut a ring from the bottom of such a shade for a few pence); one from a shade of about 6 inches in diameter will be most useful. It should be then cut in halves, making two semicircular rings; one of these must be silvered [1] on the inner and the other on the outer surface, the silver, of course, being protected by varnish. These rings are very fairly circular and give good results. Drawing-board; pins; and scale.

10. **Convex Surface.**—Place the convex semicircular piece of glass on the drawing-paper and run a pencil round its silvered surface. Place a pin, P, about 2 inches from its surface, and looking in the mirror, insert pairs of pins, Q_1R_1, Q_2R_2, ... to appear in a line with P as seen in the mirror. Remove the mirror, join the points Q_1R_1, Q_2R_2, and produce the lines until they meet. Draw a curve to touch all these lines. This curve is called the *caustic*.

The lines such as Q_4R_4, Q_5R_5, ... which are near the pin P, will be found to meet one another nearly at the same point, S. This point is called the *image* of P. It is obvious that S is only the image of P for light incident nearly normally on the surface, as the lines, such as Q_1R_1, which are at some distance from P, do not pass through S. In the determination of the formula giving

[1] See Appendix 1. It is, however, sufficient to *black* the semicircles with some black varnish, one on the outer and one on the inner surface, as the images of the pins can be quite easily seen against the black, though of course not nearly so well as in a silvered mirror.

the position of S, it is always assumed that the light is incident nearly normally, and this experiment shows the reason for that assumption.

The caustic will meet the circle at the point T, where the tangent from P touches it.

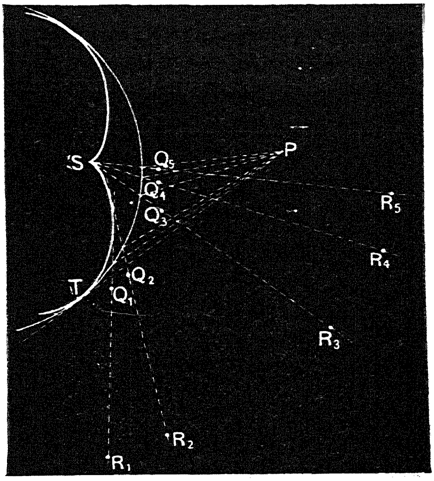


FIG. 18.—Caustic formed by a Convex Mirror.

11. **Concave Surface.**—(*a*) Place the concave semicircular piece of glass on the paper and insert a pin, P, at about 1 inch from the concave surface. Run a pencil line round the surface of the mirror. See that the mirror is just touching the line and insert pairs of pins, Q_1R_1, Q_2R_2, ... to appear in a line with the image of P. Then, as in the last experiment, it will be found that the image S is behind the mirror for rays which are incident near the centre of the mirror.

Instead of inserting the pins Q_1R_1, Q_2R_2, ... it is just as easy, and a great deal quicker, to place a scale upon the paper with its edge pointing to the image of P as seen in the mirror, and to run a line along the edge of the scale, thus obtaining in one operation the same lines that will be produced finally by joining the pins Q_1R_1, etc., and there is no risk of joining the wrong pairs of pins. Continue to draw these lines for rays incident at points of the mirror further and further distant from the centre. It will be found that at a certain distance from the centre the image

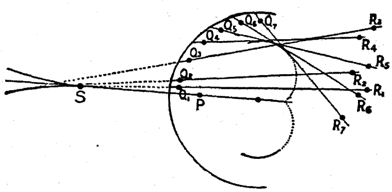


FIG. 19.—Directions of Rays from a point P after Reflection in a Concave Mirror.

of the pin seems to broaden out and become indistinct; this is because the rays forming the image at this part of the mirror are, after reflection, almost parallel to one another, the image formed by those rays being at infinity.

Still further from the centre the lines will be found to meet on the near side. Remove the mirror and produce those lines which meet behind the mirror. Notice the shape of the caustic, which has two branches as in Fig. 19, and that the image is only formed at S when the light is incident nearly normally.

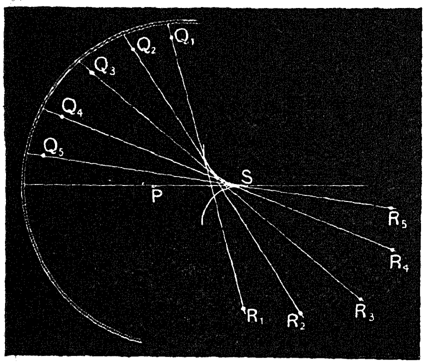


FIG. 20.—Rays from a pin P after Reflection in a Concave Mirror. They form a caustic of which the cusp S is the conjugate focus of P.

(*b*) Place P at a distance of about $2\frac{1}{2}$ inches from the same mirror, and repeat. The image S will now be formed on the other side of the centre of curvature of the surface of the mirror. Notice that in this case the point S is only the image for rays very nearly normal, and that those rays which are reflected from the extremities of the semicircular glass are a considerable distance from S. The distance of the reflected ray from S is called its *aberration*, and the distance measured along the line joining S to P—the axis—is called the *axial* or *longitudinal aberration* or the *spherical aberration* of the mirror. (Fig. 20.)

(*c*) Place the pin P at about 6 inches from the surface, and repeat.

(*d*) Place P at exactly 3 inches[1] from the surface, that is, at the centre of the circle. In this case it will be found that the lines joining the pairs of pins, Q_1R_1, Q_2R_2, ... always pass through P. There is therefore no aberration, and the image of P coincides with the pin itself.

[1] If the diameter of the mirror is not 6 inches, these distances must be altered to suit it.

12. **Wave Surface.** *Apparatus.*—As before; also set square, tracing paper, and drawing pins.

Draw the rays as directed in Experiment, § 11 (*a*). (The drawing made in that experiment may be used.) Pin a piece of tracing paper over the drawing and, starting from a point on the central reflected ray about ½ inch on the near surface of the mirror, draw a short line at right angles to that ray until it meets the next reflected ray, as in Fig. 21. Continue this line by a short line at right angles to the second ray until it meets the third reflected ray. Again continue it by a line at right angles to this third ray until it meets the fourth ray, and so on, until the surface of the mirror is reached. Continue the line in a similar manner on the other side of the centre until it again meets the mirror. This line indicates the position and shape of the reflected wave, after it has travelled back a short distance from the mirror.

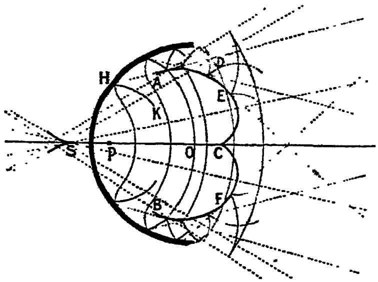


FIG. 21.—Five Wave Fronts of the Light reflected from a Concave Mirror. HK is a portion of the first wave from P that has not yet been reflected by the mirror; similar portions of the next three waves are shown, they are arcs of circles of which the centre is at T. The concave portion of the second wave near A becomes concentrated at the point on the caustic where the wave through O meets it; hence this third wave bends abruptly at the caustic. The part ECF of the caustic belongs to the other half of the mirror.

Beyond the point where this wave meets the mirror, the wave from the point P, not yet having reached the mirror, will still lie on a sphere whose centre is at P. Thus it may be continued a short way by describing arcs of circles with P as centre.

In the same way draw wave surfaces which pass through points on the central ray at distances of 1 inch, 2 inches, 3 inches, etc., from the mirror. Note that these latter reflected waves have a curious cusp, as seen at D in Fig. 21.

Course of the Rays refracted through a Curved Surface.

13. **Convex Surface.**—*Apparatus.*—We shall require a glass ring cut from the bottom of a shade such as that already used as a

cylindrical mirror (see page 14). Cement this on a circular glass plate, with Canada balsam, having first ground the edge with emery and turpentine. A convenient diameter for this trough will be about 6 inches. Stand two or three pins upright in small discs of sheet lead. These can then be placed anywhere within the trough.

Place the trough upon the drawing-board and one pin within it about 2 inches from the edge. Pour water in to the depth of about half an inch, and looking through from without, place pairs of pins, one of each pair near the trough and the other about 10 cms. away, so that they seem in a line with the one inside. We now want to mark the outline of the trough. If the glass

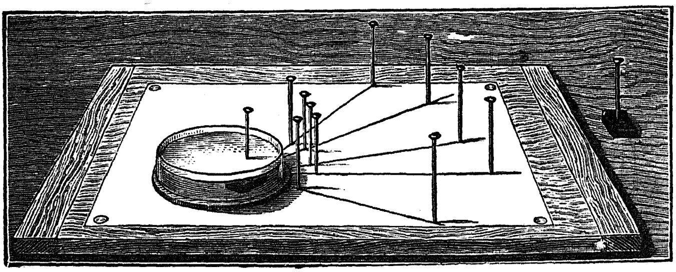


FIG. 22.—Refraction at a Curved Surface.

plate does not project beyond the ring, this is quite easy. But otherwise, knowing its diameter, a circle may be described with compasses set to the right size passing through two points on its circumference (determined by actual measurement). Each pair of pins must now be connected by a line, and the lines produced until they meet. The caustic may then be drawn touching all these lines. The geometrical image of the pin is the apex of this caustic.

Measure the distance of the actual pin within the trough from the surface, and that of its image from the surface, also the radius of the trough. Make a drawing on a reduced scale in your notebook, and enter the measurements upon it.

14. **The Course of Parallel Rays, represented by Pairs of Pins, through this Trough, and the Principal Focus.**—It will easily be seen that only those rays which pass normally through the surfaces go at

all accurately through the geometrical focus, and that there is a considerable aberration for rays that are far from the central line.

15. **Paths of Rays refracted through a Cylindrical Lens.**—*Apparatus.*—The lens may be made by cementing strips 3 or 4 inches long, cut from rings similar to those already used but of a diameter of 8 or 10 inches, on suitable bases. In this way troughs to represent a plano-convex or concave, double convex, meniscus or double concave lens may easily be formed. Instead of these troughs, a piece of cylindrical glass lens may be used. A strip a quarter of an inch deep, cut from a cylindrical lens of about 10 inches focus and $2\frac{1}{2}$ inches diameter, is very suitable. Half a cracked lantern-condenser may be used.

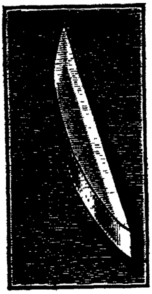


FIG. 23.—Cylindrical Lens.

Stand the lens upon a piece of ruled foolscap so that its plane may be perpendicular to the ruling. Looking through the lens, place pairs of pins, one close to the lens and one at about 10 cms. distance, to appear in a line with

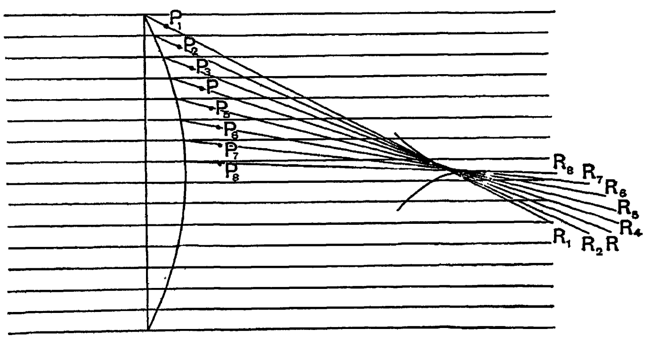


FIG. 24.—Refraction through a Lens.

each line on the foolscap, as seen through the lens. Draw the trace of the surfaces of the lens on the paper. Move the lens and connect up the pins. The caustic may now be drawn in, to touch these lines. In the case of a plano-convex lens, the rays should be traced through in both directions, and the distance of the

geometrical focus from the surfaces measured, and also the difference in shape of the caustic observed. It will be seen that the aberration is very much greater—as indicated by the angle of the caustic being more obtuse—when the parallel rays enter the lens by its plane surface than when they enter by its curved surface.

Focal Lines.

16. **Focal Lines by Refraction at a Single Plane Surface.** *Apparatus.*—Drawing board, pins, ruler and scale; two small blocks carrying a vertical and a horizontal wire respectively (see Fig. 26); a trough of water (*e.g.* a pie-dish); a piece of zinc or a card with a horizontal and a vertical line ruled on it, and a support to hold it upright in the water; blocks to support the drawing-board.

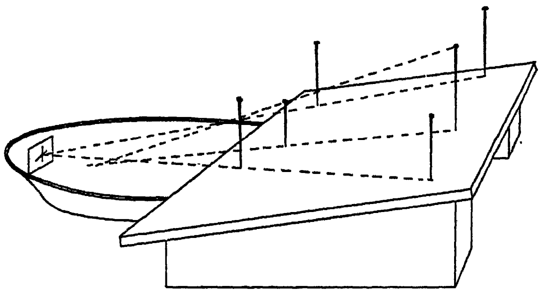


FIG. 25.—Focal lines, by refraction.

Fill a trough nearly to the brim with water. Support the card in a vertical plane at one end. At the opposite end of the trough set up the drawing-board on blocks in such a position that its plane shall appear to coincide with the horizontal line on the card, that is, so that on putting the eye in this plane and looking along the board in a line that just grazes the surface, the horizontal line on the card shall be just visible.

The final adjustment is most easily affected by raising or lowering the card slightly in the water.

Two pins—a long one and a short one—must now be inserted in the board and adjusted until the line joining their *heads*, when produced shall also appear to meet the horizontal line on the card as in Fig. 25. This is most easily done by inserting the pins

in approximately the correct positions, and then bending the long one either forward or backward until correct.

It is obvious that the image of the horizontal line must coincide with the point of intersection of the line joining the pins and the plane of the board.

Next insert two pairs of pins as shewn in Fig. 25, so that the lines joining each pair may each seem to pass through the image of the vertical line on the card. The point of intersection of these two lines must therefore fix the position of this image.

Measure the horizontal distance from the board to the card; The heights $h_1 h_2$ of each end of the board above the table; the height h_3 of the surface of the water above the table; and the depth h_4 of the horizontal line on the card below the surface of the water.

Without disturbing the drawing-board, carefully remove the trough of water. Then, looking along the feet of the pins, adjust the

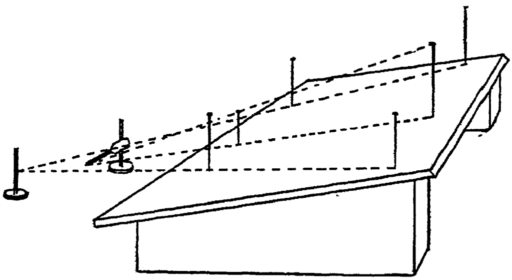


FIG. 26.—Focal lines.

vertical wire to coincide with their point of intersections (Fig. 26). In the same way adjust the horizontal pin (by varying both its height from the table, and its distance from the board) until it coincides both with the plane of the board and the line joining the tops of the long and short pins first inserted. These two wires are then in exactly the positions occupied previously by the images of the two lines on the card. Measure their distances from the board.

Note that the position of the image of the horizontal line is determined by the intersection of rays which lie in a vertical plane, and that the position of the image of the vertical

line is determined by the intersection of lines which lie in the plane of the board, *i.e.* very nearly by the lines which lie in a plane normal to the line.

See that the positions, found by experiment, agree with their theoretical ones.

17. **Focal Lines by Refraction at a Single Curved Surface.** *Apparatus.*—A reading microscope with a low power objective (*e.g.* a 2 inch), and a scale on the course adjustment; a lamp chimney of circular cross-section (an ordinary beaker will do for one portion of the experiment); a white card, or a piece of wood or metal painted white. This should be as long as the chimney with one vertical line down it lengthwise, and two horizontal lines, one of them just level with the greatest horizontal diameter of the chimney; a cork to close the lower end; calipers; water; pins; clamps.

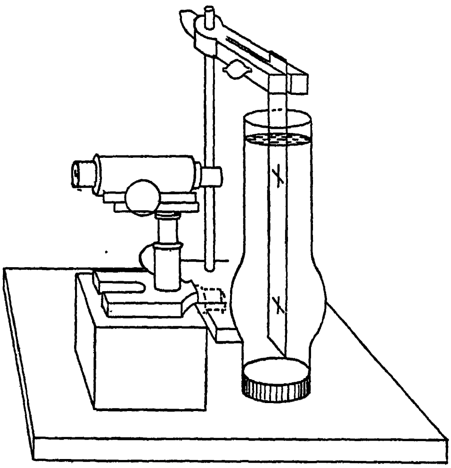


FIG. 27.—Focal lines.

Insert two pins in the card in the central line, and let their heads project exactly the same amount from the face of card (say 2 cms.).

Put the cork in the chimney, fill it with water, put it upright and fix it. Set up the card in the chimney, let the pins touch the chimney at two points in its cylindrical portion, and let one of the cross-lines be just level with the greatest horizontal section of the chimney. Adjust the reading microscope to view in succession each of the crosses formed by the horizontal lines on the card with the vertical line, measuring in each case the distances of the images of the horizontal and vertical lines from the front of the glass, by racking the microscope body back until the glass itself comes into focus.

As in the last experiment, the position of the image of the *vertical* line will be determined by the refraction in a plane normal to it, that is a *horizontal plane*. At both positions this refraction is refraction at a curved surface, of which the radius of curvature can be at once determined by measuring the diameter of the chimney there.

The actual distance of the vertical line from the glass is also known (being the height of the pin's head above the face of the card). Thus the position of the image of this line can be calculated from the ordinary formula for a curved surface,

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{r}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

When μ is the reciprocal of $1 \cdot 33$, r is half the diameter of the chimney, and u is the height of the pin, v_1 should agree with the result of the experiment.

At the lower position the difference in the radii of the glass in the two positions will have to be added to the height of the pins to get the true value of u.

Again, the positions of the images of the *horizontal* lines will be determined by the refraction in the *vertical* plane. In the first case the vertical section of the chimney is a straight line, thus the position of the image is given by the ordinary formula for refraction at a plane, viz.:

$$\frac{\mu}{v_2} - \frac{1}{u} = 0;$$

u and μ having the same meaning as above, v_2 is the apparent distance of the image from the surface.

In the second case the vertical section is neither a straight line nor (in general) a circle of the same diameter as the horizontal section of the chimney there. To find it approximately, cut a thin card (visiting card) along two straight lines which intersect at an angle of about 150°.

Hold this card against the chimney in a vertical plane, and mark carefully the exact points at which they touch one another.

Now lie the card on a sheet of paper, run a pencil along the card and mark the points of contact on the paper. Remove the card, and draw normals at these points; they will intersect at the centre of the circle. The length of either of these normals

will be the radius of curvation of the vertical section of the surface and is the r of the formula,

$$\frac{\mu}{v_3} - \frac{1}{u} = \frac{\mu - 1}{r};$$

the u and μ have the same meanings as above.

For Focal Lines by Oblique Reflection at a Concave Mirror, see § 125.

Formation of the Rainbow.

Apparatus as in § 13, except that about half the circle is to be silvered on the outside.[1]

18. **Primary Rainbow.**—Place a pin, P_1, within the trough close to the silvered surface. Looking through the trough obliquely,

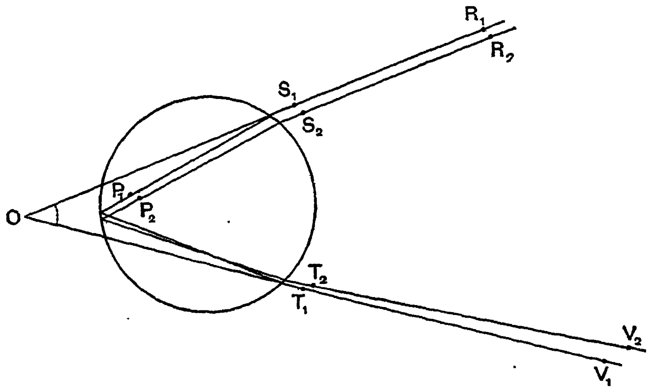


FIG. 28.—Primary Rainbow.

place two pins, R_1, S_1, to appear in a line with P_1, and two more pins, T_1 and V_1, to appear in a line with the image of P_1 and S_1, formed by reflection in the mirror. Place two other pins, S_2 and R_2, in such positions that the line joining them is parallel to the line joining S_1, R_1, and looking along this

[1] If the trough is not silvered it *must not* be blacked, as the chief reflection will occur at the outer glass-air surface. The pin itself is very difficult to see, but if a silvered bead be threaded on it, in a good light, the bright spot formed by reflection in this bead can be seen against a black background, such as a roll of black velvet, and the experiment may be performed as first described.

line, place P_2 within the trough to seem in the same line. Again, looking in the drop from the other side, place T_2, V_2, to appear in the line with the image of P_2, S_2, R_2, reflected from the silvered surface. Proceed in this way to find the direction both within and without the trough of a series of rays all parallel to R_1S_1.

It will be found that one ray, $R_1S_1T_1V_1$, is less deviated than those on either side of it. It is in this direction that the raindrop would appear brightest if illuminated by a beam of light parallel to RS. Produce the lines R_1S_1, and V_1T_1, to meet at O, and measure the angle $R_1O_1T_1$. It should be 41° 32′.

19. **Alternative Method with an Unsilvered Trough.**—If we assume that the reflection follows the ordinary laws of reflection, a silvered trough is not necessary. Place the pin P in the trough touching the far surface. Looking through the trough obliquely, place pairs of pins R_1S_1, R_2S_2, etc., to appear in a line with P. Or, which is much quicker, place a scale on the paper to appear in a line with P, and run a pencil along its edge. When the scale is almost tangential to the trough so that the light enters the surface very nearly at grazing incidence, it will be seen that the image of the pin broadens out. It is this position which gives the minimum deviation. On moving the eye slightly this broad image will separate into two—one moving inwards towards the pin, and the other outwards to become the tangent to the surface. Follow the latter and draw the ray. Do this on each side.

It will be found that the rays obtained from the second images of the pins, namely those that appear to move towards the tangent, cross the others. The total deviation of these is therefore greater.

When the image broadened out, the rays became practically parallel. Produce any pair of these parallel rays, one on each side, until they meet, and measure the angle. It should be twice 41° 32′ (*i.e.* 83° about).

The angle seldom comes quite right as the trough is not sufficiently circular—a very small deviation from the circle having a very marked effect when the rays strike the surface at such oblique angles.

20. **Secondary Rainbow.**—The only difference between this experiment and the last will be the selection of the direction of RS, which must slope as shown. The vessel must be so placed that the two reflections occur upon the silvered part of the trough.

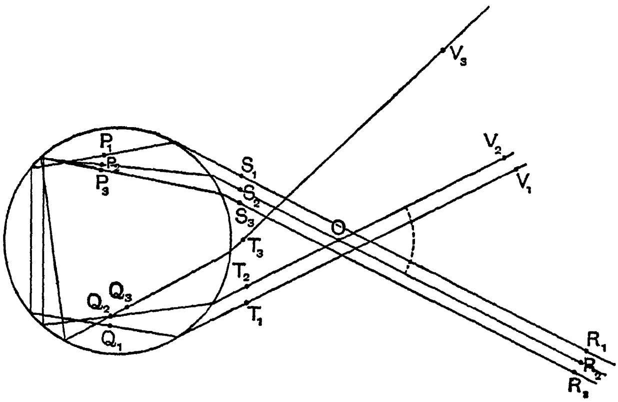


FIG. 29.—Secondary Rainbow.

A series of ruled lines such as those on a piece of ruled foolscap paper may be placed without the trough, in place of the parallel pairs of pins R, S. By looking at this system in the direction VT, it is easy to determine the one which is least deviated, and to see that the deviation of the one on each side of this is very nearly the same as that of this one, but that the deviation increases very rapidly for those still further away.

Having found the one, $R_2S_2P_2Q_2V_2$, which is least deviated, measure the angle R_2OV_2. It should be $51° 58'$.

21. **Artificial Rainbow.**—*Apparatus.*—An aspirator; glass and rubber tube; a short piece of glass tube with a bore 1 mm. in diameter; drawing-board and pins; gas flame—preferably incandescent—in a box with a small aperture, or a simple candle; blocks and beaker.

Attach the millimetre tube to the tubules of the aspirator, and support it vertically in the clip so that a vertical stream of water may flow out, which can be received in a beaker. Prop up the drawing-board so that its upper surface is on a level with this

stream. Place the gas flame at a distance of 3 or 4 feet from the jet, turning the box so that the light may fall upon the jet. Then, on bringing the eye as close to the jet as possible without intercepting the light, and looking through the jet, with the back to the light, the primary and secondary rainbows will easily be seen, each accompanied by a large number (15 or 20) of supernumerary bows. The primary rainbow is the one nearest to the line joining the flame to the jet.

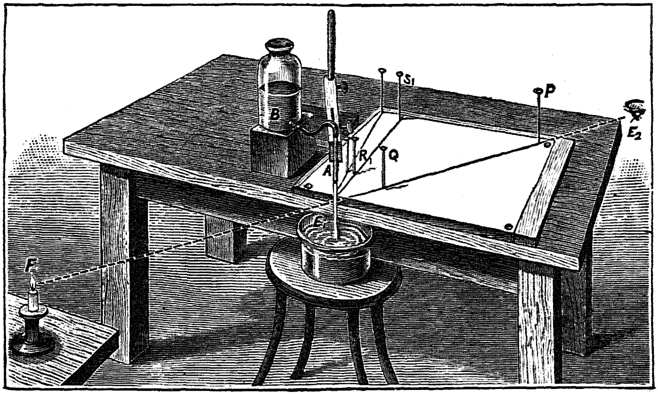


FIG. 30.—Arrangement of Apparatus to measure the Angles of the Primary and Secondary Rainbows. Standing back to the candle F, with the eye at E_1, the primary bow is seen in the direction R_1S_1. The eye is at E_2 when inserting the pins P, Q, in line with F and the jet A.

Insert in the drawing-board a pair of pins, one close to the jet and one as far away as possible in a line with the jet and the primary rainbow. Do the same for the secondary rainbow. Lastly, insert a pair of pins in the continuation of the line joining the light to the jet.

To do this go round to the far side of the board, and with the eye nearly in the plane of the board, look along the line joining the jet to the candle. In this position the surface of the paper or drawing-board will appear slightly shining; in fact, it will partially reflect the light of the candle, and a shadow of the jet will be seen along the surface. Insert one pin close to the jet in this shadow, and another at as great a distance as the board will allow.

Join the pins in pairs, and measure the angle between the lines so obtained. (As the lines will of course meet beyond the edge of the drawing-board, it will be found necessary to draw parallel lines on the board in order to measure these angles.)

Considering the simple nature of this experiment it gives remarkably accurate results. If the pins inserted in the direction of the rainbow are placed to appear in a line with the yellow of that bow, the angle given by c should be $41° 32'$, and the result can easily be obtained to half a degree. The difference between the deviation of the red, yellow, and violet can easily be measured, amounting as it does to about two degrees.

Determination of the Refractive Index of a Liquid by the Critical Angle.

22. When a ray of light travelling in a denser medium strikes the surface separating it from a rarer medium, it generally is partly reflected and partly refracted into the rarer medium, where it makes an increased angle with the normal. For a certain angle of incidence this angle, the angle of refraction, becomes 90°. If the angle of incidence be still further increased, there will be no refracted ray: the whole light will be reflected, and we obtain what is known as total reflection. The largest angle of incidence for which any of the light passes into the rarer medium is called the critical angle; and since the angle of refraction is then 90°, and the light is passing from the rarer into the denser medium,

$$\frac{1}{\mu}=\frac{\sin i}{\sin 90^\circ}=\frac{\sin i}{1};$$

and therefore

$$\sin i=\frac{1}{\mu}.$$

If a parallel plate of some other transparent substance separates the two media, when the ray reaches the critical angle from that substance to the rarer medium, the angle of incidence on the plate from the denser medium will be the critical angle for that medium and the rarer one; for instance, if a glass plate separates water from air as in Fig. 31, and if the plate be rotated until the ray, PQR, strikes the surface, R, at the critical angle, i', for glass and air, then it strikes the surface at Q at the critical angle, i, of water and air.

For, if μ be the refractive index from water to air, and μ' the refractive index from glass to air, the refractive index from water to glass will be $\frac{\mu}{\mu'}$, and

$$\frac{\sin i'}{\sin i}=\frac{\mu}{\mu'}.$$

Thus, if $\sin i' = \frac{1}{\mu'}$, we shall also have $\sin i = \frac{1}{\mu}$. The normal to the glass plate, therefore, makes an angle equal to the critical angle with the incident ray. Therefore, if we set the glass normal to the ray, and rotate it until the critical angle is reached, and measure the angle through which it is turned, the reciprocal of the sine of this angle will be the refractive index of the water.

Instead of attempting to set the glass at right angles to the ray, it will be easier to turn it into the position shown by the dotted lines, in which the ray is again incident at the critical angle. The angle between the two positions of the glass will now be double the critical angle.

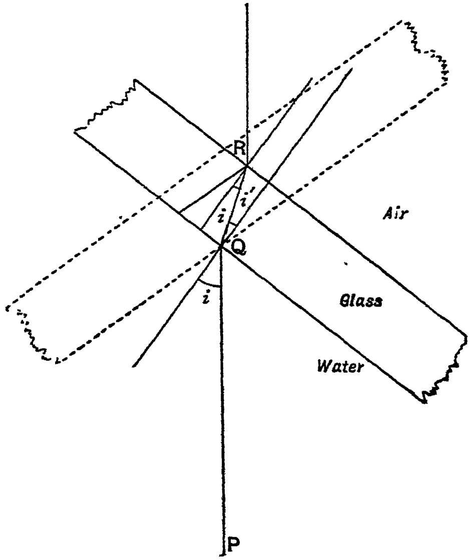


FIG. 31.—Critical Angle.

Apparatus.—A small glass tank about 2 or 3 inches cube—preferably with parallel sides, though this is not very important; drawing-board; pins; millimetre scale; two pieces of thin patent plate about 2 inches square; tinfoil; shellac varnish; and a wooden stool about 6 inches long, tall enough to put over the trough.

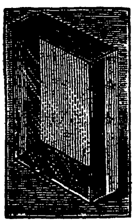


FIG. 32.—Glass Plates enclosing a layer of Air.

Cut a piece of tinfoil the size of the glass plates and remove the greater part of the centre, leaving a narrow strip all round. Clean the two pieces of glass, and placing this tinfoil mask between them (Fig. 32), attach them together top and bottom with cotton, and shellac varnish the edges, so as to make them air-tight. A layer of air will thus be

enclosed between the two glass plates—the thinner this layer of air the better. Cut a slot in the wooden stool parallel to its long side, as in Fig. 33, and insert the upper end of the double plate above described, fixing it in position with wedges and the shellac varnish. To each foot of the stand attach a needle to act as a pointer.

Place the drawing-board in front of a window, and put the trough on the centre of the drawing-board. Nearly fill it with water. Insert two pins, P and Q, perpendicular to the face of the trough. Place the stool over the trough so that the double

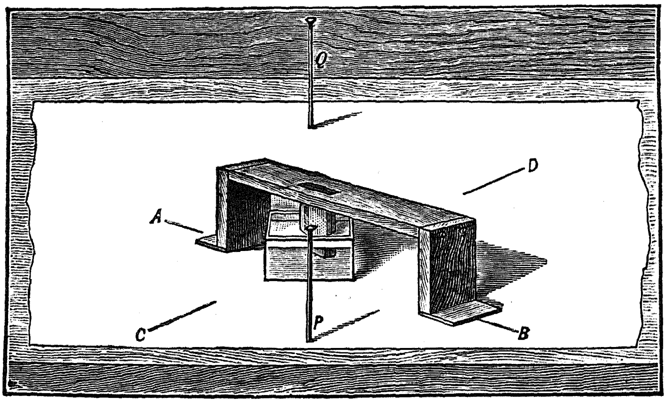


FIG. 33.—Refractive Index of Water by the Critical Angle.

glass plate containing the air-film may dip into the water. Then, looking through the trough in the direction PQ, the pin Q should be visible through the water and the glass plates. Now turn the stool round, and in a certain position it will be found that the pin Q will disappear, and one half of the field will become dark. Mark the positions of the needles on the paper as at A, B, when the blue colour, with which the dark part of the field terminates, just reaches Q. Now turn the stool round until the pin Q is again on the point of disappearing, and mark the lines C, D on the paper.

The ray after passing through the first glass plate and the air-film will, until the critical angle is reached, enter the second

glass plate, and will emerge from it into the water parallel to the original incident ray, so that the light coming from Q passes right through to the eye at P. But when the critical angle is reached, the ray on entering the air-film passes along at grazing incidence, and strikes the tinfoil instead of the second glass plate, and no light from Q will reach the eye. If the glass plates are separated by a larger space, the light from Q will be cut off before the critical angle is reached.

The blue colour indicates the disappearance of the yellow, and shows that we have reached the critical angle for yellow light; the refractive index obtained should therefore be the refractive index of water for yellow light.

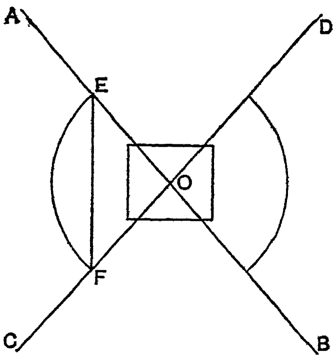


FIG. 34.—Measurement of Critical Angle.

Now remove the stool and trough, join the lines AB, CD (Fig. 34), meeting at O. With centre O, draw a circle, cutting these lines at E and F. Join E, F. The critical angle will be half the angle EOF.

And therefore $$\sin i = \frac{\frac{1}{2}\mathrm{EF}}{\mathrm{OE}};$$

or $$\mu = \frac{2\mathrm{OE}}{\mathrm{EF}}.$$

Determination of the Refractive Index of a Liquid by the Critical Angle from the Liquid to Glass.

23. *Apparatus.*—A cubical glass block or a right-angled glass prism, the refractive index of which is known; drawing-board, pins, protractor; a piece of ground glass or paper screen that can be set up vertically; gas flame—preferably incandescent.

Draw a long line AB on the drawing-board. Put a drop of the liquid, the refractive index of which is to be found, on one face of the block, and place the block on the drawing-board with this face, CD, vertically over the line AB. Look in the

direction PQ through the face DE of the block, and place the ground glass, or paper screen, S (which must be well illuminated), so that its image is seen by reflection in the face CD. At the

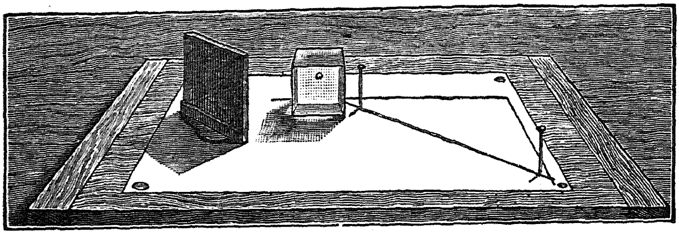


FIG. 35.—Refractive Index of a Drop of Liquid.

point where the drop of liquid is placed on the face CD, the reflection will generally be less perfect; on moving the eye towards B, at a certain angle, the liquid will begin to disappear,

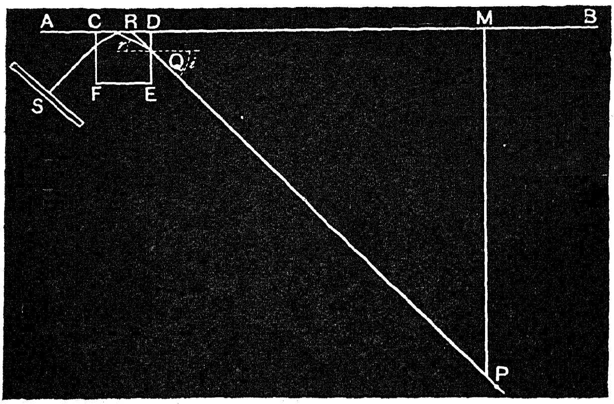


FIG. 36.—Refractive Index of a Drop of Liquid.

and the reflection become total. When the drop has only partly disappeared, a vertical blue line will be visible. Insert two pins, P and Q, on the line joining the eye to this blue line. In this direction, the yellow light has just reached its critical angle.

Thus, calling i and r the angles of incidence and refraction at the surface DE, since the ray is incident at CD at the critical angle, we have

$$\cos r = \frac{n}{N};$$

n being the refractive index of the liquid, and N that of the glass; also

$$\frac{\sin i}{\sin r} = N,$$

therefore $$\sin r = \frac{\sin i}{N}.$$

Square and add, we get

$$\sin^2 r + \cos^2 r = 1 = \frac{n^2}{N^2} + \frac{\sin^2 i}{N^2};$$

therefore $$n = \sqrt{N^2 - \sin^2 i} \quad \text{.........................(i)}$$

Produce PQ to meet AB at R, and drop a perpendicular, PM, on AB. Then

$$\sin i = \frac{PM}{PR}.$$

Thus, as N is known, n can be found by substituting in equation (i).

Determination of the Refractive Index of a Substance by Measuring the Apparent Thickness.

24. **Liquid.** — *Apparatus.* — A shallow vessel will be required to contain the liquid, preferably of glass, and a microscope with a low power objective; it should have a scale on the coarse adjustment.

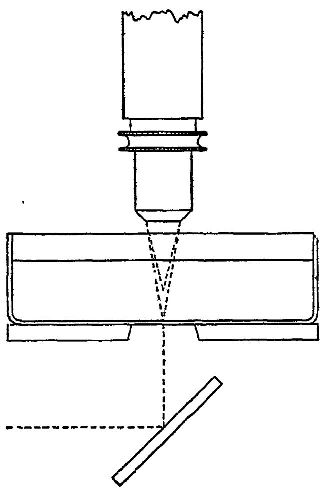


FIG. 37.—Refractive Index of a Liquid.

The microscope is set up with its tube vertical, and is focussed upon the bottom of the empty vessel. The reading on the scale attached to the coarse adjustment is observed. [In default of such a scale, the height of some definite point on the tube of the instrument can be read with an ordinary millimetre scale.] Without moving the vessel the liquid

is poured in to a depth of 1 or 2 cms., depending upon the focal length of the objective, and the microscope is focussed upon the image of the bottom as seen through the liquid, and its position again noted. Lastly, it is to be focussed upon the surface of the liquid; for this purpose a little lycopodium powder may be scattered upon the surface. The refractive index is the ratio $\dfrac{\text{the real depth}}{\text{the apparent depth}}$.

25. **A Glass Block.**—*Apparatus.*—A cubical block of glass about $1\frac{1}{2}$ inch edge; a microscope with a low power objective, its body either horizontal or vertical—preferably the former; blocks and millimetre scale.

If the block is new and free from scratches, it will be necessary to make its surface visible, either by fingering it, or by attaching a small piece of stamp-paper. Set it up so that the stamp-paper can be seen in the microscope after refraction through the block, and focus the microscope upon it. If there is a scale attached to the rack of the microscope, take the reading upon it; if not, measure to some fixed point on the tube with the millimetre scale. Withdraw the microscope until the front surface of the block is in focus, and measure the amount the objective has been moved, again using either the scale on the instrument, if it has one, or the millimetre scale. The amount of the motion is the apparent thickness of the block; its real thickness can be determined directly with the millimetre scale. The refractive index is the ratio $\dfrac{\text{the real thickness}}{\text{the apparent thickness}}$.

26. **A Micro Cover Glass.**—*Apparatus.*—Some pieces of micro cover glass; a microscope, of which the milled head of the fine adjustment must be graduated. The actual value of the gradations will not be required. A higher power (at least $\frac{1}{2}$ inch) objective will be necessary.

Place the cover glass, whose surface must be rendered visible by some means (it is sufficient to finger it), on the stage of the microscope, so as to observe a part of its surface near the edge —the edge may be in the centre of the field. By focussing first

the lower and then the upper surface, determine its *apparent* thickness.

Fix the attention now upon the edge itself; the terminations of the upper and lower surfaces will be visible owing to the large cone of rays which the objective admits, and therefore, the *actual* thickness of the surface may be measured, using the same fine adjustment screw. The ratio of the real to the apparent thickness will be the refractive index.

As we are dealing only with the *ratio* of the real and apparent thicknesses, it is sufficient to measure them in divisions of the milled head, it is not necessary to determine their absolute values in centimetres.

Selection of a Piece of Parallel Glass, and Rough Determination of the Angle between the Surfaces of a Piece of nearly Parallel Glass.

27. If a distant bright point be observed at very oblique incidence in a piece of parallel unsilvered or silvered glass, images are formed by reflection both at the upper and lower surfaces, and also after three, five, or more internal reflections. If the surfaces of the glass are truly parallel, these images will all approximately coincide, and for an object at an infinite distance, will do so absolutely, for the rays after reflection will all be parallel, and will therefore appear to come from the same point. If a piece of ordinary glass be held up so that a distant flame is observed at almost grazing incidence, a series of images of the flame will nearly always be visible which gradually diminish in brightness. If the glass be now rotated in its own plane, this series will be seen to rotate also. This series is due to the two surfaces not being truly parallel.

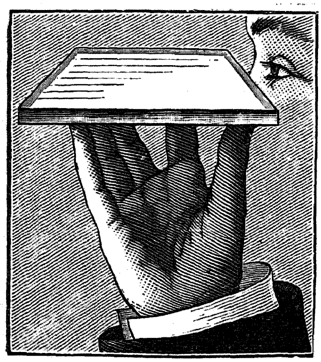


FIG. 38.—Examination for Parallelism.

Let S (Fig. 40) be the distant bright point and AB and CD be the upper and lower surfaces of the mirror, and, to simplify the

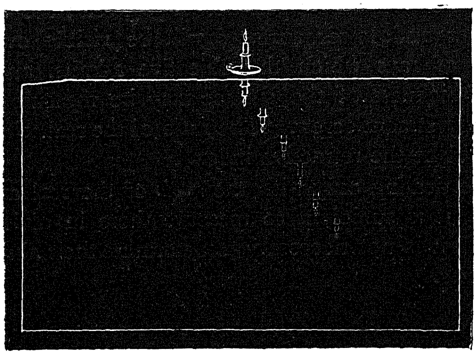


FIG. 39.—Series of Images of a Distant Candle seen by Oblique Incidence in a nearly Parallel Mirror. (This figure should be looked at obliquely.)

figure, we will neglect the refraction so that we may treat AB and CD as two ordinary mirrors. Rays from S which enter the eye, after

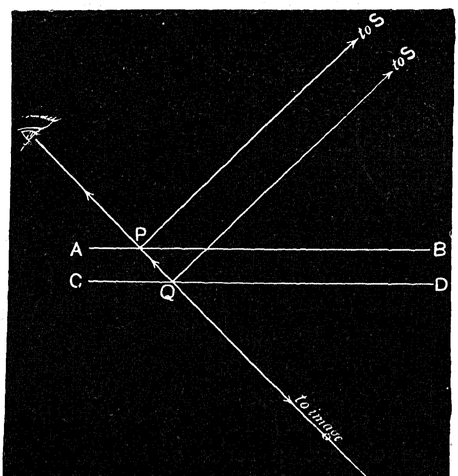


FIG. 40.—Single Image of a Distant Flame formed by Two Parallel Mirrors.

reflection at these mirrors (since the lines SP, SQ are parallel and the surfaces AB and CD are also parallel), must obviously either

coincide or be parallel. Thus, as S is distant, only a single image will result.

In Fig. 41, let the surfaces AB and CD make a small angle with one another, but suppose them each to be perpendicular to the plane of the paper so that their line of intersection is normal to the paper.

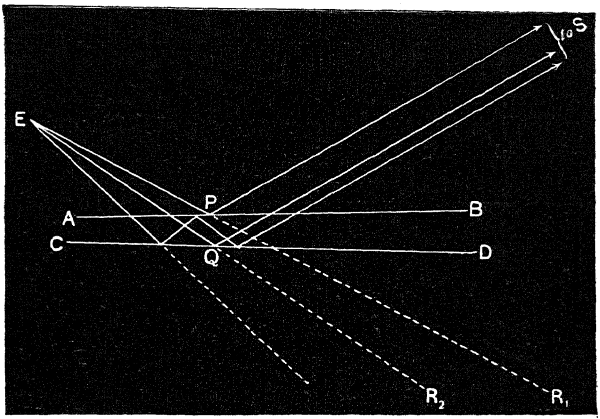


FIG. 41.—Multiple Images of a Distant Flame, formed by Two Mirrors not quite parallel to one another.

Let R_1, R_2, be the images of the source formed by the surfaces AB and CD respectively. The angle S_1PR_1 is twice the angle S_1PB, the angle S_2QR_2 is twice the angle S_2QD; therefore, the angle R_1ER_2 must be twice the angle between AB and CD. It can easily be shown that, if R_3 is the image formed after three reflections, the angle R_3ER_2 is also equal to twice the angle between the mirrors and so on. A series of images of the distant object will thus be formed at equal angular intervals.

If the glass be turned round a little in its own plane, so that the line of intersection of the surfaces is no longer perpendicular to the plane of incidence, the images formed by successive reflections will no longer lie in the plane of the paper. They will appear to tail-off either to the right or the left.

Apparatus.—Pieces of plate-glass, candles, and a metre scale.

28. **Examination and Selection.**—Light one candle and put it as far away as the size of the room will permit. Examine the

appearance of the image or images, in one of the pieces of glass held very obliquely, so that the candle seems only a short distance from its image formed in the mirror. If a series of images is formed, rotate the glass and observe the rotation of the images. They make one complete revolution by the time the plate has been rotated once in its own plane. Also move the plate in its own plane so that the reflections occur at different parts of its surface. If the surfaces are not perfectly plane (so that the angle between them varies from point to point), this will be shown by the images closing up or opening out. If only a single image is visible or if the series of images be nearly coincident,

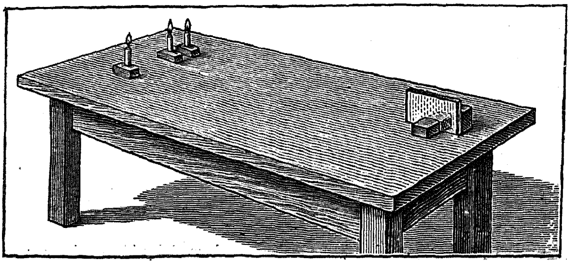


FIG. 42.—Angle between Surfaces.

the plate is either parallel or nearly so. In the latter case it may happen that some part of the plate may be found to be parallel. Should such a part be found, it should be marked and put aside, as it will be valuable for other experiments.

29. **The Angle between the Surfaces.**—Support the mirror with its plane vertical. Rotate it in its own plane until the images of the distant candle are in a horizontal line. Then arrange two other candles at about the same distance from the observer as the first, to appear (as seen through the glass if unsilvered, or seen just over the edge of the glass if silvered) to coincide with the first two images of the first candle. Measure the distance apart of these last two candles (a) and the distance from them to the eye (d). Then the angle between the two surfaces will be given approximately by $\frac{a}{2d}$ in circular measure.

ADDITIONAL PRACTICAL EXERCISES TO CHAPTER I

1. Stand two mirrors on a paper so that their surfaces are both vertical and at right angles to one another. Stick a pin in the paper about 3 inches in front of one mirror and 1 inch in front of the other mirror. Follow with pins the course of two rays from this pin to an eye placed to observe the image which has been formed after two reflections in the mirrors, producing the rays in each case until they meet, and so locate the positions of the successive images. See that the images lie on a circle whose centre is at the intersection of the mirrors, and passes through the original pin.

2. Reduce the angle between the two mirrors in the last experiment to 60°. Note the number of images formed, and see that when the angle is not exactly correct, the diagonally opposite image of the pin seems suddenly to alter its position as the head is moved sideways, and that this alteration occurs when the line joining the image to the eye passes from the one mirror to the other. Follow the path of two rays which are forming this image from the pin to the eye. Again see that the successive images lie on a circle.

3. Place two mirrors facing and parallel to one another, and about 2 inches apart. Insert a pin between the mirrors and about half an inch from one of them. Looking in the mirrors a little to one side, note that a series of pairs of images are produced by these successive reflections. By the method § 2 (*b*) find the positions of these successive images.

4. Trace a ray in the last experiment from the pin to the eye which has suffered three reflections at the parallel mirrors.

5. In Fig. 11 the two pins seen through the prism do not appear close to the surface of the prism, and do not reach down to the lines AB, AC. Why is this? Examine which is the pin marked P in the figure, and trace the rays by which the pin is seen in this position.

6. From the measurements in Experiment, § 6, calculate the refractive index of the prism from the formula:

$$\mu=\frac{\sin\frac{1}{2}(\delta+A)}{\sin\frac{1}{2}A},$$

δ being the angle FOS, and A the angle BAC of the prism (Fig. 12). The sines of the angles are to be taken from the table of natural sines given at the end of the book.

7. In Experiment, § 7, draw a series of pairs of lines RP, SQ. See that (if the prism is isosceles, for instance, an equilateral one) the lines RP and SQ make equal angles with BC, and thus the prism acts in a sense as a mirror. Produce the pairs of lines to meet. They do not meet in the same plane. Note that the pin S may be placed on the other side of the line BC produced, and the reflections will still take place.

8. From the measurements made in Experiment, § 13, calculate the refractive index by means of the formula:

$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r},$$

or

$$\mu=\frac{\frac{r}{u}-1}{\frac{r}{v}-1},$$

in which u, v, and r are the distances of the pin, image, and the centre respectively, from the surface of the trough at which refraction takes place when the incidence is nearly normal, and are to be reckoned positive when they are measured towards the incident light. As this is the case here, they will all be positive in the above formula.

9. In Experiment, § 14, calculate the geometrical focus, and compare its position with that found by experiment (assuming the refractive index of water to be 1·33) by making two applications of the formula:

$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}.$$

10. Place the concave semicircular glass on a sheet of paper as in Experiment, § 11. Light a wax match and hold it 1 inch, 2½ inches, 3 inches, and 6 inches from the surface. Note that the light reflected from the concave surface is much brighter along the caustic. Note that the paper is a little brighter within the caustic than it is outside. Hold a lead-pencil against the inner surface. It will form a shadow on the paper, which comes to a point on the caustic. Move the pencil along the surface. See that the point of the shadow always lies on the caustic, and the shadow is always tangential to the caustic. The shadow, of course, marks the direction in which the light it cuts off would have been reflected, and therefore shows that the reflected light passes through the caustic. As the shadow is pointed, it shows that the light reflected from the whole breadth of the pencil is concentrated into one point, and therefore explains the greater intensity there.

11. Instead of the stream of water of Experiment, § 21, use a glass rod, and find the angles of the primary and secondary rainbow for glass. (The angle of the secondary bow will be found greater than a right angle.)

12. Place two right-angled prisms with their hypotenuses in contact to form a cube. On looking in one face of the cube only objects reflected in the hypotenuse will be visible. Insert a drop of water between the two faces in contact, the part wetted will become transparent.

13. Place this block on the drawing-board as in Figure 36, and find the direction PR in which the block just ceases to be transparent. Measure the lines RM, PR, as in that experiment, and calculate μ_w from the equations:

$$\mu_w=\mu_g \sin\theta \quad \text{and} \quad \sin(\theta-45)=\frac{1}{\mu_g}\cdot\frac{\text{RM}}{\text{PR}}.$$

CHAPTER II

MIRRORS AND LENSES

Position and Nature of the Image formed by a Concave Lens.

30. *Apparatus.*—Concave lines of about 15 cm. focal length; stand to hold the lens vertical; a card with large printed letters upon it, also supported in a vertical plane.

Place the lens at a distance of 5, 10, 15, 20, ... cms. from the card, and examine the image seen through the lens. Move the eye to the left, and, treating the lens as a window (p. 4), determine whether the image is in front of or behind it. Observe also the nature and character of the image—whether it is erect or inverted, magnified or diminished—at each distance.

FIG. 43.—Concave Lens.

Enter the results thus:

Distance of object from lens.	Is the image on the same side of the lens as the object?	Is it erect or inverted?	Is it magnified or diminished?	Is it nearer the lens than the object?

To make the entries in the last column, we must remember, as pointed out above, that when viewing objects at different

distances from the observer that are practically in a line with one another, a movement of the observer to right or left will cause the *farther object to appear to move relatively to the nearer one in the same direction as the observer: i.e.* if he moves to the left, the farther object will move to the left relatively to the nearer one. They may both move to the left or right, but in either case the further one will finally be to the left of the nearer one. The object as seen outside the lens, and its image as seen through the lens, will be found both to move to the left across the lens if the observer moves to the left; but the motion of one of them across the lens will be greater than that of the other. The object is therefore moving relatively to its image. If this relative motion is to the left, the object is further off than its image.

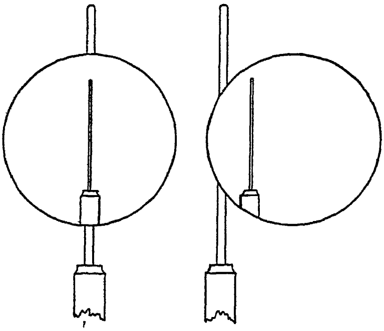


FIG. 44. FIG. 45.

Images in a Concave Lens.

FIG. 44.—Central position when a knitting needle is used as an object.

FIG. 45.—If it appears as in this figure, since the observer has moved to the left, and the object is now to the left of the image, the object must be further away than the image.

The Position and Nature of the Image of an Object formed by Refraction through a Convex Lens.

31. *Apparatus.*—A convex lens of about 15 cms. focal length; stand and printed card as before; small gas flame and screen; knitting needle in a vertical stand.

Place the lens at a distance of 5, 10, 15, ... cms. from the card, and standing at a distance of 2 or 3 metres from the lens observe the image as seen through the lens.

Enter the results in columns as in Experiment, § 30.

It will be found that at some distances the image is erect and at other distances it is inverted. Also at some distances it will be magnified, whilst at other distances it will appear diminished;

it is also sometimes on one side of the lens, and sometimes on the other. At one distance the image will be indistinct. This distance should be noticed.

At those distances at which the image is formed on the observer's side of the lens the image is real and can be received upon a screen; but there would not be sufficient light from the card to see this image. To find the image, place the knitting needle between the observer and the lens. On moving to the left, both image and knitting needle, being on the near side of the lens, will appear to move across its surface towards

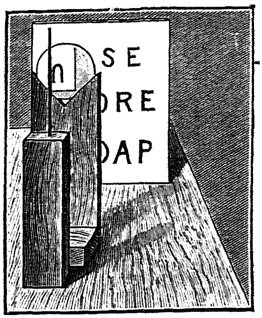


FIG. 46.

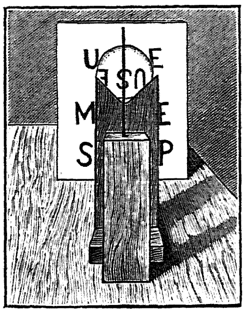


FIG. 47.

In Fig. 47 the observer has moved to the right, and the needle has moved from the S to the U, or has moved to the right relatively to the lettering. Thus, the needle is farther off than the image, and must be moved towards the observer.

the right. If the knitting needle is exactly in the same position as the image, it will move at exactly the same rate as the image across the lens, and will not seem to move relatively to the image itself. But this is not likely to occur at the first trial. The knitting needle will either be some distance in front or behind the image. If the knitting needle is farther from the observer than the image, when he moves to the left it will seem to move across the image towards the left. As beginners usually find some difficulty in distinguishing the *relative* motion of the knitting needle across the image from the motion of knitting needle and image across the lens, it is advisable to sketch the position of the knitting needle and the lettering, first when the observer is straight in front of the lens and again when he has

moved some distance to the left. On comparing the two sketches it is easy to see if the needle has moved across the lettering to the left; if so, it is too far from the observer, and must be moved towards him. By repeating this several times, it is possible to find a position for the needle, in which this relative motion ceases, and then it and the image really coincide.

If we now replace the knitting needle by a screen and the printed card by a gas flame, a clear image of the gas flame should be formed upon the screen.

The Position and Nature of the Image formed by Reflection in a Convex Mirror.

32. *Apparatus.*—Convex mirror about 20 to 30 cms. radius of curvature, with stand, small gas flame, knitting needle in a vertical stand.

Set up the convex mirror in a vertical plane, and place the gas flame at distances 5, 10, 15, ... cms. from a card, and observe its image as seen in the mirror.

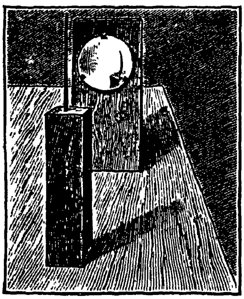


Fig. 48.—Convex Mirror.

Enter the results in columns as before.

It will be found that the image is always formed behind the mirror. Place a knitting needle also behind the mirror; of course, only the upper and lower parts will be visible, the centre being obscured by the mirror itself. Looking at the image of the gas flame, place the knitting needle so that the gas flame seems in the same vertical line as the knitting needle. Move the head sideways to the right or left, and notice if the gas flame moves across the mirror at the same rate as the knitting needle; that is, whether it still seems in a line with the ends of the knitting needle even when the eye is considerably to one side. If not, consider whether the knitting needle is too far back or too near the mirror, using the rule given on p. 6, and move it accordingly. Do not move the needle by guess-work only and try to find its position by rule of thumb, but think

first the direction in which it should move. If this is not systematically done, it will be found that not only will a great deal of time be wasted in obtaining even the approximate position, but the position will never be found with the same amount of accuracy as when the movement of the needle is made intelligently.

Observe whether the needle or the flame is nearer to the mirror.

The Position and Nature of the Image formed by Reflection at a Concave Mirror.

Apparatus.—The concave mirror of about 20 to 30 cms. radius of curvature, and of as large a diameter as possible (a *shaving*-mirror does excellently); stand, gas flame, knitting needle, and screen; black card, vase, and flowers.

33. **Real and Virtual Images.**—Proceed as described for the convex lens, placing the flame at 5, 10, 15,... cms. in front of

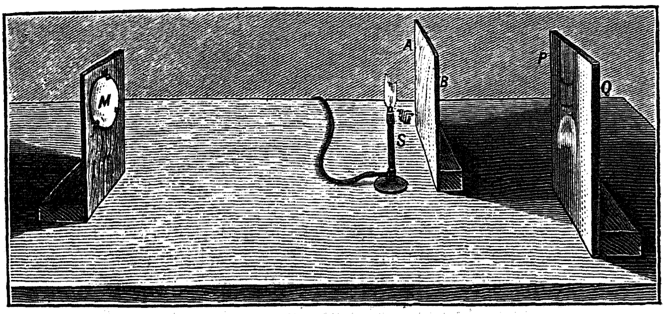


FIG. 49.—Real Image formed by a Concave Mirror.

the mirror, and observe it from a distance of 2 or 3 metres. Then, as in that case, the image will sometimes be on one side of the mirror and sometimes on the other, and at some distance will become indistinct. When the image is formed on the observer's side of the mirror, it will be real and can be received upon a screen. Find the position of the image by setting up a knitting needle and using the parallax method as described for

the convex lens, and finally replace the knitting needle by the screen. If the position has been correctly found, the image should appear well in focus. One difficulty will be found in this case; namely, that the direct light from the flame as well as that reflected from the mirror will both reach the screen. This can only be avoided by placing the flame and screen slightly on opposite sides of the axis so that the reflection from the mirror is slightly oblique, and then a card can be interposed to shade the direct light of the flame from the screen, as in Fig. 49, p. 45.

Notice that as the flame is brought nearer the mirror, the screen, in order to keep the image in focus, will have to be gradually withdrawn.

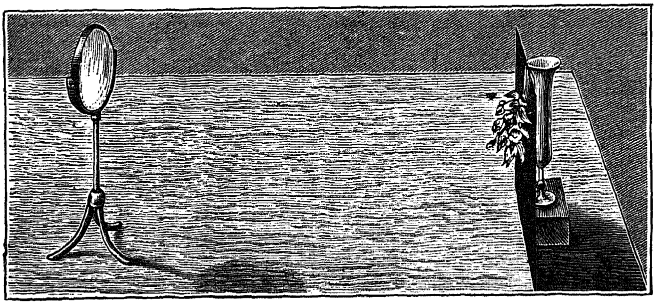


FIG. 50.—Formation of Aërial Image by a Concave Mirror.

34. **Centre of Curvature.**—Try to obtain a position in which the image is the same size as the object. It will be found that this occurs when the screen is the same distance from the mirror as the flame. This distance is the radius of curvature of the mirror, which may therefore be at once measured. If the flame is at a greater distance, the screen will have to be placed between the flame and the mirror (of course, it must be put slightly on one side of the axis and the flame slightly on the other, or the light cannot reach the mirror, and therefore no image can be formed on the screen).

35. **Optical Illusion.**—Arrange a small flower vase on a block, so that its mouth may be on a level with, and near the centre of

curvature of, the mirror (Fig. 50). Against this, between it and the mirror, stand a black card which also reaches up to the same level. Suspend a small bunch of flowers against the back of the card. Adjust the distance, and inclination, of the mirror until the image of the bunch of flowers is formed above the vase; and then, on looking into the mirror from a little distance, it is difficult to realise that it is only the image that is seen.

ADDITIONAL EXERCISES ON CHAPTER II

Apparatus.—Some long focus concave and convex lenses (old spectacle lenses, which may be obtained from any opticians, of powers $\frac{1}{4}$ to 2 dioptres, will do excellently).

1. Hold a convex lens close to the eye, and move it up and down in a vertical plane. Look through at some moderately distant object. The image of this object will appear to move up and down also. Notice whether the movement is in the same direction as that of the lens; for instance, whether the image moves up when the lens is moved up.

Draw a diagram showing the course of rays from the object to the eye through the upper edge of the lens.

See that the displacement observed agrees with that which you would expect from your drawing.

2. Repeat this experiment with a concave lens. The motion of the image will be in the opposite direction to that caused by a convex lens. Make a drawing showing the course of rays through its upper edge, and compare the result of the experiment with your drawing.

3. Sort a series of lenses, separating the convex from the concave by this method. It will be found very difficult to distinguish a very weak convex lens from a concave lens by the appearance, but quite easy by this method.

4. Arrange the convex lenses in order according to their powers; that is, according to the amount of the displacement they produce. If two or more lenses produce equal displacements, they are of equal power, and should be put together.

5. Next arrange the concave lenses in a similar manner. Attempt to pair a convex and a concave lens; *i.e.* to find a convex and a concave lens which will, when placed in contact and moved up and down before the eye, produce no apparent displacement of the image. Such a pair will have equal focal lengths.

6. If in the fourth exercise there were two or more convex lenses of equal power, and if there is a concave lens to pair with one of them, test it with the others, and see if it pairs with them also.

Hence test the equality of their powers. (If there is no concave lens that exactly compares, try with the one which is nearest.)

7. Suddenly place a piece of the plate glass used in Experiment, § 27, in front of the eye, or suddenly remove it, and see if there is any apparent displacement of the image of an object seen through the glass, as compared with its actual position. If there is, it shows that the two surfaces of the glass are not exactly parallel. Note the direction of the displacement, and hence determine in which direction the surfaces slope together. Compare the result with the appearance of the distant flame observed as in Experiment, § 27.

CHAPTER III

FOCAL LENGTH OF MIRRORS AND LENSES—OPTICAL BENCH

The Optical Bench and Fittings.

36. A very simple form of optical bench can be made by attaching a metre scale to one side of the upper face of a board about 5 inches broad and 1 inch thick, and supporting lenses on short blocks of wood in uprights cut with **V** slots, as indicated in the

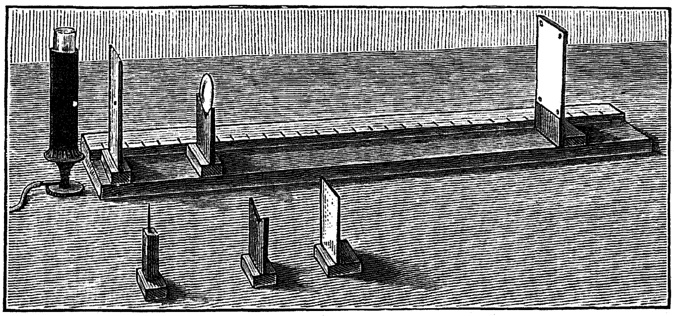


FIG. 51.—Simple Optical Bench.

figure. Or the lenses can be held between two upright parallel pieces of wood which are attached somewhat loosely at the bottom to a block. At the top they are drawn together by an elastic band. The lens is held by its edges in **V** grooves cut in the inner faces of the uprights (Fig. 51).

The *object* can be a piece of thin board about 6 inches square, supported on another block, with a $\frac{1}{4}$-inch hole exactly on

a level with the centres of the lenses; to the front surface of this attach a white card with drawing-pins, having a clean hole a little more than $\frac{1}{8}$-inch in diameter opposite the centre of the larger hole in the wood, with a vertical and a horizontal cross-wire, for which a very fine glass capillary tube does excellently.

A lamp, or better an incandescent gas flame, is placed behind this. The receiving screen can be another similar board and card, but without any hole.

Owing to the warping of the wood caused by the heat of the lamp, it is better to make the first screen of zinc (the one with the hole in). This is bent into the shape shown in Fig. 52, and a card slipped in the turned over part, AB, so that there is an air space of about $\frac{1}{8}$ inch between the card and the zinc. The zinc can be cemented in a groove cut in the wooden block (Fig. 52). The heights of the centres of the lenses and screens need not be more than about 3 inches. [It will be well to make them the same as that of the saccharimeter described on p. 461, so that the parts may be interchangeable.] The blocks should be of the same width, so that when pressed against the metre scale, the centres shall be in a line. Each size lens will require a separate **V** block in order that the centre may remain at the same height; but as these are very easily made, and only about four lenses will be necessary, this is no serious matter.

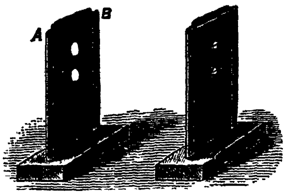


FIG. 52.—Cross-wire Screen with two holes. (When only one is required, the other is to be covered.)

AB shows the shape to which the zinc is bent, and the other figure shows the card in position.

Spectacle lenses may be used if desired. The most useful will be a 10-inch and a 20-inch convex lens, and a 14-inch and a 20-inch concave lens. They should all have the same diameter.

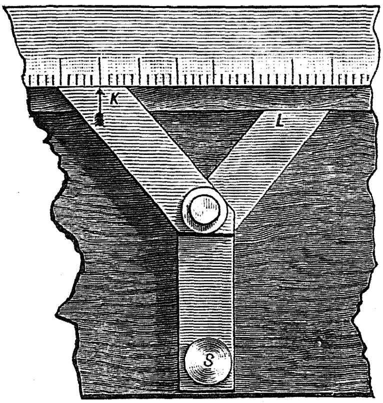
If special lenses are bought for it, 2 inches diameter will be found a convenient size.

A convex and a concave mirror, each of about 12 inches focus, will be wanted; also a plane mirror of good plate-glass, which must be mounted to stand vertically in a block.

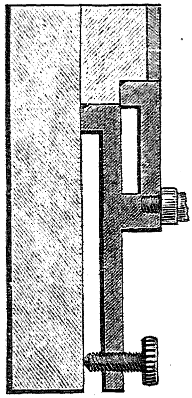
One stand will be required to hold both convex lenses in contact, and another the 10-inch convex and the 14-inch concave together. If an achromatic lens about 2 inches in diameter and 6 to 10 inches focal length be obtained (such as are used for

marine glasses), the cost will probably be no more than that of a convex and concave lens. It will not only answer the purpose for which those lenses are wanted, but will be useful in other ways, and will be much better for the *auxiliary lens* in the methods which depend upon the use of a convergent beam.

A better form of bench, allowing the stands to be brought closer to one another and allowing the centres to be adjusted in line, is shown in Fig. 53 in plan and section.



Plan.



Section.

Fig. 53.—Improved Optical Bench.

The section of the base shows a ledge, upon which one foot of the stand rests. A second foot rests on the base-board, and the third foot is supported by a screw, S, also resting on the base. It will be noticed on examining the drawing of the stand itself, that this will allow the two stands to be brought very close to one another; the foot K of one of the stands that rests on the ledge, sliding above and across the foot L of the other that rests on the base-board. The screw S allows the stand to be tilted, and thus produces the effect of a transverse motion, enabling the lens to be set nearer or further away from the edge of the bench. Each stand has a tube about 6 inches long, strengthened at the top with a brass cap, and is split and sprung inwards, to grip smoothly an inner sliding tube which carries the lens or other fitting.

For this bench it is preferable to mount the lenses and the various mirrors each on a separate inner tube, so that they may

be interchanged readily, and so that they are less likely to be broken.

The back view of the screen next the lamp is shown, to illustrate the method of attaching it to the inner tube. ABCD

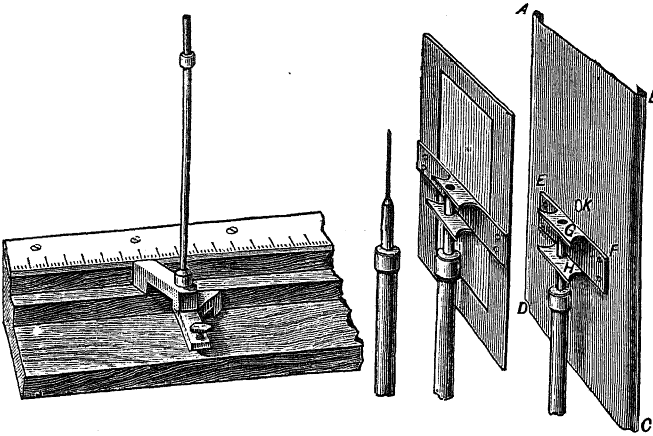


FIG. 54.—Optical Bench, Stand, Needle, and Screens.

(Fig. 54) is a piece of zinc bent as described before. To the back of this is screwed a thin brass plate, EF. Two brass plates, G, H, are silver-soldered to this, and a tube passes through a hole in each of these and is soldered in. K is the half-inch hole through the zinc and exactly opposite it is the small hole in the cardboard sheet.

The receiving screen is mounted on the inner tube in a similar manner, and consists merely of a hollow frame cut out of sheet brass, on which paper is stretched. A piece of good drawing paper is damped and left for about ten minutes to stretch; the front of the frame is then pasted and placed upon the damp paper; some weights are put on, and it is left a day or two to dry. When dry it will be found to be stretched tightly and perfectly flat.

37. Still another form of bench is shown in Fig. 55, in which also the stands may be brought close together. The bench consists of three stout metal tubes of nickled steel, let into blocks at their ends. The stands rest on a **V**, and flat on the upper rods, and on a **V** on the lower rod. A thumb-screw on a swinging arm of the stand, and tightening on the under side of the lower rod, clamps the stand.

For holding any odd lenses that one may wish to experiment with, a very convenient stand can be made by joining a pair of curved

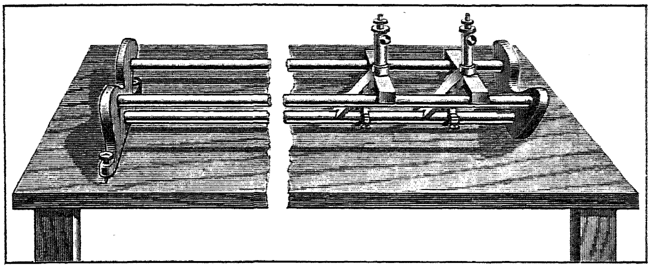


FIG. 55.—Optical Bench of Steel Tubes and two Stands.

pieces of brass into a cross-piece, fixed to the top of a brass rod that fits in the various stands. A nut with a conical upper surface screws up and down on this rod, and as it is screwed up, the edges of the curved pieces are forced together. These edges have a **V** cut on their inner edge, and therefore will clamp a lens of any size (Fig. 56).

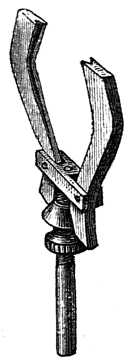


FIG. 56.—Lens Holder.

A steel rod with square ends about 8 or 10 inches long will be required for finding the index error (p. 59). This should be mounted at right angles on another (inner) tube.

Each of the stands should have an arrow on the leg K by which its position may be read, and the upper face of this leg must be on the same level as the upper face of the scale (Fig. 53).

38. An optical bench, upon which a lot of work can be done, can be made very easily as shown in the accompanying drawings. Upon a board about $4'' \times 1''$ and of any convenient length, say two feet, are attached a straight tube A about $\frac{5}{8}''$ diameter, such as a piece of brass gas barrel, and a strip of plate glass about $2''$ wide. The rod and the glass must be

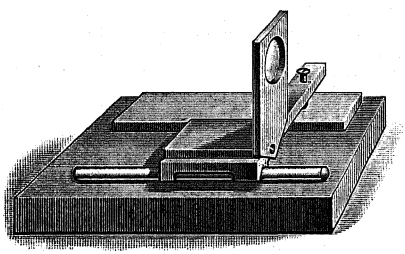


FIG. 57.—Home-made Optical Bench.

parallel to one another. They may be attached to the board by plasticene, red wax, or sealing-wax, or they may be drilled and screwed on. Two such boards may be built up in one line or making an angle with one another. They can be temporarily attached to the table with small lumps of plasticene under their corners.

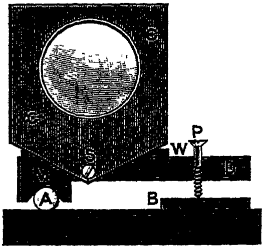


FIG. 58.—End view of Home-made Bench.

Temporary lens stands can be made of wood as shown in Figs. 57 and 58 A small piece of wood D has two blocks M, N glued on, which have a **V** cut in them to slide on the rod A of the bench (Fig. 58). A screw P is put through the other end, which will rest on the glass B. The lens is either inserted in a turned recess in C and held in place by a spring of brass wire, which fits in the recess, or it may be simply attached with a few pellets of plasticene round its edge. The screw P enables the lens to be raised or lowered, and rotating C on the screw S moves the lens transversely across the bench. Thus it can be centred. A weight W of sheet lead is necessary to keep the stand from tipping over. The heavier the lens stand is the steadier it is; thus stands of similar design in cast-iron are much to be preferred to wooden ones. They need not be filed up. P and S can be two set screws; the feet will of course be cast on; a washer can be put between C and D to allow C to turn smoothly; C may be made of wood.

39. For my own experimental work I have found a piece of angle brass, supported horizontally at a convenient height on a kind of heavy retort stand, with the **V** upwards, very convenient. The lenses must be mounted in short lengths of brass tube ("telescope tube"), which must all be of the same diameter. They will then always be coaxial. I use tubes of two sizes, the one sliding in the other; the lens is ground to such a size that it just fits in the outer tube and shoulders against the inner tube; the lens is held in, either by a wire spring ring, which goes tightly into the outer tube, or by another very short length of the inner tube. Any number of lenses can be put up in a row, and will at once be coaxial, and have their planes normal to their common axis. They can be slid to and fro, or lifted off, or put back instantly. In this way experimental microscopes, telescopes, etc., can be set up in a few minutes, or the best arrangement of lenses for any particular purpose can be easily tried. Stops can, of course, also

be mounted in the same way, in the same or similar tubes. A two-foot length of one-inch angle brass, and tubes $1\frac{1}{8}''$ diameter, $1''$,

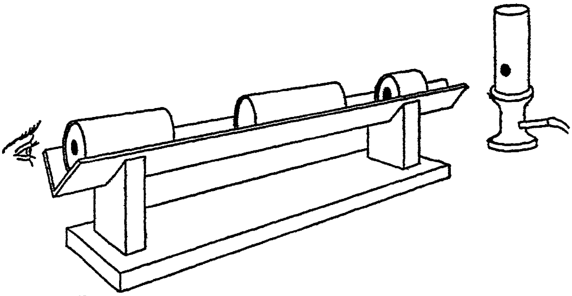


FIG. 59.—V Bench for Lenses of Uniform Diameter.

$2''$, and $4''$ long, are convenient sizes for most purposes. I have also found $\frac{1}{2}''$ angle brass and $\frac{5}{8}''$ tubes very good.

Rule of Signs.

40. In all formulæ, measurements from the surface of the lens or mirror towards the light will be considered positive, those from the surface away from the light, negative. To apply this, imagine yourself standing at the surface where reflection or refraction is occurring, and turn so that the incident light falls upon your face. If by walking forwards from the surface, you come to any point, the distance of that point from the surface is to be considered positive. But if to reach a point it is necessary to turn round and walk in the other direction, that distance is to be reckoned negative.

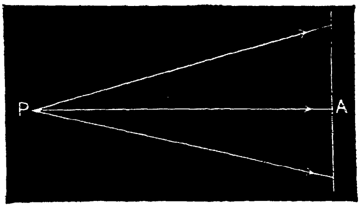


FIG. 60.—Divergent Incident Beam.

EXAMPLES.

i. A divergent beam falling on a lens or mirror (Fig. 60).
Suppose A to be the surface of the mirror and P the bright point. Applying the above rule, we see that the distance AP is positive.

ii. A convergent beam falling on a mirror or lens (Fig. 61).
If a beam travel in the direction BQ and fall upon the surface A,

the point of the cone of light that we are dealing with is Q. To reach this point starting from the surface A, you would have to turn your back upon the light, and thus the distance AQ is negative.

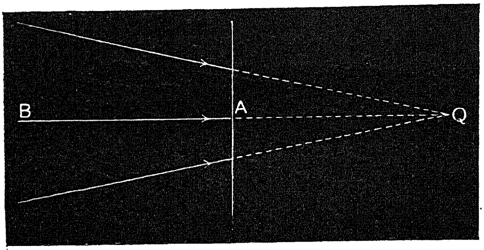


FIG. 61.—A Convergent Incident Beam.

iii. Suppose a beam of light falling upon the lens A, afterwards to diverge so that it appears to come from a point R. Then R is the

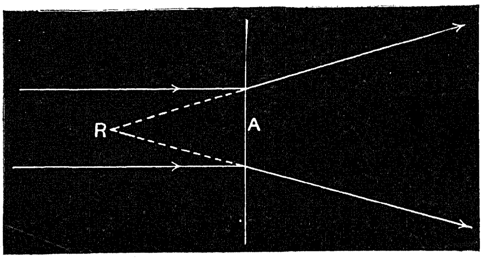


FIG. 62.—Emergent Diverging Beam.

vertex of the cone of rays that is leaving the lens. To reach R from A we should be walking towards the incident light, and the distance AR is positive.

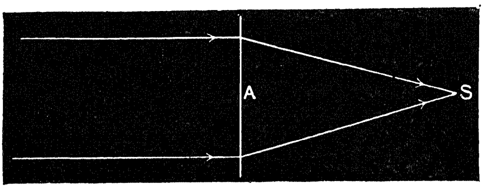


FIG. 63.—Emergent Converging Beam.

iv. Light converging after passing through the lens.

Suppose it to converge to the point S. Then to reach S we shall have to walk away from the light, and the distance AS is negative.

In the formula, u always is used to refer to the incident cone of light, and is the distance from the surface to the vertex of that cone. v in the same way is used to refer to the emergent cone—that is, the cone after reflection or refraction as the case may be. So that in Fig. 60, the distance AP is u. In Fig. 61, AQ is numerically equal to u, but being negative, it really is $-u$. In Fig. 62, $AR=v$, and in Fig. 63, $AS=-v$.

If the incident light is parallel, the emergent light either comes from, or proceeds to, a point which is called the principal focus of the lens, and the distance AR or AS is the focal length. It is obvious from Figs. 62 and 63 that the focal length of a concave lens will be positive and that of a convex lens negative.

v. Using these conventions, the formula for a mirror is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=\frac{2}{r},$$

where r is the radius of curvature of the surface; and for a lens it is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

If we use U, V, F, R to stand for the reciprocals of these measurements in *metres*, the formulæ become

$U+V=F=2R$ for the mirror, and
$V-U=F$ for the lens.

F is called the *power* of the mirror, or lens as the case may be, U and V measure the *curvatures* of the waves at the surface of the lens or mirror, and R that of the surface of the mirror itself, the curvature of a sphere of radius 1 metre being taken as unity.[1] In words these formulæ express that the sum of the curvatures of the incident and reflected waves is equal to the power of the mirror, and is also twice its curvature; and that the increase of curvature caused by refraction through a lens is equal to the power of the lens. From this way of stating it, it is obvious that if a second lens is placed close against the first one, the curvature will be again increased by the power of that lens, and thus, that the power of two lenses in contact is equal to the sum of the powers of the individual lenses, the power of a convex lens being taken, of course, to be negative. The power of a lens as defined above is said to be so many *dioptres*.

The following way of stating the formula is instructive.

When a spherical wave of light falls normally upon a lens, the change in the curvature is always the same. This change in the curvature is called the power of the lens.

The measurements on the optical bench will be those of u, v, and f, respectively. These must be converted into U, V, F, by the table of reciprocals given at the end of the book.

[1] Do not confuse *curvature* with *radius of curvature*. If a spherical surface has a small radius, its curvature will be very large; but the surface becomes a plane, and therefore has no curvature, when the radius becomes infinite. The curvature is the reciprocal *in metres* of the radius of curvature.

Focal Length of Convex Lens.

41. **First Method.**—*Apparatus.*—Optical bench; cross-wire and receiving screens; flame; 10-inch lens; thin metal rod 6 or 8 inches long, *e.g.* a knitting needle, for the index error; squared paper.

Set up the gas flame, the screen with the hole and cross-wire, also the 10-inch lens and the paper screen in a line, so that the light from the cross-wire after passing through the lens falls upon

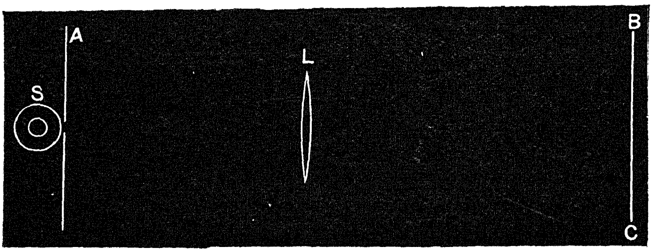


FIG. 64.—Focal Length of a Convex Lens.

the paper screen. A bright patch will be seen on the latter. Move this to and fro, and see if an image can be formed upon it of the cross-wire. This will not be possible if the lens is not at a greater distance from the cross-wire than its focal length. To obtain an image on the far side of the lens, the curvature of the emergent light has to be negative. The curvature of the incident light is positive, and if, therefore, the curvature of the incident light is greater than the power of the lens, it would be impossible for that lens to render it negative. The curvature must be reduced by increasing the distance from the lens to the cross-wire until it is less than the power of the lens. By placing the eye close to the lens and looking through it at the cross-wire, some idea may be formed of the behaviour of the light. If the curvature is still positive, it will be possible to see the cross-wire distinctly through the lens. It will merely appear magnified. When the curvature becomes negative so that a converging beam is falling upon the eye, it very soon becomes impossible to focus

this clearly on the retina, and if the light be received on the screen, the image can be found at a short distance. If the lens is one of very low power indeed, it may not be possible to make the curvature of the incident ray small enough in the length of the bench, and its power will have to be determined by an indirect method. Supposing however the image found, the reading of the scale arrows on each of the stands must be taken.

Index Error.—The difference between the readings on the cross-wire stand and the lens stand would be the distance u, if there were no "index errors." In the same way, the difference between the readings on the lens stand and the receiving screen stand would be $-v$. But the distance from the screen to the lens will not, as a rule, be exactly the distance from the one arrow to the other: it will differ from it by a constant amount, the "index error." This constant amount must be found by the steel rod.

Withdraw the lens stand from the cross-wire screen, and insert the rod horizontally between them on a line with the centre of the lens, and adjust it just to touch the card by the cross-wire at one end and move up the lens lightly to touch it at the other end. Their true distance is now exactly the length of the rod.

Carefully lift out the rod without disturbing anything else, and place it on the scale, with one end exactly opposite the arrow on the stand of the cross-wire screen. If the other end of the rod is exactly opposite the arrow on the lens stand, there is no index error. If not, note how much the distance is from the end of the rod to the arrow. If the needle is longer than the distance between the arrows, the error is positive, and you will write "index A to B 2·3 cms." (say). If the needle is shorter than the distance apart of the arrows, the error is negative. This error is to be added with its proper sign to all the readings of the distances AB, that are subsequently taken.

Repeat the operation with the lens and receiving screen.

A series of readings of the distances u and v can now be taken.

Enter the results thus :

SCALE READINGS.

Cross-wire.	Lens.	Screen.	u	v	U	V	V - U = F	f

The column F is the power of the lens in *dioptres*.

U, V, F are found from tables of reciprocals at the end of the book.

The mean of the columns F and f should be taken

42. The graphical solution should also be found.

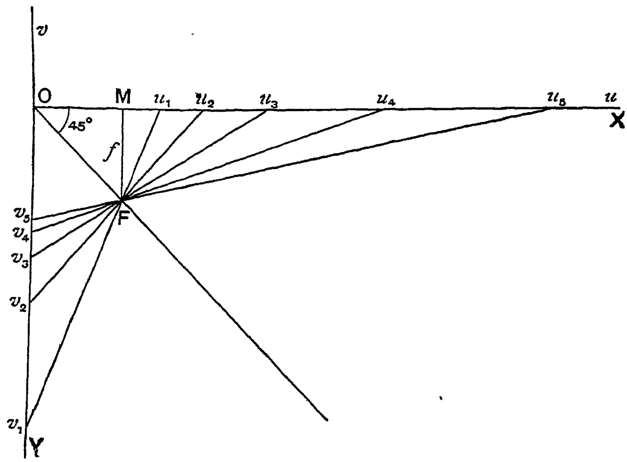


FIG. 65.—Graph of a Convex Lens. FM is the focal length, and is negative.

The values of u_1, u_2, u_3, u_4 (Fig. 65) are to be marked off along XO, and the values of v_1, v_2, v_3, v_4 (being negative) are to be measured off along OY. Join the corresponding points. These lines should all meet a line, OF, bisecting the angle XOY, in the same point, F.

The mean position of this point must be found, since the lines will not all meet in OF in exactly the same point. Then the

length of the perpendicular FM should agree with the mean value of f found from the last column.

The best results will be obtained when u and v are nearly equal to one another, as the errors of measurement are then less important.

43. **Second Method.**—*Apparatus*—As before, with the addition of a plane plate-glass mirror and stand.

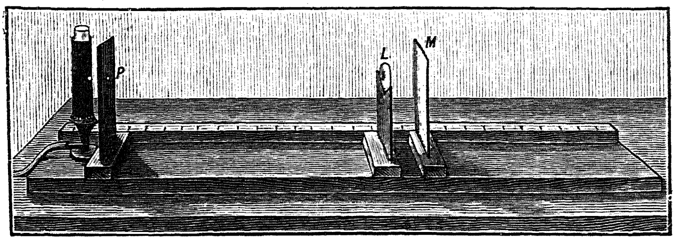


FIG. 66.—Mirror Method for determining the Focal Length of a Convex Lens.

Place a plane mirror, M, close behind the lens, and adjust the distance of the lens L from the cross-wire until the image of the cross-wire is formed on the screen P itself close by the side of the cross-wire, so that the light is practically returning along its own path. To do this it must be striking the mirror nearly perpendicularly, or as it is a plane mirror, the light falling upon the mirror must be parallel. It has, therefore, no curvature, and the power of the lens must be equal and opposite to the curvature of the incident light. The distance of the lens from the cross-wire must therefore be the focal length of the lens. The distance between the arrows corrected for index error will be the focal length.

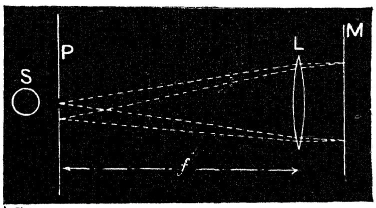


FIG. 67.—Focal Length of a Convex Lens. Mirror Method.

This is the most accurate method as well as the simplest.

44. **Third Method.**—*Apparatus.*—The same as for the first method.

If the cross-wire, lens, and screen be adjusted as in the first method, to form an image of the cross-wire P on the screen at Q with the lens at A, it will be found that there is another position, B, for the lens at which an image of P is again formed at Q. That is to say, that with the cross-wire P and the screen Q at a given distance apart, a, there will be two positions of the lens, A and B, at which the image will be sharply formed. Also the distance AP will be equal to the distance BQ, and AQ equal to BP, so that in the formula

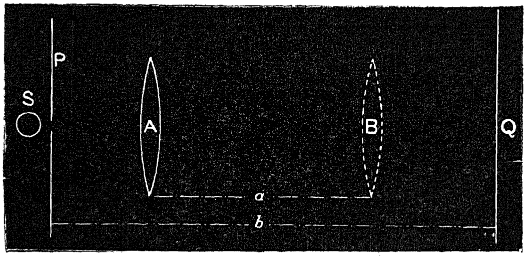


FIG. 68.—Conjugate Positions of a Convex Lens.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

the magnitudes of the u and v can be interchanged.

(It must be remembered that v is $-$AQ or $-$BQ by the rule of signs, p. 56.)

For this reason the points P and Q are frequently called *conjugate foci* of the lens A.

If $$PQ = b = AP + AQ = u + (-v),$$

and $$AB = a = AQ - AP = (-v) - u,$$

$$b^2 - a^2 = -4uv;$$

but $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

or $$f = \frac{uv}{u - v} = -\frac{b^2 - a^2}{4b}.$$

If, therefore, we measure the distance b between the two

screens, and a between the two positions of the lens, we can calculate f.

It will be obvious that the measurement of a will be independent of index errors, as it is merely the distance the arrow attached to the lens stand is moved, and it will not therefore involve any determinations of the exact position of the optical centre of the lens. It does, however, involve index error in the measurement of b. (This method is nearly correct for the measurement of the focal length of an ordinary photographic lens. The following method is theoretically correct for such a lens.)

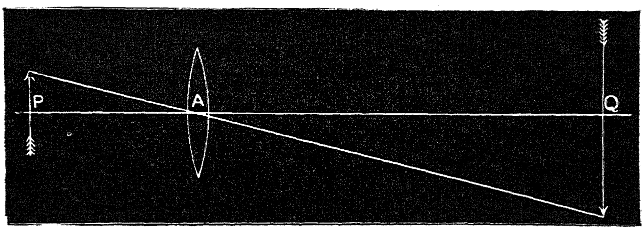


FIG. 69.—Magnification of Image.

45. **Fourth Method: Abbe's Method.**—*Apparatus.*—Optical bench; receiving screen; gas flame; pair of dividers; millimetre scale; in place of the ordinary cross-wire screen, the screen with two holes, vertically one above the other, about 2 mms. in diameter, and 2 cms. apart, with a horizontal cross-wire stretched over each (Fig. 52).

When the image of an object placed at P is formed on a screen at Q, the ratio of the size of the image to the object, the *magnification*, is equal to the ratio $\frac{v}{u}$. If this ratio is negative, the image is inverted. This is obvious from similar triangles, if it is remembered that the light passes through the centre of the lens without bending.

Since $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$$

$$\frac{1}{u}=\frac{1}{v}-\frac{1}{f}=\frac{f-v}{vf};$$

therefore $$\frac{f-v}{f}=\frac{v}{u}=m_1, \text{ the magnification.}$$

If, without moving the lens, the screen Q be withdrawn a known distance, d, so that v becomes v', we have

$$\frac{f-v'}{f}=m_2, \text{ the new magnification.}$$

Subtracting (and remembering that $v=-AQ$, and that m_1 and m_2 are negative quantities) we have

$$\frac{d}{f}=m_2-m_1, \text{ or } f=-\frac{d}{m_1-m_2}.$$

To determine the magnification, focus the two cross-wires on the receiving screen, and set the dividers to the exact distance apart of the images of the two cross-wires. Measure this distance, a, on the millimetre scale—if possible estimating to fractions of a millimetre. Note the position of the receiving screen; withdraw it a definite distance from the lens *without moving the latter*, and adjust the cross-wire screen, until the images of the cross-wire are once more sharply in focus. Again take the distance apart, b, of the cross-wires with the dividers. Very carefully measure the actual distance apart, c, of the cross-wires themselves. The distance d is, of course, the amount the screen Q is moved as measured by the arrows on its stand.

Numerically, m_1 will be the ratio $\frac{a}{c}$, and m_2 will be $\frac{b}{c}$: thus the focal length is given by

$$f=-\frac{dc}{b-a}.$$

There are no index errors to be allowed for, and it is true for a compound lens.

46. **Fifth Method.**—For a lens of long focus, that is, a lens of very low power.

Apparatus.—The same as for the first method, and in addition a long focus convex lens and stand (if obtainable, the short focus convex lens is much better replaced by a photographic, or even a lantern objective); squared paper.

Remembering that the power of a lens is the difference between the curvatures of the incident and refracted waves, it will be obvious that if a lens be interposed in a converging beam, *i.e.* one whose curvature is already negative, it will increase that

negative curvature if the lens is a convex one, and the change in the curvature will be the power of the lens.

Set up therefore the cross-wire screen at P, an auxiliary lens, L (preferably a good photographic objective, or failing that a good

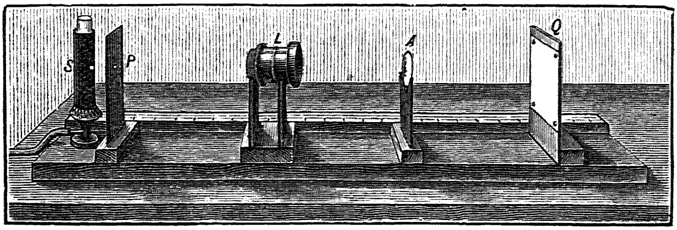


FIG. 70.—Auxiliary Lens Method.

achromatic lens), and screen at Q. Adjust the distance so that an image of P is formed at Q. If now a convex lens, A, of low power, be interposed between L and Q, it will increase the curvature of the light falling upon it, and cause it to focus at some point, Q′, nearer L.

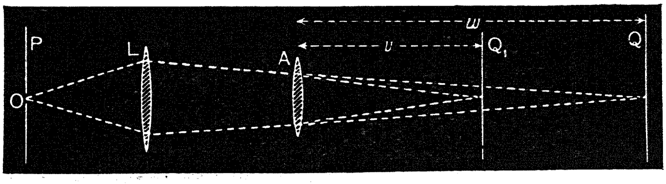


FIG. 71.—Auxiliary Lens Method for a Low-Power Convex Lens.

Note the position of the screen at Q, and then move it forward to Q′, again noting its position. Read also the position of A. Find the index error between A and Q.

The curvature of the incident ray was equal to $-\frac{1}{AQ}=U$.

The curvature of the emergent ray $= -\frac{1}{AQ'}=V$.

Therefore the power of the lens, $F=V-U$, can be found.

This method may also be used to find the focal length of any concave lens, and, with a good photographic objective as an auxiliary lens, is the best method to employ.

47. **Sixth Method.**—*Apparatus.*—Telescope; 10-inch lens and stand; cross-wire, stand, and optical bench; knitting needle; rod for index error.

Focus a telescope on a distant object. Place it to receive the light coming through the lens A from the cross-wire P, and adjust the distance of the lens A from the cross-wire until the cross-wire is seen sharply in focus. If the telescope has been

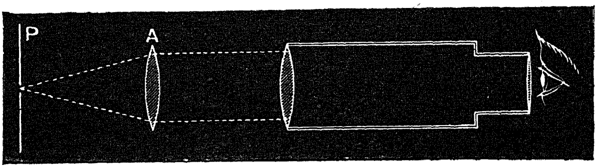


FIG. 72.—Telescope Method of obtaining the Focal Length of a Convex Lens.

properly adjusted, the light entering it will be parallel light, and thus the cross-wire is at the principal focus of the lens A, and the distance AP is its focal length.

Focal Length of Concave Lens.

48. **First Method.**—The focal length of a concave lens is best found with the aid of an auxiliary lens, and is similar to Experiment, § 46, for a convex lens.

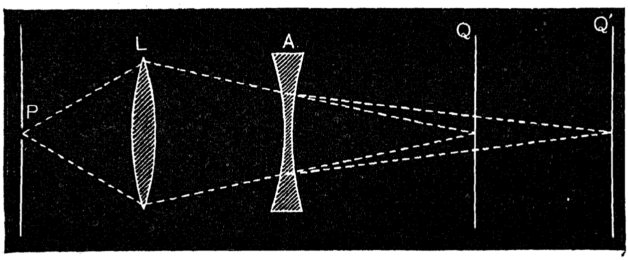


FIG. 73.—Focal Length of a Concave Lens. Auxiliary Lens Method.

Apparatus.—Optical bench; cross-wire and receiving screen; photographic or lantern objective—or, failing this, a convex lens of 6 to 10 inches focus—on a stand; gas flame; concave lens and stand; squared paper.

Set up the screen P and the auxiliary lens L to produce an image of P on the screen at Q. Note the position of this screen. Withdraw the screen to the end of the bench at Q'. Introduce the concave lens, and adjust it until the image of P is in focus at Q'. Note the position of the lens A, and find the index error from A to Q. As the light is coming from left to right, the distances AQ and AQ' will both have to be reckoned negative. The vertex of the incident cone of light is on Q. Thus AQ is $-u$ and AQ' will be $-v$.

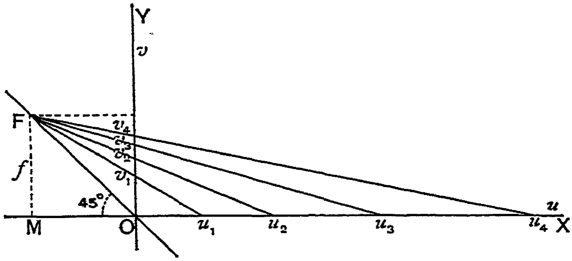


FIG. 74.—Graph of a Concave Lens. FM is the focal length, and is positive.

Several readings must be taken, and tabulated as in Experiment, § 41.

The graph will be that shown in Fig. 74.

49. **Second Method.**—As the power of a lens measures the increase in the curvature of the wave passing through it, two lenses in contact will increase the curvature by an amount equal to the sum of their powers, and this will be, of course, the power of the combined lens. So that if F_1, F_2, F, be the powers of the separate lenses and of the combination respectively, $F = F_1 + F_2$, and if any two of these are known, the third can be calculated.

Apparatus.—Optical bench; cross-wire and receiving screens; gas flame; convex and concave lenses, and a stand which will support them in contact with one another, the convex lens being stronger than the concave; rod for index error.

Place the concave lens the power of which is required in contact with a convex lens of known power, and determine the

power of the combination; that of the concave lens can then be calculated. If the power of the latter is less than that of the convex lens, the combination will act as a convex lens, and its focal length can be determined by any of the methods given for a convex lens, preferably the second method, both as being the most accurate and the simplest. In the same way the focal length of the convex lens alone can be found, and thus that of the concave obtained.

As the combination of these lenses will not have negligible thickness, there will necessarily exist some uncertainty as to the

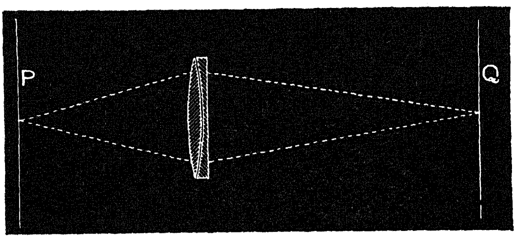


FIG. 75.—Focal Length of a Concave Lens by combining it with a stronger Convex one.

point to which measurements are to be made. If the surfaces of the convex lens are about equally curved, the measurements should be made from its centre approximately, and if the index error is taken from its front surface, half the thickness should be added. If the lenses are of fairly long focal length, and the thickness small, the error produced by neglecting it will not be very great.

In performing the experiment, the two lenses will both have to be mounted in contact on one support so that they may be moved about together.

If there is any appreciable distance between the lenses, the power of the combination is not the sum of the powers of the individual lenses.

The formula then becomes—

$$F = F_1 + F_2 + aF_1F_2,$$

or

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2},$$

where a is the distance between the lenses, and the combination will have to be treated as a "compound lens," see § 45 or 131.

When substituting in the formula, it must be remembered that the power of the convex lens is negative. The answer, being the power of a concave lens, should be positive.

50. Third Method, by the use of a Virtual Image.

Apparatus.—Optical bench; concave lens; knitting needle and needle on stands; plane plate-glass mirror and stand.

Let P be the screen with the cross-wire, or better, a knitting needle in a vertical stand, and A the concave lens. To the

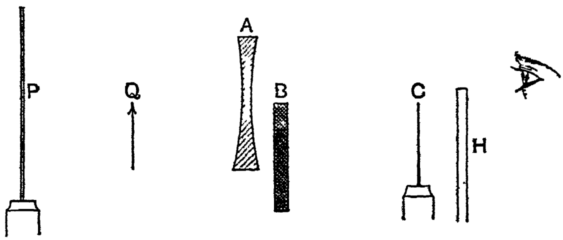


FIG. 76.—Virtual Image Method for a Concave Lens.

right of A, place vertically a silvered mirror, B, covering the lower half of the lens, and facing to the right, so that an eye placed as in the figure will be able to see the reflection in this mirror of a pin at C. This image will, of course, be formed at Q, as far behind the mirror as C is in front of it.

At the same time, through the upper half of the lens, the image of the knitting needle, P, will be visible somewhere between P and A. By altering the position of C, Q, the image of A may be formed at any desired point, and therefore may be caused to coincide with the image of P, seen in the upper half of the lens. When these two images coincide, a movement of the eye will not alter their relative position.

To adjust C, therefore, place the eye in such a position that the image of the pin, seen in the mirror, coincides with that of the knitting needle seen in the upper half of the lens. [The image of the pin will be much more easily seen if a piece of white paper, H, is placed behind it so that it appears as a dark

line on a white surface.] Move the eye to the right or left, and if the images do not remain coincident it shows that they are not really formed in the same position. If on moving the eye to the right, the image of the pin moves to the right of the image of the needle, it shows that the former is behind the latter. To improve the result, the pin, C, must be pushed a little closer to the mirror. In this way it is not difficult to obtain the coincidence of the images of C and P.

The distance AP will be the u of the formula, and AQ will be v. AP can, of course, be read off directly, correction being made for the index error.

AQ = BQ − AB, which is BC − AB. The readings should be taken from the silvered surface of the mirror, or more accurately still, from the point in the thickness of the mirror one-third its thickness from the silvered surface.

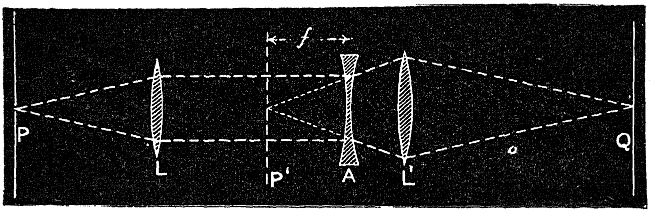


FIG. 77.—Direct Measurement of the Focal Length of a Concave Lens.

51. **Fourth Method, with the aid of two auxiliary Convex Lenses.**

Apparatus.—Optical bench; concave lens; cross-wire and receiving screens; the two convex auxiliary lenses, of 6 to 10 inches focal length (half-plate photographic lenses, if possible).

Adjust the first convex lens L so that the light from the cross-wire P shall emerge parallel. This is most conveniently done by placing a plane mirror behind it, as in Method, § 43 (p. 61). Place the concave lens A to receive this light, and then with the second convex lens L′, form an image of P on a screen Q. Note the position of A. Remove A and L and shift P forward to P′ until its image is again focussed at Q. Then the light which emerged from A must have been apparently coming from P′, and as the light falling on A was parallel, AP′ is the focal length of the lens.

Read the position of P, therefore, and the distance AP′ corrected for index error will be the focal length of the lens. Whether this method will give good results will depend upon the lenses L and L′. They should be telescopic or photographic objectives.

Focal Length of a Concave Mirror.

Although the theory of the concave and convex mirrors always precedes that of the lenses, yet as the experimental determinations of the focal lengths of lenses are not only more important but are more easily performed, there is an advantage in placing them first in the practical work. This also allows us to use lenses in determining the focal lengths of mirrors.

52. **First Method.**—*Apparatus.*—Optical bench; concave mirror; cross-wire screen.

Adjust the mirror facing the screen P at such a distance that an image of the cross-wire is formed on P, close by the side of the wire itself, and that the light which leaves P is returned, after reflection at the mirror, almost along its original path.

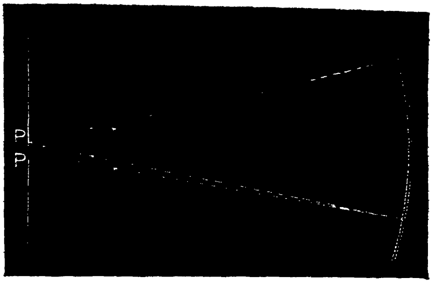


FIG. 78.—Radius of Curvature of a Concave Mirror.

If the image and object had perfectly coincided, each ray of light must have been returned exactly upon its path, and thus have struck the surface of the mirror normally. But lines which are normal to a spherical surface are radii of the sphere of which it is a part. So that the cross-wire is at the centre of curvature of the surface, and the distance from it to the mirror is the radius of curvature of the mirror r, and is double the focal length of the mirror.

This is the most accurate method of determining the focal length of a mirror as well as the simplest. It has the further

advantage that the image and object can be kept practically on the axis of the mirror, that is, on the line joining its centre of curvature to the centre of its surface.

53. **Second Method.**—*Apparatus.*—Optical bench; concave mirror; cross-wire and receiving screens.

Interpose a screen between the cross-wire P and the mirror, and let the edge of the screen reach just about to the line joining P to the centre of the mirror (the axis of the mirror), withdraw the mirror and tilt it a little until the image of the cross-wire is formed on this screen close to its edge (as in Fig. 49). In this way the object and image will be very nearly on the axis of the mirror. The distances from P and Q to the mirror will be the u and v in the formula for the mirror.

By using a receiving screen not more than half an inch in diameter, the image can be kept on the axis of the mirror.

Repeat this experiment with the screen Q at different distances.

The results can be entered thus:

Position of			PA corrected for index error.	QA corrected for index error.	$\frac{1}{u}=U.$	$\frac{1}{v}=V.$	U+V=F.
P.	Q.	Mirror.					

The mean of the last column, which should be nearly constant, gives the power of the mirror.

Also obtain the focal length graphically as in Fig. 79.

54. **Third Method.**—*Apparatus.*—Optical bench; concave mirror; silvered and unsilvered plane mirrors; cross-wire and receiving screens.

Between P and A, Fig. 80, interpose a flat plate of thin unsilvered patent plate-glass. The light reflected from this glass falls upon the screen Q.

Adjust Q until the image is in focus. The distance of the

image will now be the sum of the distances AM and MQ, which can be read with an independent scale.

The disadvantage of this method lies in the fact that there will be a reflection from both the front and back surfaces of the

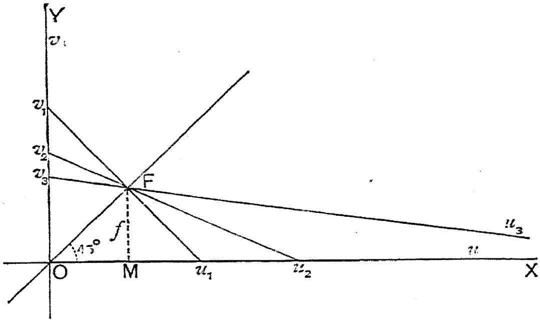


FIG. 79.—Graph of a Concave Mirror. F is the point in which the lines u_1v_1, etc., meet OF; FM is the focal length of the mirror.

glass plate, resulting in a certain amount of uncertainty in the position of Q. By using a silvered mirror from which some of the silver has been removed over a small area in the centre

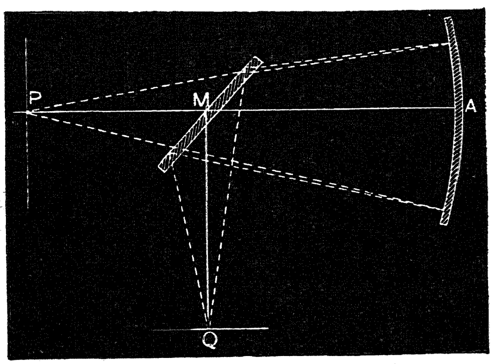


FIG. 80.—Focal Length of a Concave Mirror.

through which the light from P shall pass, this difficulty will be overcome, since the light reflected from the silvered surface will be much more intense than that reflected from the unsilvered portion. In this case, the distances AM, MQ must

be measured to the silvered surface, or more accurately, as before, to a point in the mirror a third of the thickness from the silvered surface.

In the above experiment we have supposed the distance AMQ to be greater than the distance AP. If the mirror is arranged

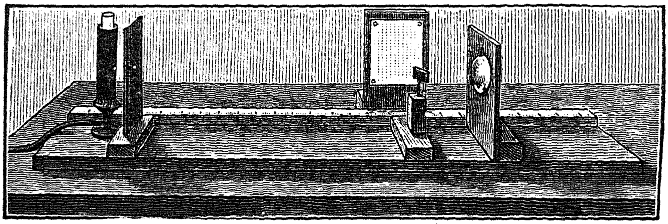


FIG. 81.—Concave Mirror and Small Plane Mirror.

so that AP is the greater, it will be best to use a small silvered mirror on the axis to reflect the light to the screen Q. In this case, the mirror may be of speculum metal, and the distance AMQ can then be found accurately.

This method is introduced as illustrating the principle of the reflecting telescope.

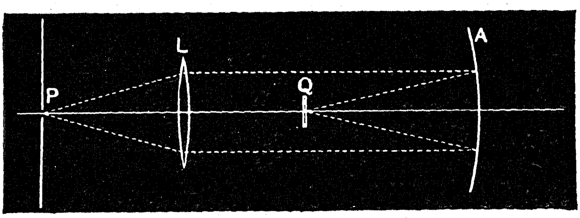


FIG. 82.—Parallel Beam Method for a Concave Mirror.

55. **Fourth Method.**—*Apparatus.*—Concave mirror; optical bench; a small white receiving screen, about 1 inch diameter, on stand.

Allow the light to pass through the lens L, placed at its focal length from P so that a parallel beam falls upon the mirror A. Receive the reflected light upon a very small screen Q placed on the axis of the mirror.

The distance AQ will be the focal length of the mirror.

If the lens L is a good one (*e.g.* a photographic objective), this will give good results.

56. **Fifth Method.**—*Apparatus.*—Optical bench; a needle and a knitting needle, or two knitting needles, on stands.

Place two needles P and Q in stands of the optical bench and put P in front of the mirror. An image of P will be visible in the mirror. If P is close to the mirror, this image is erect and magnified. At a greater distance, it will be seen inverted, and may be either magnified or diminished according to the distance.

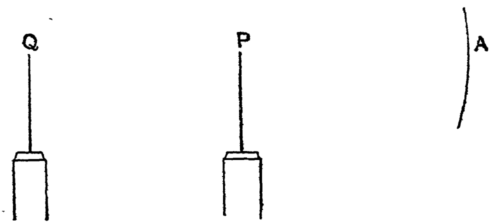


FIG. 83.—Parallax Method for a Concave Mirror.

Place it at such a distance that it is inverted and somewhat magnified. Now put the needle Q in such a position that it appears to coincide with the image of P. That is to say, get P and Q in a line on the axis of the mirror. It may be necessary to rotate the mirror slightly and to adjust the heights of P and Q. The image of P should appear inverted with the point of the needle exactly in a line with, and touching the point of, the needle Q itself.

Move the eye sideways. If the image of P and the needle Q still appear coincident, they are really together. But if they separate, Q must be moved nearer or farther from the mirror until the coincidence is perfect, remembering that the one which moves in the same direction as the eye is moved, is the farther from the eye. The distance PA and QA may now be measured either by using the scale on the optical bench and obtaining the index error, or by holding an independent scale against the surface of the mirror and reading the distances

of P and Q directly. They will be the a, u, and v of our formula.

Focal Length of Convex Mirror.

The convex mirror does not form a real image of a real object, and thus we cannot obtain its power and curvature directly, but must employ methods similar to those adopted for the concave lens.

57. **First Method.**—*Apparatus*—Optical bench; cross-wire and receiving screens; convex mirror; a short focus convex lens (preferably a photographic objective).

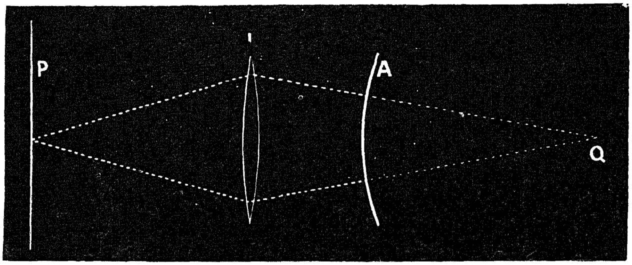


FIG. 84.—Auxiliary Lens Method for a Convex Mirror.

With an auxiliary lens L, form an image of the cross-wire P on a screen Q. Note the position of Q. Interpose the mirror A between L and Q. The light will now be reflected back, and by adjusting A it will be possible to form an image of the cross-wire on the screen P by the side of the cross-wire itself. When this is the case, the light being returned upon its path must strike the mirror A normally, and thus the rays must be radii of the surface A, and would, if produced, meet at its centre. But they do focus at Q when the mirror is not in the way. Thus Q is the centre of curvature of the mirror, and the distance AQ is its radius of curvature.

If the mirror A has a very small curvature, it will be necessary to put L in such a position that Q is at the end of the bench. Generally the lens should be so adjusted that A is fairly close to it to give the best results.

With a good lens this is the best method.

58. **Second Method.**—*Apparatus.*—Optical bench; convex and plane mirrors on stands; needle and knitting needle on stands.

Place a plain silvered mirror B in front of the lower half of the convex mirror A. At some distance from it place a large

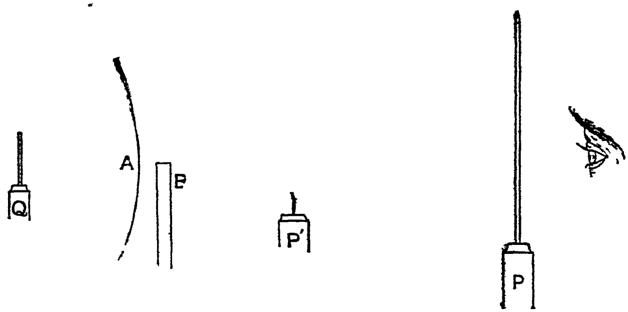


FIG. 85.—Parallax Method for a Convex Mirror.

needle, a knitting needle for instance, P, in a stand of an optical bench. An eye placed behind this at E will see a diminished image of the needle formed in the upper half of the concave mirror at Q.

If a small needle P′ be placed at a short distance in front of B, an image will be formed by B of this needle, at an equal distance behind it. By adjusting the position of P′, it can be so arranged that this image formed in the plane mirror B and the image of P formed in the upper half of the convex mirror A may coincide at Q. This coincidence may be determined as usual by moving the eye to and fro, when no change in the relative positions of the two images should occur.

Then $AQ = BQ - BA = BP' - BA$.

This is the v of the formula, and is negative.

The distance AP is the u of the formula, and can be measured directly.

A piece of white paper to the right of P will make the images in the mirrors more easily distinguished by providing a plain white background for them.

Three or four readings should be taken and the results entered in columns as in the case of the concave mirror. Experiment, § 53. Also obtain the focal length graphically as in Fig. 86.

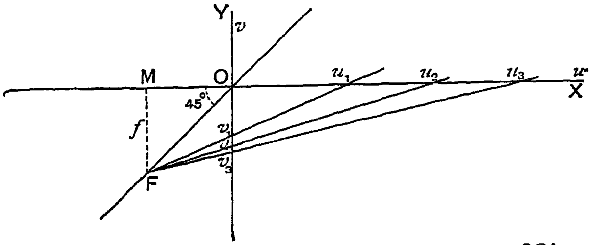


FIG. 86.—Graph of a Convex Mirror. The lines u_1v_1, u_2v_2, etc., meet OF in a point. The focal length is represented by FM, and is negative.

59. **Third Method.**—*Apparatus.*—Convex mirror on stand; glass scale and Bunsen clip or stand for same; card on stand; small cork borer to make the holes in the card.

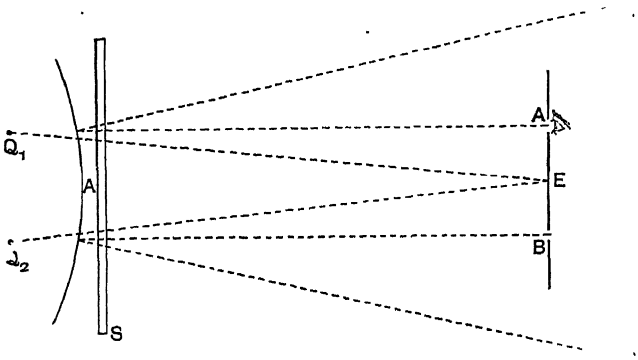


FIG. 87.—Focal Length of Convex Mirror by Virtual Image Method.

Set the mirror up facing the window.

Let P_1 and P_2 be two of the window bars. A reduced image of these will be formed in the mirror at Q_1 and Q_2.

Place a glass scale, S, close in front of the mirror, and, standing at a distance of 2 feet away, observe with one eye the number of divisions subtended by the image on the scale. This will not give the true size of the image owing to the

convergence of the rays from Q_1 and Q_2 to E, but it may be treated as a first approximation. Now make two small holes in a piece of card at this distance apart, place the card in a clip with the holes horizontal, and observe Q_1 through the hole A, and Q_2 through the hole B. The size of the image will now be found to a second approximation and may be taken as correct. Let it be b. Measure the distance apart of P_1 and P_2, a. Then m, the magnification, which as the image is erect is a small positive quantity, is given by

$$m = \frac{b}{a} = \frac{-v}{u} = \frac{-f}{u-f}.$$

The distance from A to the window can be easily measured, and therefore (which is of course negative) is given by

$$f = -\frac{mu}{1+m}.$$

Instead of two window bars, a pair of candles may be used.

60. **Fourth Method for an Unsilvered Mirror,** *e.g.* one of the surfaces of a lens.

Apparatus.—Convex lens; fusible metal, test-tube, and a beaker of water; rubber ring; metal or cardboard ring.

Place a few sticks of fusible metal in a test-tube and put the latter in a vessel of hot water. In a short time the metal will melt. At the same time warm the mirror carefully by putting it in cold water and gradually adding hot water, so that it may not crack. Make a paper, or cardboard, ring to fit tightly round the edge of the lens so as to make a small vessel into which the melted metal may be poured. The surface the curvature of which is wanted is to be the bottom of this vessel.

FIG. 88.—Method of Casting a Mirror.

Or, better still, place an indiarubber ring on the surface and press a short piece of brass tube down on this so that it makes a water-tight cell. Pour in the fusible metal. It will very soon set, and, when quite cold, a smart blow with a piece of wood will separate it from the lens. (It is not advisable to try this for the first time with a very valuable lens, as even if the heat

does not crack the lens, it is sometimes difficult to separate the two surfaces.)

This metal casting will now be found to make a very good bright concave mirror, the curvature of which can be determined by any of the methods given.

61. **Fifth Method.—To find the curvature of the convex surface of a convex lens.**

Apparatus.—Optical bench; cross-wire screen; convex lens on stand.

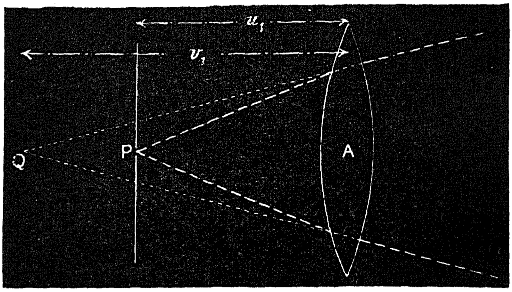


FIG. 89.—Curvature of Convex Surface of a Convex Lens.

Place the lens on the optical bench near the cross-wire.

A patch of light will be seen on the screen P. This is caused by the reflections from the front and back surfaces of the lens. Gradually withdraw the lens. If the surface A, farthest from the screen, is convex, and the lens be gradually withdrawn, the light reflected from this surface will at a certain distance be focussed on the screen P, and, by tilting the lens, the image may be formed close to the cross-wire.

As the light is now reflected back upon its original path, it must be striking the far surface A of the lens normally, and therefore, *in the glass it must be travelling along radii of this surface.* Owing to the refraction at the near surface of the lens, this will not be the case in the space between P and the lens. The ray, as we have seen, is striking the surface A normally, the light which emerges—and the greater part of the light does emerge—will do so without bending, and the point from which it

seems to come must therefore be the centre of curvature of that surface. But the point from which the emergent light appears to come is also the "image." Thus, when the light which strikes the back surface of the lens is reflected back along its path to P, that light which emerges is coming both from the image of P and from the centre of curvature of the surface, and these two points are coincident.

Now if we know the focal length of the lens, and the distance of the object, we can, of course, calculate the distance of the image. Therefore:

(1) Find the distance $AP = u$.

(2) Find the focal length of the lens f (by placing the plane mirror behind it and obtaining the image on the screen as in Method, § 43 (p. 61)).

Then the radius of curvature of the surface AQ, or v, is given by $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ (remember that f, being the focal length of a convex lens, is negative).

62. **Sixth Method.**—If the curvature of the surface is very slight indeed, it may be found with great accuracy by the method of Newton's rings (p. 290).

63. **Radius of curvature of a surface of a very small lens or mirror, e.g. a part of a micro-objective.**

Apparatus.—An adapter to screw on the end of the tube in place of the micro-objective, furnished at its lower end with a screw to carry the latter. This adapter has an aperture, CD, on one side, and it also has a plane semi-silvered mirror, B, set at an angle of 45° to the axis, to reflect the light from the lamp at A down through the objective. The lamp must have a zinc chimney with a small hole at A, across which one or two fine wires are stretched.

The distance from A to the mirror B must be made approximately equal to the distance from B to E, the focal plane of the eye-piece. Place the surface S to be tested on the stage, and adjust it so that its centre of curvature may be in line with the axis of the microscope. The light from A should now be reflected down by B and returned up the tube by S, so that a certain amount of light should be perceived in the eye-piece.

On racking the tube up or down, when the convergent cone of rays emerging from O has its vertex at the centre of curvature of S, each ray will strike the surface S normally and will be returned along its path, and the reflected cone will converge to foci both at A and E. Thus an image of the aperture A with its cross-wires will be seen. When this occurs the centre of curvature of the surface S is conjugate to A or E.

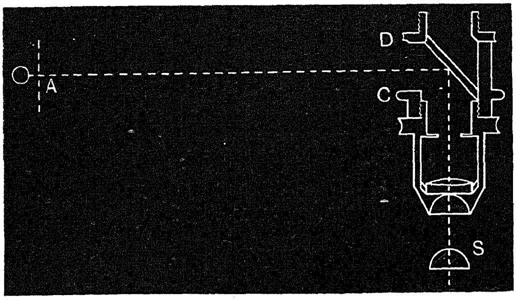


FIG. 90.—Radius of Curvature of a Small Mirror.

Scatter a few starch grains on the surface S and focus them. The surface itself is therefore now at the conjugate focus to E. The distance the tube has to be raised or lowered, to pass from the position in which the image of A is in focus to that in which the starch grains are so, is the radius of curvature of the surface; if it has to be *raised* it is convex, if *lowered* concave.

Ophthalmometers.

An ophthalmometer is essentially an instrument for measuring the curvature of a convex mirror of small radius. In § 59 it was proved that $f = -\frac{mu}{1+m}$; thus if the size of image produced by the mirror of an object of known dimensions at a known distance from the mirror can be found, f (or r) can be easily calculated. In measuring the curvature of the cornea of the eye, no method which depends upon the eye remaining stationary while two independent readings are taken is of any use; all ophthalmometers which have proved successful have obtained the size of the image by the method of doubling.

64. Let MN be a double-image prism, and suppose A and B two points on the image. Then, if A and B are viewed through the prism, each will appear double as AA′, BB′. If now the distance between the image AB and the prism MN is increased (either by moving the object, or the prism) so that A and B are now at C and D relatively to the

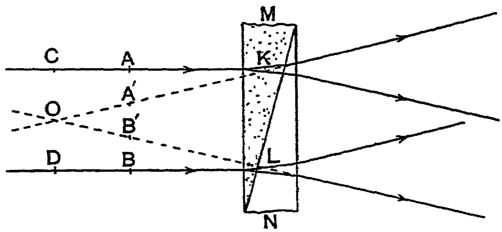


FIG. 91.—Doubling produced by Double-image prism.

prism, the other images will coincide at O. The angle AKA′ is a constant of the prism; thus if the distance CK is known, the distance CD (=2.CO) is also known. In this way the size of the image can be found, and the measurement is almost independent of a small motion of the eye, as this causes A and B to move together (it is affected slightly as the surface of the cornea is not a perfect sphere).

The doubling can be produced in a great number of ways, of which the following are some:

(*a*) Helmholtz used two thick plates through which the light passed obliquely, the plates being inclined in opposite directions.

(*b*) Hardy used a bi-plate (see p. 272).

(*c*) Two equal prisms placed bases or apices together.

(*d*) A Wollaston or a Rochon prism.

(*e*) Two similar de-centered lenses, each lens being cut across a chord, and either the two larger segments or the two smaller ones being put together to make one double lens.

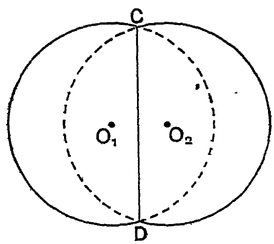


FIG. 92.—Doubling by Lens Segments.

The above all give fixed separations at a fixed distance. There are two giving a variable separation.

(*f*) A pair of rotating prisms, which revolve over one another in opposite directions, causing the image to move in a straight line. The rays forming the other image go by the side of them, or through a hole in them.

(*g*) Probably the best is that invented by Sutcliffe. It consists of a pair of weak cylinders of equal strength crossed (*i.e.* with their axes at right angles). Such a pair is equivalent to a simple spherical lens. Then if one of these cylinders is cut into three strips C, D, E, at right angles to its axis, and the middle one D is de-centered, it will produce a doubled image, of which the doubling can be varied at pleasure, since it depends upon the amount of the decentration.

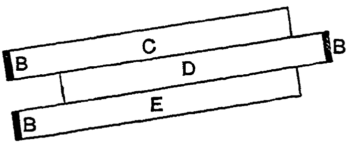


FIG. 93.—Cylinders C, D, E. The base or thick end is at B in each case.

65. One of these systems is put between the objective and eye-piece of a low-power microscope (magnifying about ten diameters), and thus the eye-piece views the doubled image. In the case of the systems (*a*) the plate or prism may be moved nearer or farther from the image to vary the doubling, and in every case the size of the image is to be varied until the selected points just coincide. The experiment is usually conducted in a darkened room, and the object is some illuminated design; Sutcliffe uses a circle with hooks. He splits *both* cylinders, and so is able to measure the curvatures in two meridians at right angles simultaneously. The appearance when adjustment is perfect is shown in Fig. 95. The curvature is marked on the scale.

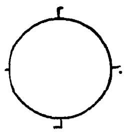


FIG. 94.—Mire.

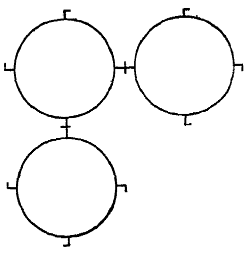


FIG. 95.—Adjusted image of Mire.

As an ophthalmometer is chiefly of use to obtain a *difference* of curvatures in two meridians, the absolute curvature is comparatively unimportant, and the graduations are, as a rule, more or less on an arbitrary scale. For accurate work it would have to be calibrated. To do this it should (*a*) be pointed at a millimetre glass scale (which will of course appear double). Then it should be adjusted so that the overlapping is exactly 1, 2, 3 mm., etc., the corresponding readings on the scale taken, and a curve plotted, connecting the size of image and the scale readings. (*b*) The actual size of the object (or *mire* as it is called) must be measured. Then if (*c*) the mire

is set up at a known distance from the mirror of which the radius is required, and (*d*) the size of the image it forms is measured (using the calibration curve previously plotted), the actual magnification can be found. From it the radius of curvature, r, is obtained, since $r = 2f = \frac{-2mu}{1+m}$ [$= -2mu$ nearly if m is small], where m is the magnification of the mirror under test, r is a small positive quantity. f is of course negative.

The Magnification produced by a Lens.

66. *Apparatus.*—In place of the ordinary cross-wire screen we shall require one with two holes, one vertically above the other, about 1 inch apart, each with a single horizontal cross-wire (Fig. 52).

Adjust the lens so that its axis shall be on a line normal to the screen and cutting the screen at a point halfway between the two cross-wires, and form an image of these magnified successively to 1 inch, 2 and 3 inches on the screen. With a millimetre scale measure in each case the exact distance apart of the images of the cross-wires on the screen. Find the distances from the lens to the cross-wires, and to the screen, correcting for index error as usual. Measure also the actual distance between the cross-wires.

Enter the results thus :

Position of			u	v	Ratio. $\frac{v}{u}$	Size of		
Cross-wire.	Lens.	Screen.				Object.	Image.	Ratio.

Loss of Power of a Lens in Water.

67. *Apparatus.*—A convex lens about 10 or 12 inches focal length and a trough with parallel sides large enough to contain it; the optical bench.

Determine the focal length of the lens in air and then place it in the trough, seeing that the lens is vertical. Fill the trough with the liquid and again determine the focal length of the lens, using the mirror method (§ 43). As, in this method, the light on the mirror side of the lens is parallel, if the lens is placed close to one side of the trough and the mirror behind the other side, the thickness of the trough will have no appreciable effect on this determination, and we may assume the focal length to be the distance from the lens to the screen. The focal length of the lens in the water is given by the formula:

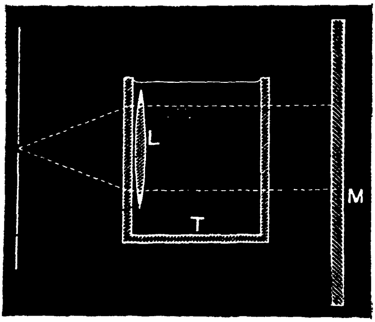


FIG. 96.—Power of a Lens immersed in a Liquid. L, lens; T, trough; M, mirror.

$$\frac{1}{f}=(\mu_w-\mu)\left(\frac{1}{r}-\frac{1}{s}\right).$$

The focal length in air is given by:

$$\frac{1}{f1}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right).$$

So that if the refractive index, μ, of the lens is known, that of the water, μ_w, can be found.

Refraction at Spherical Surfaces.

68. *Apparatus.*—A globular flask (such as is used for the preparation of oxygen or for vapour density determinations); a support for the flask; a screen with a hole about $\frac{3}{16}''$ in diameter furnished with a cross-wire; a white receiving screen; gas flame; meter scale; calipers; black paper, scissors and water.

Take the diameter ($2a$) of the spherical body of the flask; fill it with water, and support it with its centre on a level with the cross-wire. Set the cross-wire up near the surface of the flask, and the flame beyond it; let the light, after passing through the

hole in the screen, pass along a diameter of the flask, and receive it on the white screen. Cut a circular hole about $\frac{3}{4}''$ diameter in a piece of black paper, and attach the paper to the flask so that it may act as a "stop" to limit the aperture of the light emerging from the flask. The receiving screen can now be so adjusted that an image of the cross-wire is focussed upon it. Measure the distance of the cross-wire from the surface of the flask, add to it the radius of the flask a (already measured), and so obtain the distance of the cross-wire from the centre of the flask p, measured towards the incident light. In the same way measure the distance from the flask to the receiving screen, and add the radius to it to obtain its distance from the centre of the flask. As, however, it is measured away from the incident light it must be called a negative quantity $(-q)$.

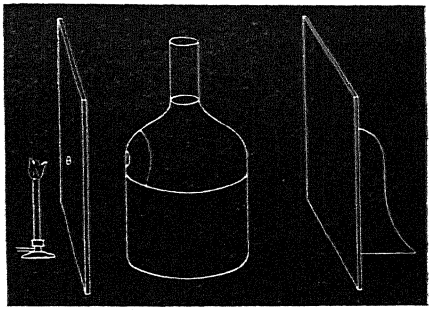


FIG. 97.—Refraction through Spherical Globe of Liquid.

Then the refractive index of the water (μ) is obtained from the formula:

$$\frac{1}{p}-\frac{1}{q}=\frac{2}{a}\left(1-\frac{1}{\mu}\right).$$

For if O is the centre of a sphere filled with liquid of refractive index μ, and P be a bright point, the image Q_1 of P is given by

$$\frac{\mu}{p}-\frac{1}{q_1}=\frac{\mu-1}{a}, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

where $p=OP$, $q_1=OQ_1$, and a is the radius of the sphere.

When the light emerges from the second surface, substitute $\frac{1}{\mu}$ for μ, $-a$ for a, and q_1 for p in order to find the position of the final image Q, and

$$\frac{\frac{1}{\mu}}{q_1}-\frac{1}{q}=\frac{\frac{1}{\mu}-1}{-a},$$

or

$$\frac{1}{q_1}-\frac{\mu}{q}=\frac{\mu-1}{a}, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

adding (i) and (ii) and dividing by μ,

$$\frac{1}{p}-\frac{1}{q}=\frac{2}{a}\left(1-\frac{1}{\mu}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

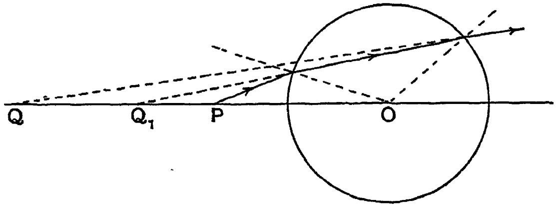


FIG. 98.—Refraction through a Sphere.

Repeat with other distances, and tabulate results thus :

Radius of Sphere.	Distance of Object		Distance of Screen		$\frac{1}{p}$*	$-\frac{1}{-q}\overset{*}{=}\frac{1}{q}$	Sum of last two Columns.
	From surface.	From centre $=p$.	From surface.	From centre $=q$.			
							Sum Mean value of $\frac{1}{p}-\frac{1}{q}=$

$$\therefore \quad 1-\frac{1}{\mu}=\frac{a}{2}\left(\text{mean value of } \frac{1}{p}-\frac{1}{q}\right)=$$

$$\frac{1}{\mu}= \qquad \therefore \ \mu= \ *$$

ADDITIONAL EXERCISES ON CHAPTER III

1. Arrange a convex lens and a plane mirror to form an inverte image of the cross-wire on the screen the same size as the origina.. See that rotating the mirror moves this image ; also that the distance of the mirror from the lens is immaterial.

2. Place the plane mirror at a little more than four times the focal length of a convex lens from the screen. Interpose the convex lens.

* From table of reciprocals, p. 510.

A bright patch will be seen on the screen. Focus this by adjusting the distance of the lens from the screen. It will be found, that when in focus, the image coincides exactly with the original cross-wire, and therefore becomes invisible. Explain the formation of this image.

Breathe upon the mirror, and see that the cross-wire is focussed on the surface of the mirror.

3. See that there is a second position of the lens in which the patch again comes in focus, and that this occurs when the cross-wire is again focussed on the mirror.

4. Move the lens a little so that the image is thrown a little out of focus, and is not quite coincident with the original; by passing a knitting needle over the cross-wire, see that the image of the needle moves in the same direction, and therefore that the image is erect.

5. Slightly rotate the mirror, and see that the image does not move. Explain this with the aid of a diagram.

6. Pair a convex and a concave lens as described on p. 47, Ex. 5. Determine the focal length of the convex lens by the mirror method. (Experiment, § 43.) The focal length of the concave lens will of course be numerically the same. Measure it by one of the methods described in this chapter, and compare your results.

7. Place a convex lens sufficiently far from the screen to form a real image. Place a concave mirror in a line with the centres of the cross-wire and lens, but beyond this image. If the mirror receives the light directly, a bright patch will appear on the screen. Adjust the distance of the mirror until this patch comes into focus. It will be found that when this occurs, the image coincides with the original cross-wire, moving the lens either nearer or farther, causing it to separate on the screen.

8. See if tilting the mirror alters the position. The distance from the mirror to the image is of course the radius of curvature of the mirror.

9. Place a convex lens and a concave mirror in contact. Adjust them to form an image of the cross-wire by the side of the cross-wire. The light striking the concave mirror must now be returning along its path, and therefore it is striking it normally, and must be coming from its centre of curvature. Call the distance from the lens to the screen u. Measure the focal length of the lens, and determine the conjugate focus of the cross-wire by substituting in the formula. The value of v so determined will be the radius of curvature of the concave mirror. It can then be determined directly as in Experiment, § 52, and the results compared.

10. Combine a weak concave lens with a concave mirror, and form an image of the cross-wire by the cross-wire itself, as in the last experiment. Measure the distance from the screen to the lens, and the radius of curvature of the concave mirror by Experiment, § 52. Calculate the focal length of the concave lens by substituting these for u and v respectively in the formula.

11. On the graph of Experiment, § 42, set off any distance OA along the line OX, and double that distance OB along the line OY. Join the points A, B, and draw through F a line parallel to AB cutting the axes OX and OY in C and D respectively. Then OC and OD represent the distances from the lens to the cross-wire and receiving screen, at which the image will be twice the size of the object. Adjust the lens and screen to these distances, and test your result.

12. Adjust a concave mirror, the screen with two cross-wires, and the receiving screen to form a real image, as in Fig. 49; and hence measure the magnification produced by a concave mirror. Enter the results as in Experiment, § 66.

13. Repeat Experiment, § 58, substituting two knitting needles side by side at about 1 inch apart for the single one P, and adjust two needles for the one at P′, to coincide with the two images of the knitting needles. Measure the distances apart, both of the needles P, and of the needles P′; use them as the sizes of the object and image, and enter the results as in Experiment, § 66.

14. Set up a convergent meniscus lens with its convex side facing the cross-wire. As the lens is convergent, it will be possible to obtain an image, by the reflection from the further surface of the lens, upon the cross-wire screen close to the cross-wire, as in Experiment, § 61. In this case, as the light is being normally reflected from a convex surface, the light which proceeds through the lens will converge to a real image, and may be received on a screen as usual. The distance from this screen to the lens will be the radius of curvature of the surface. Calculate it from the formula as in the experiment referred to, and compare the results.

CHAPTER IV

OPTICAL INSTRUMENTS

The Astronomical Telescope.

69. *Apparatus.*[1]—A long focus convex lens 15 to 30 inches focus; a short focus convex lens of 1 or 2 inches focus, which need only be half an inch in diameter; stands for same; knitting needle on stand; some printed matter at the far end of the room—for instance, a poster.

Set the long focus lens up to face the printing. Stand at some little distance and look into the lens. An inverted image of the printing will be seen. That this image is on the observer's side of the lens can be proved by moving the head, when the image will move across the lens in the opposite direction. Adjust the knitting needle to coincide with this image, by the method of parallax (p. 3). Lastly, place the short focus lens in a line with the other lens and knitting needle, and adjust its distance from the knitting needle so that the latter may be in focus when the eye is placed close against the lens. Then, as the knitting needle has been made to coincide with the image of the distant printing, this printing will be in focus also, and an enlarged and inverted image of a part of the poster will be seen.

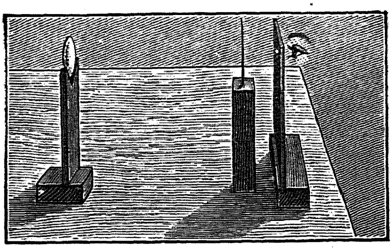


FIG. 99.—Principle of the Astronomical Telescope.

[1] The experiments in this chapter, § 69 to § 72, can be more easily performed with the apparatus of § 39. It is, however, a better exercise for the student to do them as here described.

Galileo's Telescope.

70. *Apparatus.*—A long focus convex lens; knitting needle on stand; a concave lens half an inch in diameter, and about 2 inches focus.

Adjust the knitting needle to coincide with the image formed by the 15-inch lens as before. Then insert the concave lens between the knitting needle and the long focus lens, at about its focal length from the needle. Remove the needle, place the eye close to the concave lens, and the image will be at once seen, and a slight adjustment of the lens will bring it into focus.

The Terrestrial Telescope.

71. *Apparatus.*—A long focus convex lens 15 to 30 inches focus; two short focus convex lenses of 1 or 2 inches focus and half an inch in diameter; stands for same; two knitting needles on stands; some printed matter at the far end of the room.

Adjust the knitting needle as before, to coincide with the image of the poster formed by the long focus lens. Place one of the short focus lenses at about double its focal length from this needle. It will form a real image of the knitting needle on the other side of the lens at about an equal distance. Adjust the second knitting needle to coincide with this image. Lastly, place the second short focus lens at such a distance from the second needle, that the needle may appear sharply in focus when the eye is placed close to the lens. Then, as the needles have in succession been adjusted to coincide with the images of the poster formed by the long focus lens and the first short focus lens, the image of the poster also should be in focus when observed through the second short focus lens. It will be found to be erect. To succeed in these adjustments the lenses must be all exactly in the same line, and care must be taken that the needles are made accurately to coincide with the images.

The Compound Microscope.

72. *Apparatus.*—Two convex lenses of about 1 or 2 inches focus, and half an inch in diameter; some small print on a stand; knitting needle.

Place one of the short focus lenses at about double its focal length from the print. A real inverted image will be formed. (If the image is not inverted, the lens is too close to the print, and the image will not be real.) Adjust the knitting needle to coincide with this image, and the second lens to view the knitting needle. An enlarged image of the print should be visible at the same time.

The magnification will depend upon the focal length of this second lens and upon the ratio of the distances of the image

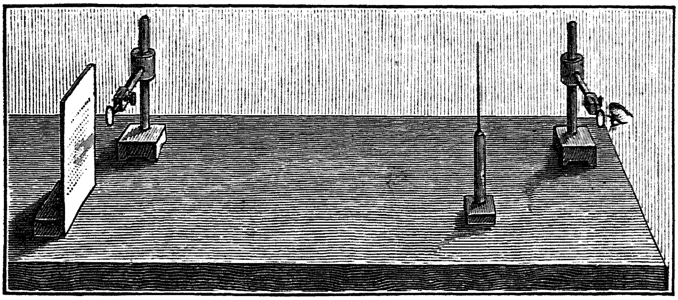


FIG. 100.—Principle of the Compound Microscope.

and object from the first lens. By bringing the print a little nearer to the first lens and withdrawing the second lens to a greater distance at the same time, the magnification can be increased; but with ordinary lenses the image becomes very distorted if the distance of the image is more than two or three times that of the object.

Optical Lantern.

73. *Apparatus.*—The optical bench with convex lenses of 10″ and 6″ focal length and about 2″ diameter; cross-wire and receiving screen; a lantern slide—preferably one containing black lines and clear spaces, for instance, a diagram; elastic bands; cardboard.

Put the 10″ lens, L, at about the middle of the optical bench, and with the receiving screen, S, at the far end find the position at which the cross-wire screen must be placed so that its image

is focussed on the screen S. Clip the lantern slide, B, against the face of the 6″ lens, C, with rubber bands, and replace the cross-wire screen by this lens and slide, so that the slide, B, may be in such a position that its image will be focussed on the screen S. Hold a piece of card, or paper, against the surface of the lens L and adjust the cross-wire screen at such a distance behind the lens C that the cross-wire is focussed at L.

On removing the paper a clear image of the lantern slide should appear on the screen S, and should be evenly illuminated.

To see the necessity for focussing the cross-wire screen on the lens L, cut a hole about $\frac{1}{4}''$ in diameter in a card and clip the

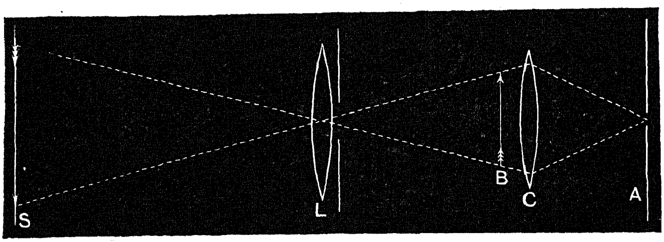


FIG. 101.—Path of Rays through the Optical Lantern.

card with a rubber band against the surface of the lens L, so that the hole in the card may be against the centre of the lens.

It will now be found that if the screen A be moved a little forward, the edges of the image on the screen S will become dark; whilst if the screen A be moved back to a greater distance from C, a shadow will form in the centre of the image on S. There is only one position for A in which neither of these shadows occur, and a very small movement either nearer or farther will cause them. As in the ordinary optical lantern the lens C has a much larger diameter, namely about 4 or $4\frac{1}{2}$ inches, this is even more noticeable than when the lens has a diameter of only 2″.

If another card be cut having a hole only about $\frac{1}{8}''$ in diameter, it will probably be found impossible to avoid shadows for any position of the screen A. This is due to the fact that the lens C does not converge the light from A again accurately to a

point, so that it cannot be made to go through a hole as small as that mentioned. And with the 4″ lens at C, or *Condenser* as it is called, this fault is still more noticeable; unless the lens L has a large aperture, it is sometimes very difficult to avoid these shadows. When a convex lens cannot be made to converge the light from a point exactly to a point again, the inability is said to be due to *Spherical aberration*.

The Magnifying Power of a Telescope.

Apparatus.—Telescope; metre scale; a pair of dividers.

74. **First Method.**—Place a metre or 2-metre scale at one end of a room. From the other end observe it through the telescope, and adjust the latter until the divisions on the scale are clearly focussed. Now open both eyes, and whilst looking with one eye through the telescope at these divisions, with the other eye look at the scale itself.

A difficulty will often be found in seeing the scale with the one eye, and the divisions with the other, clearly focussed at the same time. This may be partly due to a difference in illumination, if the objective aperture is too small for its magnifying power. But it is probably due chiefly to the accommodation of the eye.

When observing the divisions in the telescope with the one eye closed, it was possible to see them clearly when the image formed by the telescope was at any distance within the range of vision, that is for a normal-sighted person anywhere between 8 inches and infinity, and the telescope may therefore have been adjusted so that the image of the scale was formed at a short distance. In order to see this clearly, therefore, the eye had to be accommodated to that short distance. When on opening the other eye the attempt was made to see the scale itself at the far end of the room, and its image at a short distance, at the same time, it is obvious that the attempt must fail, as we cannot accommodate one eye to a great distance and the other to a near distance simultaneously. But if, while observing the scale in the telescope, we imagine it to be at the end of the room, and focus the telescope with this idea clearly in our minds (especially if at

the same time we open the other eye, and so fix the accommodation of the eye to that distance), it will be possible to adjust the focus so that the image shall be actually formed at that distance.

The coincidence may be tested by slightly moving the head, still keeping both eyes open. If the image is now really formed at the far end of the room, a motion of the head will not displace it relatively to the objects situated at that distance, and the focus should be adjusted until this is the case.

If now the image of the divisions of the scale do not quite coincide with the position of the scale as seen by the other eye, the direction of the telescope should be adjusted until they do coincide, and the number of divisions seen in the telescope which correspond to the whole length of the scale must be determined.

The magnification is then obviously the length of the scale divided by the number of divisions it subtends in its image.

75. **Second Method.**—Focus the telescope on infinity, then pointing it at the sun, receive the emergent light on a piece of paper placed at a short distance from the eye-piece. If the telescope has been focussed at infinity, the light will emerge as a parallel bundle of rays. If a piece of ground glass be placed to cut the emergent bundle of light, there will be a bright circular patch of light on the glass. This is called the "Ramsden Circle."

Measure the diameter of this bundle of rays and the diameter of the objective.

The magnification will be the ratio of the one to the other.

Or, place a diaphragm over the objective, pierced with two small holes on a diameter at a known distance apart, d_1. Two bundles will emerge from the eye-piece which correspond to these holes. Measure on a good glass scale their distance apart, d_2. The ratio of the distance apart of the holes themselves to the distance apart of the emergent bundles will be the magnification

$$m = \frac{d_1}{d_2}.$$

Caution.—On no account observe the sun directly through the telescope.

The Tangent Lens Gauge.

A very simple instrument for measuring the radius of curvature of a convex surface described by Mr. C. J. Birch, *Phil. Mag.*, 1897.

76. *Apparatus.*—The whole instrument can be made from patent plate glass. On to a strip of about 6 inches long and

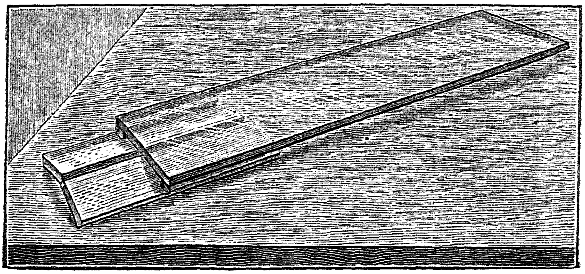


FIG. 102.—Tangent Lens Gauge.

$1\frac{1}{2}$ inches broad at one end are attached, with marine glue, two other strips each about 2 inches long and $\frac{3}{4}$ inch broad, making a small angle with one another. The instrument is shown in Fig. 102. The end view figure (Fig. 103) shows the way in which the glasses are to be tilted to one another. The two inclined plates will touch the lens at two points, and a small system of Newton's rings will be formed at each point of contact.

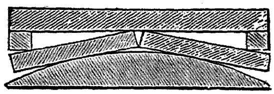


FIG. 103.—Section.

The apparent distance between these points of contact is to be measured with a reading microscope. This will be the distance *a*. The true distance between these points will be slightly less than *a* owing to the refraction through the glass. The amount *e* to be subtracted will, however, be a constant, which can be measured once for all by measuring the *apparent* distance between two needle points, seen through the glass plates, and their *true* distance apart when the glass is removed. Subtracting this constant amount from *a*, we obtain the true distance, *b*, between the two Newton's ring systems. As the angle, θ (Fig. 104),

between the glasses is a constant, if R is the radius of curvature of the surface:

$$b = 2R \,.\, \sin\frac{\theta}{2}.$$

and therefore $$R = \frac{b}{2\sin\frac{\theta}{2}},$$

or putting A for $\frac{1}{2\sin\frac{\theta}{2}}$, which is a constant of the instrument, we may write $R = A\,(a-e)$.

The angle θ may be found either by means of a surface of known radius of curvature, or by setting the instrument up

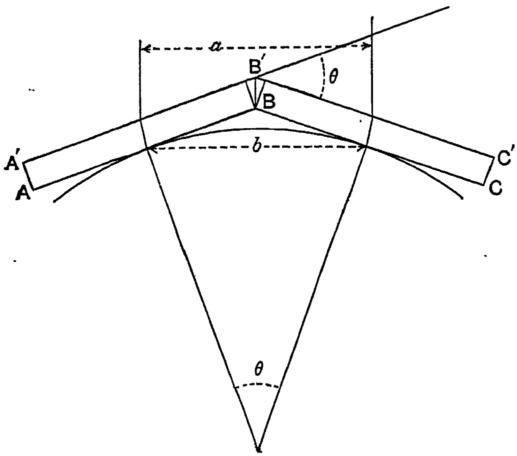


FIG. 104.—Tangent Lens Gauge.

vertically on the spectrometer table, and reflecting the light from the surfaces AB and BC in turn. The angle the telescope of the spectrometer has to be moved through will be 2θ. To be certain that the reflection takes place from the front of the surface AB, and not at the back surface A′B′, the latter may be breathed on to destroy the image reflected from it.

If the angle is so chosen that a is fairly large, it may be measured with an ordinary glass scale with sufficient accuracy,

and then the instrument becomes a very simple one and will give better results than the ordinary cheap spherometers.

The Sextant.

Apparatus.—A sextant, artificial horizon, and metre tape.

The artificial horizon may be a plane mirror or piece of black glass, and if so, it must be mounted on levelling screws, and a level will be needed as well. A mercury bath may be used in place of the mirror and level, if the observations are being made in a place free from vibration; in a room in which other students are working, or in a town, a mercury bath is practically useless.

77. **Adjustments.**—Assuming the circle to be accurately divided, and to be concentric with the arm, there are still several adjustments.

(*a*) Each mirror must be perpendicular to the plane of the circle.

(*b*) They should be parallel to one another when the arm is at the zero division.

(*c*) The axis of the telescope should also be parallel to the plane of the instrument.

To see that the mirrors are perpendicular to the plane of the instrument, look obliquely in the mirror attached to the arm, at the image of the divided circle. If the plane of the mirror is perpendicular to the circle, this image will appear to be a continuation of the circle itself. If not, the image and the circle will be inclined one to the other.

Then, placing the pointer at zero, point the instrument at some very distant object, two images of this will usually be seen in the telescope at the same time, the one transmitted directly through the unsilvered half of the small mirror, and the other formed after two reflections, one from each mirror. If the mirrors are perfectly parallel, these two images will coincide. If they do not, see if they may be made to coincide by a slight movement of the arm, and if this is sufficient to produce coincidence, take the reading and treat it as an index error. If the reading is very much wrong, the adjusting screws of the smaller mirror may be altered. There are usually two of these; one is the end of a tangent screw which rotates the

mirror on an axis perpendicular to the plane of the circle, and this screw will adjust the index error. To adjust it, set the arm at zero by the scale, and turn this screw until the images coincide. If the instrument is held with its plane vertical, a movement of this screw should move the images relatively to one another in a vertical direction. If, while the plane of the instrument is held vertical, the images are separated from one another horizontally, the adjustment must be made by the screw at the back of this mirror, which alters its inclination to the plane of the instrument. The student should never interfere, however, with either of these adjustments; he should be content with reading the index error.

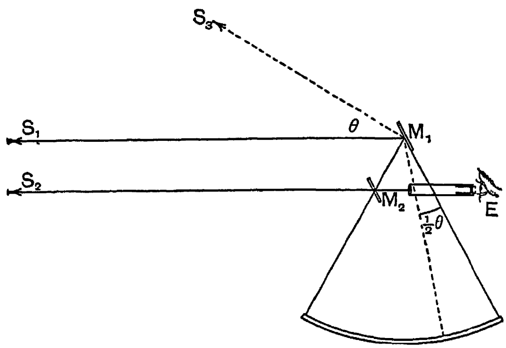


FIG. 105.—Angle between Two Distant Objects.

78. **Graduation.**—Let M_1 and M_2 be the movable and fixed mirrors respectively, and E the eye-piece of the telescope; then when M_1 and M_2 are parallel to one another, it is obvious that parallel rays S_1M_1 and S_2M_2 incident on the mirrors M_1 and M_2, respectively, will, after reflection, coincide in the ray M_2E. If M_1 be turned through any angle, the ray S_1M_1 will be turned through twice that angle; thus the ray from an object S_3 making an angle θ with the rays from S_1 will be brought into coincidence with the ray from S_1 when the mirror has been turned through an angle $\frac{1}{2}\theta$. In order to read the angle θ directly, the graduations on the arm are always marked in *half* degrees; thus the angle which is really only 60° is numbered 120°. The readings on the scale give the angle θ directly.

In finding the zero of the instrument, it is necessary that the rays S_1M_1 and S_2M_2 shall really be parallel, and it is useless, therefore, to attempt to adjust the instrument with an object that is not a very great distance—even a distance of 200 yards is insufficient.

The instrument may, however, be used to find the angle between two objects, S_2 and S_3, if that angle is fairly large, without appreciable error; for it is obvious from the figure, that the angle found is the angle subtended at O, the point of intersection of the lines S_3M_1 and S_2M_2, and therefore, that if the angle θ is of any appreciable size, the position of this point, O, is practically that of the observer. But when S_3 is very near S_2, this point may be a considerable distance behind the observer.

79. Determination of the Angular Elevation of a Point.

Let P be a point and HL the artificial horizon. With a sextant we can measure the angle between the lines PO and OQ. But—

$$\angle POQ = \angle POK + \angle KOQ = \angle POK + \angle PQL.$$

If P be sufficiently distant in comparison with the length of the line OQ, these angles may be supposed equal, and we may assume that the angle POQ is twice the angle POK. If the object is not at a very great distance, the sextant should be used as near the horizon glass as possible, so that the distance OQ may be kept small.

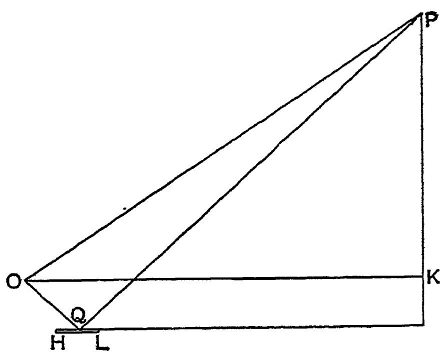


FIG. 106.—Measurement of Altitude.

We have first to level the horizon glass. If this is mounted on three levelling screws as usual, the level should first be placed parallel to the line joining two of these screws, A, B (Fig. 107) and the horizon glass adjusted until the bubble is in the centre. The level should then be turned through 180° to see if the bubble is still in the centre. If not, the level is not perfect, and the new

position of the bubble should be observed. The correct position of the bubble will probably be half way between this latter one and the former, and the horizon glass should be adjusted until the bubble takes this position. Again turn the level and see whether the bubble now is in the same place. If not, note the mean position, and adjust the horizon glass to get the bubble there. In this way, with a few trials, even if the level is not perfect, the horizon glass can be adjusted so that the bubble is in the same place in its two positions.

Now place the level at right angles to its previous position, and adjust the third levelling screw C until the bubble comes to the place which has already been found to be its correct position. Reverse the level, and see if it remains in the same place. Once more place the level parallel to the line joining the first levelling screws, A, B, and see whether it is still correct; generally a slight adjustment will be necessary, and if so, the third levelling screw, C, must also again be adjusted.

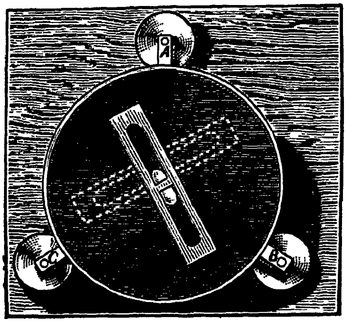


FIG. 107.—Artificial Horizon.

The bubble should now remain in the same place in the level however the latter is turned. If it is intended to use a silvered mirror instead of a black glass one, as the reflection will take place at the back surface of the mirror, either a piece of mirror whose front and back surfaces are parallel must be selected as described already; or the mirror must be placed in such a position that the series of images formed by it at grazing incidence (p. 36) lie in a direction at right angles to the plane in which the observations with the sextant are to be made, for the want of parallelism will then have the least effect. If now the upper surface be levelled, the lower surface, although not level, will yet be parallel to the horizon in the plane of incidence.

Holding the sextant in the right hand, with its plane vertical, point it down so that the telescope is directed to the image in

the horizon glass of the distant point the altitude of which is to be determined, and rotate the movable arm until the image formed after the two reflections in the mirrors of the sextant is brought to coincide with the image formed by the horizon glass. If the horizon glass be only a black mirror, its image will probably be very much fainter than the one formed by the sextant mirrors, and it may be necessary to place one of the dark glasses in the path of the rays forming the latter. If the axis of the telescope can be moved further from the plane of the instrument so as to receive a greater proportion of the light through the unsilvered half of the small mirror, this will be a better way of equalising the two images. When the coincidence is perfect, the angle can be read upon the scale. To this must be added the zero error of the instrument, if there is one, with its proper sign, and the sum will give the angle POQ. The altitude required is half this.

It is a good exercise to find the altitude of two points vertically above one another, and also to find the angular distance between the two points; the latter should equal the difference between the two altitudes, and thus the observations can be checked against one another.

Optical Lever.

80. *Apparatus.*—This is an instrument for measuring the thickness of thin slabs, such as a piece of micro-covered glass. It consists

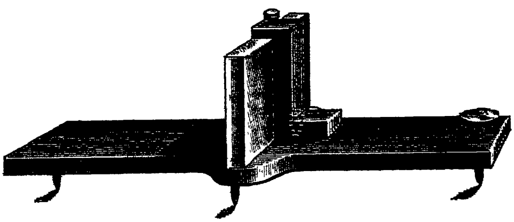


FIG. 108.—Optical Lever.

of a bar of brass, of any convenient size, say about 6 in. long and 1 in. wide, standing upon four points. Two of these are placed cross-wise about the middle of its length, and there is one at each end. One of the latter may be the point of a screw, so that the

amount it projects can be varied to suit the thickness of the slab that is to be measured. Mounted cross-wise in a vertical plane on the top of the bar is a plain mirror.

A metre scale; a telescope on stand; and a flat surface upon which the lever shall stand will be required (a piece of plate glass does fairly well, but it is necessary to mark on the glass the position upon which the points shall rest, so that it may always rock upon the same surface, and then any slight curvature of the glass will not affect the result).

An optical lever about a quarter of the size above given is convenient for lens measurement.

At some known distance, say a metre, the scale is put up vertically facing the mirror, with the telescope mounted horizontally by the side of it also facing the mirror, and adjusted so that the image of the scale formed by the mirror shall be in focus. A small weight, say 10 grms., is used to put alternately on the one end or the other of the bar, so that it may rest at one time on the two centre points and the front one, or, by putting the weight at the opposite end, upon the two centre points and the back one.

The scale readings are taken with each position of the weight in order to determine the amount the mirror rocks upon the bare surface. Then the slab, the thickness of which is required, is inserted under one of the end points, and the amount that the mirror rocks again determined by means of the scale readings.

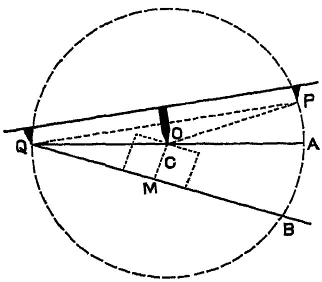


FIG. 109.—Optical Lever.

Theory.—If the thickness e of the glass C (Fig. 109) to be measured is very small the following approximation will give sufficiently accurate results. Calling the angle AQB between the position of the lever when the front point is on the slab and when the glass is interposed, θ; θ, is given by:

$$\sin\theta = \frac{e}{a}, \text{ or } \theta = \frac{e}{a} \text{ nearly,}$$

where a is the distance QO from the line joining the two central points to the front point Q.

The reflected light is always turned through twice this angle; thus if d is the distance from the mirror to the scale, l the apparent movement of the scale as seen in the telescope:

$$\frac{l}{d}=\tan 2\theta=2\theta \text{ nearly}.$$

Thus, $$e=a\theta=\frac{al}{2d}.$$

Mirrors of Fixed Deviation.

81. When light is reflected successively by two mirrors inclined at an angle to one another, the final deviation depends only upon the inclination of the mirrors to each other and is independent of the angle of incidence of the light upon the first mirror; also if the angle between the mirrors is kept constant, the mirrors may be rocked about their line of intersection, and if the plane of the incidence of the light is normal to this line, the direction of the reflected beam will not be affected by the motion.

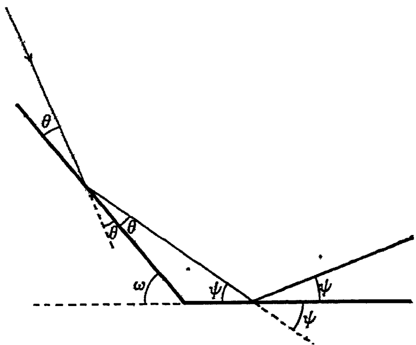


FIG. 110.—Successive Reflection at Two Plane Mirrors.

For if θ be the angle the ray makes with the surface at incidence on the first mirror it is obvious that the light is rotated counter-clockwise through an angle 2θ.

Also, if ϕ be the angle the ray makes with the surface at incidence on the second mirror the light is further turned counter-clockwise through an angle 2ϕ. Thus the total amount the light is deviated from its original direction is $2\theta+2\phi$.

Let ω be the angle between the plane of the mirrors. Then, ω is the external angle of a triangle of which θ and ϕ are the internal opposite angles, and therefore $\omega=\theta+\phi$. Therefore the light is turned through an angle 2ω.

To ensure that the angles between the mirrors shall really be constant, the mirrors may be the surfaces of such a prism as shown in Fig. 111. In the latter case, to avoid unequal deviation

produced by the refraction in and out of the prism, the surfaces must be symmetrically inclined to the light, and then it will be easily seen that the effect of dispersion will disappear.

If the two mirrors are parallel to one another, and the light is reflected from them successively, they may be rotated in any direction, not only about a normal to the plane of incidence, but may also be turned obliquely to that plane; the light will still emerge parallel to the incident light, and the image of a distant object will not be affected by any motion of the mirrors.[1]

To bend the light through an obtuse angle or a right angle the light paths should be allowed to cross as in the figure. It is quite easy to design a prism to produce any deviation required, for draw the lines AO and OB to represent the direction of the incident and the desired beam, produce the lines to any two points C and D, so that OC = OD. Join CD. Then the path of light should be ACDB obviously. Therefore, the mirrors must be normal to the bisectors of the angles at C and D, and the entrant and emergent faces of the prism may also most conveniently be normal to the rays AC and BD. The prism can be completed by an unpolished surface QR.

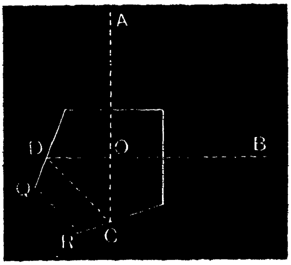


FIG. 111.—Prism turning a Ray of Light at a Right Angle.

82. These prisms have been extensively used in range finders. For instance, Major Forbes has made use of this device in the construction of a portable range finder, which can be folded up without risk of making it inaccurate. In this instrument the light from a distant object falls upon two of these prisms, A and B, which are some six feet apart. Each is constructed to bend the light through a right angle, and placed so that the emergent light is bent inwards. Midway between them are two other prisms C, D, placed to reflect the light again through a right angle, along lines parallel to their original directions and about $2\frac{1}{4}''$ apart. These

[1] The emergent ray will be parallel to, but not in the same line as the incident ray; thus the image of a *near* object is not coincident with the object, but will be a little to one side of it; it is on a line through the object, normal to the mirrors, at a distance from the object equal to twice the distance apart of the mirrors.

beams enter the objective of an opera glass. Thus the observer looking through the opera glass sees the distant object with an increased stereoscopic effect, in fact the same effect as would be produced if his eyes could be put six feet apart. In each

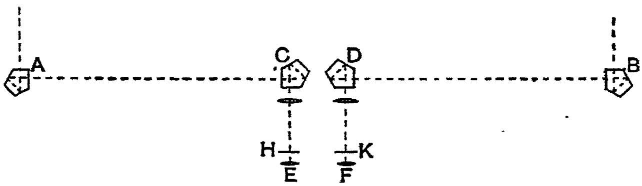


FIG. 112.—Range Finder.

eye-piece of the opera glass there is a photograph of a balloon with a trailing rope, one of which, K, is mounted with a screw and may be moved transversely, and this transverse motion gives it the appearance, owing to the stereoscopic relief of moving to a greater or less distance from the observer. The screw is turned until the balloon appears immediately over the object of which the distance is required. The screw is graduated so that its readings give the distance directly.

The same principle underlies the theory of the sextant. If two distant objects are brought into coincidence (the telescope being removed), it will be found that the instrument may be rocked to and fro in its own plane without destroying the coincidence.

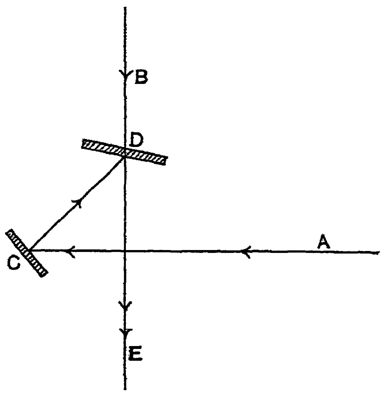


FIG. 113.—Optical Square.

Also in the "optical square" this principle is employed. Light from a distant object entering along AC (Fig. 113), as before, after two reflections at C and D respectively, emerges along DE, and light from an object entering along BD goes straight through. Thus an eye at E can view both objects superposed, if D is semi-silvered, or half-silvered. By providing C with a graduated screw to alter its inclination, angles that are nearly a right angle can be measured.

Selection of an Optically Flat Micro-cover Glass, or other Unsilvered Glass Surface.

83. Arrange an optically flat glass plate (such as the surface of a prism or cube, or even a piece of plate glass) in front of a sodium flame. See that it is clean and free from dust. Clean the cover glasses, and drop them on the surface one by one. The black and yellow Newton's fringes will be visible in the light reflected from the surface, and will generally form a pattern such as a hyperbola, or an ellipse, owing to the curvature of the glasses. A few glasses will have nearly parallel or circular fringes; these should be laid aside.

If also a thin one is desired, it can be easily sorted out from the others by Boys' method of *ringing* the glasses on the table, when, if they are all of the same diameter, the thinner ones will give a lower note than the thick ones.

To Test the Optical Flatness of a Plane Mirror Silvered on the Face.

84. *Apparatus.*—A good collimator and a telescope. The collimator, in place of the usual slit, should be furnished with a glass plate which has been silvered and had the silver removed in some definite manner, say a cross and circle. If the collimator and telescope of the spectrometer are sufficiently good they may be used provided the slit can be removed, but they generally have too small an aperture. A pair of good telescope objectives of about $2\frac{1}{2}''$ diameter and $30''$ focus would usually be better. If these are mounted at the ends of boxes about $12''$ square inside and of the necessary length, they will be useful for many purposes. They are far better thus mounted than in tubes; for when mounted in tubes there is always a large amount of light reflected from the walls of the tubes which no black will entirely eliminate, and this light is practically absent with the boxes suggested. The boxes can also be made at a very small cost. If the boxes are both the same size they can lie on the table, and whether it is

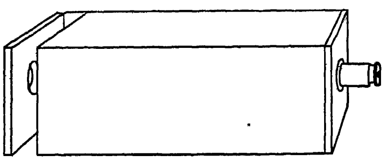


FIG. 114.—Mounting of Collimator and Telescope.

desired to point them at one another or to set them obliquely, no other support will be necessary. The one to be used as a telescope should have at the eye-piece end a board, which has a tube fastened in it, in which the eye-piece can slide to and fro for focussing; and this board should be screwed on, so that it may be easily removed, and replaced by a camera for photographic work. The collimator should have in the same way a tube, which ought not to be less than two inches in diameter, fixed in the centre of a board, in which the tube carrying the slit can be inserted, or in this case the tube carrying the silvered glass plate. For diffraction experiments another tube carrying a pin-hole will be required.

The mirror to be tested must be mounted on a temporary wooden stand that will enable it to be placed opposite the lens of the collimator with its plane vertical.

85. **Adjustment of Collimator and Telescope.**—Focus the telescope on a very distant object through an open window, using the method of parallax described on page 6. Point it at the collimator, and adjust the silvered glass so that its cross and circle, illuminated by a lamp or gas flame behind, shall be in perfect focus as seen through the telescope.

Unless the lenses are good so that the silvered glass seems quite sharply defined, the test cannot be properly made. If the definition is not good to begin with, the lenses may be examined to see if they have been screwed up too tightly in their mount, which would strain them and spoil their figure.

It is important to see that each lens is truly perpendicular to the line joining its centre with the centre of the eye-piece or the glass plate, as the case may be. Replace the glass plate in the collimator by a pin-hole, upon which a strong light is focussed, and observe the image of the pin-hole through the telescope; if it shows any flare to one side, looking something like a comet, it is probably because one or both of the lenses are out of truth. Remove the eye-piece and look at the series of images of the pin-hole formed by reflections at the surface of the collimator and telescope lenses. These should all lie in one straight line. They must be adjusted until the images are perfectly collinear. Instead of a pin-hole, the image in a bicycle ball or in the spherical bulb of a mercury thermometer, of a small bright light, such as that

from an incandescent gas flame emerging from a small circular aperture, may be used. When properly adjusted the image of the point of light should appear as a point surrounded by circular diffraction fringes. If these rings are imperfect, or if there is a flare on one side, the lens must be again adjusted until this disappears. If the axes of the lenses are not at the same level the definition cannot be made perfect.

Now turn the tubes so that their axes are at right angles, and adjust the mirror to reflect the light from the collimator into the telescope. If the mirror is flat, the image will be still perfectly sharp.

86. **Rayleigh's Method.**—An unsilvered glass surface can be tested as follows: Thoroughly clean the surface with repeated washings with a solution of caustic soda and nitric acid, used alternately, rinsing with clean water between each. Set it up on levelling screws, with the surface to be tested upwards and horizontal. Pour upon it a very small quantity of clean water, and observe the reflections of a sodium flame from it. It will show "Newton's rings," and if the surface is optically flat these will be in straight lines except at the edges. If it is required to test the surface right up to the edges, it must be set up in a trough, and water poured in until the surface of the water is just above the surface to be examined. A very steady support is absolutely necessary for this experiment. In a town or near a railway, the trough can be placed on a stone slab supported upon tennis balls, or on a coil of rubber tubing, or suspended by ropes; this will reduce the tremors which would otherwise disturb the surface.

ADDITIONAL EXERCISES ON CHAPTER IV

1. Place a mirror on a drawing board as in Experiment 1, and insert pins P, Q, R, and S, as in Fig. 1, to appear in a line. Turn the mirror AB through an angle of about 30°, and insert two more pins, R′, S′, to appear again in a line with the pins P, Q. Measure the angle the mirror has been rotated. Measure also the angle between the line joining the pins R, S, and the line joining the pins R′, S′. See that the latter angle is double the angle the mirror is turned through.

2. Make a mirror about 2 inches in diameter with fusible metal, as described in Experiment, § 60, with a central hole of about $\frac{1}{2}$ inch.

in diameter, and of from 10 to 20 inches radius of curvature. Make also a concave and a convex mirror of about $\frac{1}{2}$ inch in diameter, and 3 or 4 inches radius of curvature. (These can be cast on ordinary spectacle lenses.)

Set up the larger mirror to form an image of a distant object (a candle flame) on a screen, and so determine the position of the image. Place the concave mirror at a little more than its focal length beyond this image, and adjust it to form an image in the hole in the larger mirror. Examine the latter with an eye-piece.

This forms a model of Gregory's telescope.

3. Repeat the adjustments, replacing the small concave by the small convex mirror, and of course placing the latter mirror at a little less than its focal length in front of the image formed by the larger mirror.

This forms a model of Cassegrain's telescope.

4. Adjust a short focus convex lens (about 6 inches) and a plane mirror to form an image of the cross-wire, as in Experiment, § 43. Rotate the mirror through angles of 5, 10, 15, 20, etc. degrees, and measure the displacement of the image in each position of the mirror. See if they are proportional to the rotation of the mirror.

5. Replace the combination of convex lens and plane mirror by a concave mirror, and repeat.

Plot the results of both Exercises 4 and 5 on squared paper. Also plot the tangents of half the angle the mirror was turned through (from the table on p. 514); *i.e.* of the angles $2\frac{1}{2}^\circ$, 5°, $7\frac{1}{2}^\circ$, 10°. See that the displacements with the concave mirror are proportional to these tangents.

6. Find the magnifying power of the microscope by comparing the image of a scale seen through the microscope with another scale seen directly at the same time with the other eye, the second scale being placed at a distance of about 10 inches. Of course, in making this comparison, the microscope must be so focussed that the image of the scale seen through it is formed at a distance of 10 inches, and not at infinity; to see if this is so, move the head slightly to and fro sideways—the image should not be displaced relatively to the scale seen with the other eye.

7. Clamp a rectangular plate-glass mirror on a stout board in such a way as to bend its surface, one pair of opposite corners being pressed down, while the other corners are wedged up slightly. Set it up facing a window at a distance of some ten feet. Observe at a distance of two or three feet the image of the window. Note and explain the distortion produced.

8. Hold a piece of window glass at arm's length so as to form, by oblique reflection, an image of a lamp-shade. Observe the appearance of the image. Repeat with a piece of plate-glass, and a piece of optically worked glass. This gives a ready method of roughly testing the quality of a surface.

CHAPTER V

DEVIATION AND DISPERSION OF LIGHT

Spectroscope.

Apparatus.—Two glass prisms;[1] three convex lenses of 6, 10, and 15 inches focus respectively; stands for same; metal screen with a slit in it about 1 mm. broad and 1 or 2 cms. long; metal screen with circular hole 2 mm. diameter; screen to receive the spectrum; gas flame; a piece of black card; and blocks. If the stands for the lenses are not adjustable, the blocks must be of such a size that the centres of the lenses, prism, slit, and the gas flame shall be all one height.

87. **To Produce a Spectrum. First Method.**—Place the flame behind the slit, and the 10-inch lens at about 12 or 13 inches distant from the slit. Adjust the screen to receive a sharp image of the slit. Interpose the prism. The light will be cut off from the screen. Keep the distance from the prism to the screen the same, and move the screen round until the light which has been bent aside by the prism is once more received upon the screen. It will be found to be coloured. If the distance from the prism to the screen has been kept exactly the same, the upper and lower ends of the coloured band will terminate sharply, the distance should be adjusted until this is the case.

Notice the order of the colours, and see which colour has been most bent aside, red or violet.

Slowly turn the prism round. The coloured band will move sideways on the screen to the left, perhaps, then become

[1] See footnote, p. 9.

stationary, and then move back to the right. See that in the stationary position it is less bent than in any other position. Notice also that in this position the colours are brighter and clearer than in any other position. This position of the prism is called one of *minimum deviation*. Cut a slit about a quarter of an inch broad in a card, and with the prism at minimum deviation, move the card to and fro across the face of the prism on which the light is falling. Notice that the coloured band, or spectrum as it is called, shifts sideways slightly, showing that the different parts of the prism produce coloured bands on different

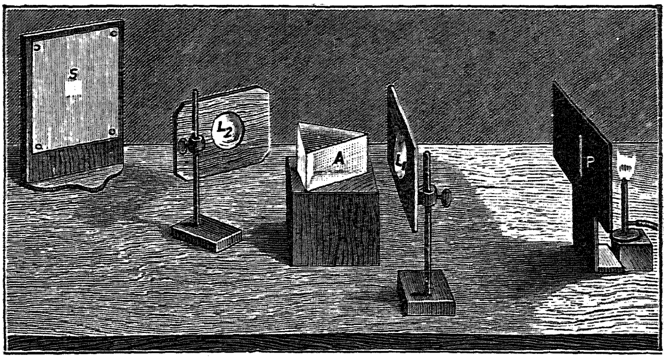


FIG. 115.—Model of Spectroscope.

parts of the screen; thus the band, when the whole prism is used, really consists of several superposed bands, and therefore at any point there are several overlapping colours. The spectrum is not *pure*. This is because the light falling upon the prism is not parallel light: it is convergent; and therefore it is falling upon a prism at different angles, and, as we saw by rotating the prism, the deviation produced by the prism depends upon the angle at which the light strikes it. It is therefore impossible with convergent light to obtain a spectrum free from overlapping colours—that is, a pure spectrum.

88. **Second Method.**—Place the 10-inch lens at its focal length from the slit. To do this experimentally, focus the slit as before on the screen with the lens at about 12 or 14 inches distance

from the slit. Gradually withdraw the screen, adjusting the lens at the same time to keep the slit in focus, until the screen has been removed to the far end of the room. In order to see the image at this distance it will be necessary to shade the screen unless the experiment is conducted in a darkened room. The light from the lens is now very nearly parallel.

Place the prism to receive it. To make sure that the light is really falling on the lens and prism, a piece of white paper may be held in turn against their surfaces.

The prism will deviate the light, and the 15-inch lens is to be put to receive the deviated light. It may be placed at any distance from the prism that is convenient; for as the light from the first lens is supposed to be parallel, the distances of the prism and second lens should not matter; but practically, it is well to keep them fairly close to one another. To see that the light from the prism is really going through the centre of this lens, again make use of a piece of white paper.

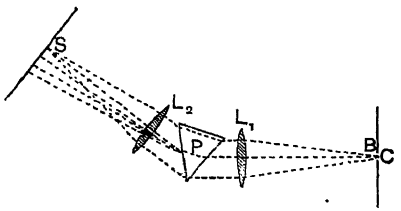


FIG. 116.—Plan of Model Spectroscope. C, slit; L_1, 10-inch lens; L_2, 15-inch lens; P, prism; S, receiving screen.

Lastly, adjust the screen to receive the spectrum, and move it nearer to, or farther from, the lens until the top and bottom of the coloured band are as sharp as possible. On rotating the prism, it will be found that the spectrum again moves to and fro sideways, reaching a stationary position at which the deviation is less than at any other—the position of minimum deviation. Set the prism to this position, see that the light is going through the centre of the second lens and that the image is properly in focus. (If, owing to the length of the slit, the top and bottom cannot be clearly focussed, place a piece of paper against the slit to cover up the lower half, and focus the edge of this by adjusting the distance of the screen.)

The colours will be more brilliant than they were in the first method, as the overlapping should now have ceased. To see if

this is the case, again slide a card with the slit in it to and fro across the face of the prism. If there is any movement of the spectrum, it shows that the first lens has not been properly adjusted for parallel light.

89. The following experiments are all due to Newton (though he used sunlight). They require a room that can be darkened.

i. **Effect of Crossed Prisms.**—Adjust the apparatus as in § 88, to obtain a bright spectrum on the screen S, substituting the screen with the 2 mm. hole for the slit. Then introduce a second prism with its axis *horizontal* so as to refract the light upwards. Note that the spectrum is not raised equally, but that the violet end is raised higher than the red, and that the resulting spectrum is formed along a line inclined to the horizon. Note that the colour that suffers the greatest deviation in the first prism is also most deviated in the second. Note also that the second prism does not decompose the colours of the first spectrum a second time; for instance, the yellow of the first spectrum is not spread out vertically into a spectrum, but only raised vertically, and it remains yellow.

ii. **Recomposition of White Light.**—Obtain the spectrum as in § 88, using the slit again. Then place the 6" lens L_3 exactly where the screen S was. Cut a hole carefully 2 cms. square in a black card and place it between the prism P and the lens L_2, with its edges horizontal and vertical respectively. Adjust the screen S to receive the light from L_3, at the conjugate focus of the card. A rectangular patch of light will be formed on S. As the lenses are not achromatic, it will probably have coloured edges, but by slightly tilting the lens L_3 this can be corrected and a clean image produced. If the aperture in the card is not too long and is evenly illuminated, and if the lenses L_2 and L_3 do not cut off any of the light, the patch on the screen S should be white. Interpose a card at L_3 to cut off some of the light (*e.g.* the red end of the spectrum formed there). The patch will appear coloured, and the colour will vary with the amount and the colour of the light cut off. This experiment shows that white light results from a mixture of all colours. Look through a second prism, held near the eye, at the patch of white light formed above on S, it will be drawn out into a spectrum; this shows that the recomposed white behaves just as the original white.

iii. Cut a comb out of card with teeth about 2 cms. broad, 6 cms. long and 2 cms. apart. Set up the apparatus of the last experiment (ii), and holding the comb just in front of the lens L_3 slowly move

it along. The colour of the patch will pass through a series of changes as each tooth is moved across the lens. Move it to and fro a little more rapidly, and the colours will succeed one another rapidly. Lastly, if it be moved quickly to and fro the colours of the patch will succeed one another so rapidly that they will be all superposed by the persistence of vision, and once more the patch will appear white.

iv. **The Colours of the Spectrum cannot be again decomposed.**—This has already been shown by one method in Experiment i.

Set up the apparatus as in § 88 to form a spectrum. Replace the screen S by one with a 2 mm. hole. At about 12″ distance put the 6″ lens L_3, then the second prism and the screen S beyond at such a distance that the lens L_3 shall focus the hole on S.

It will be found that only a nearly circular image of the hole will be formed on S of the same colour as the light emerging through the hole, and that it is not drawn out into a spectrum. Now, either by rotating the first prism, or by moving the first slit, cause the different colours of the spectrum to pass through the hole in turn. In each case the final image on S will be of the colour passing through the hole—and not a spectrum—and the position of the image will vary on S moving to right or left according as the more refrangible light (the violet) or the less refrangible (the red) is caused to fall on the hole.

Lastly, when some colour (say the green) is passing through the hole and being imaged on S, without disturbing anything hold a lighted match in front of the hole in the middle screen, so that its light may form a spectrum on S, and note that the position of this colour (the green) in the spectrum coincides exactly with that of the green from the original light.

v. Form the spectrum as in § 88 on the screen S. Look at this spectrum through the second prism with its refracting edge parallel to the edge of the first. If the prism is at the right distance from the screen the image of the spectrum will be reduced to a line again, and that line will be white. The easiest way to understand this, is to remember that if white light which was proceeding to a line at S fell upon the prism it would form a spectrum, and therefore, reversing the path, light proceeding from the spectrum, and falling on the prism, will emerge as if it came from S.

vi. Set up the apparatus as Experiment i, but using a screen with *two* holes on the same horizontal line and about 2 cms. apart. In place of a single spectrum there will now be two spectra which are end to end in the same horizontal line until the second prism is

inserted, and then each spectrum will be raised. Note that the yellow of each spectrum is raised by the same amount.

vii. In the same experiment, instead of inserting the second prism in the path of the light, hold it in front of the eye with its refracting edge horizontal; the effect on the spectra is the same.

viii. Put two isosceles right-angled prisms with their hypotenuses in contact on a small piece of board. Set up the apparatus of § 88, and insert this compound prism in place of the prism of that experiment. In a certain position of the compound prism, part of the light will go through, and part will be reflected at the hypotenuse of the first prism. Put screens to receive each of the beams. They will not be spread into spectra, but will each be coloured, the colours being complementary.

The Spectrometer.

Apparatus.—The spectrometer consists of a collimator, the function of which is to parallelise the light; the telescope rotating round a graduated circle; and a table also furnished with a graduated circle, upon which the prism or other apparatus is placed (Fig. 135).

90. **Essential Parts.**—*The Collimator.* This usually consists of a brass tube varying in length from 8 inches upwards, furnished at one end with an achromatic objective and at the other with a slit. The latter is mounted on a tube sliding in the main tube. The sliding tube should be furnished with a rack-and-pinion motion, by which the slit may be adjusted at the upper face of the lens.[1] The slit must have good edges which are usually made of steel. These should be examined occasionally, and rubbed with an oily rag. A **V** is usually mounted outside the slit to limit its length vertically. A small right-angled prism is mounted on a movable arm that may be swung in front of the lower half of the slit; and thus the light from a flame placed to one side of the end of the collimator, can be reflected down the tube, at the same time as the light from another flame, directly in front of the tube, passes through the upper half of the slit. The collimator is usually mounted rigidly upon the stand of the instrument. The lengths of the collimator tubes must be such that the slit can be placed at the focal plane of the lens.

[1] In many instruments a rack is put on the telescope tube and omitted from the collimator. It is useless, however, to be able to focus the telescope unless the collimator can also be adjusted.

The Telescope is an astronomical one, furnished at one end with an achromatic objective of the same diameter as the collimator. The eye-piece required for most purposes is a positive Ramsden eye-piece, which should slide smoothly in its tube and be focussed upon a fine cross-wire. The tube containing the eye-piece and cross-wire is moved by a rack and pinion. The telescope is mounted on an arm which rotates round a vertical axis, and in the best instruments is balanced. The arm in the cheaper instruments is nearly always too weak. The position of the telescope is read on a divided circle by a vernier (or preferably by two verniers at opposite ends of a diameter).

The Table of the instrument rotates, and its position is also read by a vernier on a divided circle. It should have a "hole, slot, and plane" upon which the prism table may be placed. This will allow of the prism table being moved and replaced in the same position.

91. *Simple form of Spectrometer for elementary students.* A collimator and a telescope can easily be set up on two bases

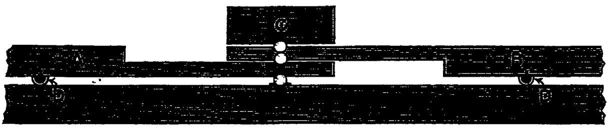


FIG. 117.—Simple mounting for Spectrometer.

similar to those described on page 53. The slit and the eye-piece can be fixed to the base, and only the lens be on a sliding stand, which should be heavy. The bases of these and of a stand for a prism can be connected as shown to form a simple spectrometer. The bases for this purpose should, however, be much narrower—say $2\frac{1}{2}$ to 3 inches wide. Two large bicycle balls, about 2″ diameter, are inserted in a tongue of wood about $\frac{1}{2}$″ thick, projecting from the end of the telescope base, B. One of these rests in a small brass washer, inserted in a tongue of wood projecting from the base, A, of the collimator. The other forms one support for the prism stand, C, which has a similar washer inserted in it. The three bases stand on a plate-glass mirror, D. Each base rests on two balls besides that forming the axis. A ball is attached with cement to the middle of the glass surface on which A rests. A divided paper circle is carefully cut out, so that the divisions just come to the edge of the paper, and is attached to the glass with a few spots of paste; if pasted all over the paper would swell. It is very important that the

scale shall be concentric with the ball. A fine wire stretched between two stout pins driven into the sides of the stands will enable the angles to be read to the fifth of a degree, by making the wire appear to coincide with its image seen in the mirror. Or a piece of celluloid, cut from the "film" used to support a photographic negative, can have a fine scratch made on it with a sharp knife, and be attached to the bases. If the scratch is on the lower side of the celluloid, and the latter is lightly pressing on the paper scale, no mirror is necessary. There should be no difficulty with such a pointer in reading to a fifth of a degree.

Adjustment of the Instrument, The Maker's Adjustments.

92. The table and its divided circle should be parallel to the plane of the divided circle of the telescope; the axis of the latter should be perpendicular to and concentric with its divided circle. The axis of the telescope should be perpendicular to its axis of rotation, and on the same horizontal level as that of the collimator. When pointed at the collimator the two axes should be collinear, and should cut the axis of rotation of the telescope arm. These last adjustments are usually effected by screws, and they may be tested by placing a piece of parallel glass, silvered on one face, on the table, and illuminating the cross-wire, which must be horizontal for this purpose. By setting the mirror perpendicular to the axis of the telescope, an image of the cross-wire can be formed upon the cross-wire itself. Rotate the telescope in its tube through 180°. If the cross-wire is truly in the axis of the telescope, the image will still be formed upon the cross-wire. If not, the cross-wire must either be adjusted or the amount of the displacement noticed and allowed for. The true position of the cross-wire is half way between the cross-wire and its image formed after the telescope has been rotated. If the table be now rotated through an angle of 180°, the image will again coincide with the cross-wire if the adjustments were perfect, but if the telescope had been

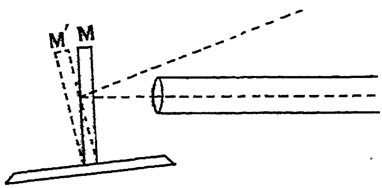


FIG. 118.—M, mirror mounted on table and adjusted perpendicular to the axis of the telescope. M′, the same mirror after the table has been rotated 180°.

inclined to the axis of rotation of the table, the image would now be displaced by four times the amount of this inclination, as can be seen from the accompanying diagram.

The adjusting screws should therefore be altered until the telescope is moved through a quarter of this amount, and the observation repeated. If the axis of rotation of the table coincides with that of the arm carrying the telescope, the adjustment will remain correct at all positions of the telescope. When the axis of the telescope has been adjusted that of the collimator can be adjusted from it by pointing them at one another, and attaching a hair to the slit at such a height that its image is formed on the cross-wire. Now rotate the collimator in its bearings through 180°, and see if the hair still coincides with the cross-wire.

The above adjustments should not be interfered with by the ordinary student, certainly not without special permission from the demonstrators.

93. **Further adjustments. The Collimator and Telescope have each to be focussed for parallel light.**

First Method.—Point the telescope through a window at a very distant object, and focus it approximately with the rack. Owing to the power of accommodation of the eye, it is possible to see the cross-wire and the image of the distant object independently, and yet be unable to see them distinctly at the same time. Beginners usually find this the case; it is because the observer knows that the cross-wire is close to him, and therefore only focusses his eye to view a near object when he wishes to look at it. On the other hand, he will focus his eye at infinity when he wishes to observe the distant object; with the result that its image will not be formed in the same plane as the cross-wire. He must try to imagine the cross-wire to be stretched over the distant object while he focusses it. In making the final adjustment, it is best to use the method of parallax, and to move the eye to and fro as far as the aperture of the eye-piece will allow, and see if the cross-wire shifts over the image.

If two students are working together at the instrument, they will frequently find a slight difference in their sight, so that after one has focussed the instrument, the other will not find it correct. If, however, the image of the distant object has really been focussed upon the cross-wire by the first observer, it is obvious

that the adjustment of the eye-piece, which would bring the cross-wire sharply into focus for the second observer, should make the distant object distinct at the same time; so that if the cross-wire has once been set at the principal focus of the objective by one observer, *no adjustment of the focussing screw* of the telescope should be made by the second observer. All that he will have to do will be to adjust *the eye-piece* to suit himself, and this will not at all interfere with the adjustment for parallelism. Thus, if each observer in turn adjusts the eye-piece to suit himself, he will be dealing with parallel light so long as the cross-wire (that is so long as the focussing screw) is left untouched.

Now point the telescope directly at the collimator, illuminate its slit, and adjust the length of the collimator tube until the slit is sharply focussed, again using the method of parallax to make sure that the slit and cross-wire are in the same plane. Then, as the telescope has been focussed for parallel light, that issuing from the collimator must be parallel.

Second Method.—If the cross-wire of the telescope can be illuminated, point the telescope at a plane mirror, so that the image of the cross-wire after reflection in the mirror may be formed in the tube of the telescope. If the mirror is normal to the axis of the telescope, and if the telescope is focussed for infinity, so that the light striking the mirror is parallel (and therefore the rays all perpendicular to the mirror), it will be reflected as a bundle of parallel rays, and be focussed again at the principal focus of the lens. Thus the image of the cross-wire will be formed upon itself. The telescope must be adjusted until this image does coincide with the cross-wire itself.

If there is no means provided for illuminating the cross-wire, by removing the eye-piece and using a piece of unsilvered glass to reflect the light down the tube, an image of the cross-wire will be visible, and by the method of parallax can be brought to coincide with the original. It is difficult, however, to obtain good results in this way.

Third Method.—Interpose a bi-plate (p. 272) between the collimator and telescope. If both telescope and collimator are focussed for parallel light the slit will remain sharp and *single*; but if the light is divergent or convergent the bi-plate will produce two images of the slit. The focus must then be adjusted until the two images coincide.

Fourth Method. Shuster's Method.—When a beam of light goes symmetrically through a prism it also goes through with a minimum deviation.

Let ABC be a prism, and consider the bundle of rays converging to O falling upon the face AB. Let $P_2Q_2R_2S_2$ be the ray which passes through symmetrically, and therefore with a minimum deviation. Had the deviation of the other rays been equal, they would, after refraction, converge to a point O_2, but as the deviation of P_1Q_1 is greater than that of P_2Q_2 it will converge to a point O_1, and as the deviation of P_3Q_3 is greater than that of P_2Q_2 it will only meet R_2S_2 at O_3. Thus a convergent pencil P_1OP_2 falling upon a prism with a less angle of incidence than that required for minimum deviation will be more convergent after emergence than before; whilst a convergent bundle

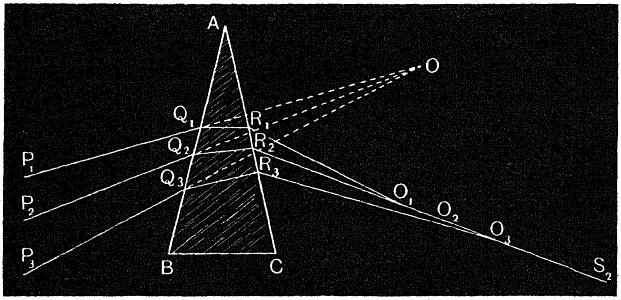


FIG. 119.—Deviation through a Prism of a Convergent Beam of Light.

P_3OP_2 falling upon the prism at an angle greater than that for minimum deviation will be less convergent,—or more nearly parallel. In the same way, by drawing a figure for a divergent pencil, it can be seen that this also will be more nearly parallel after passing through the prism if it is incident at an angle greater than that for minimum deviation. Suppose, then, we place such a prism on the table of the spectrometer, and using sodium light, we turn the telescope to view the slit through the prism. If the light from the collimator is either convergent or divergent, after it enters the telescope it will be more nearly parallel than that leaving the collimator, if the light from the collimator strikes the prism more obliquely than it should for minimum deviation; so that we have the following method for adjusting the instrument for parallel light.

Place the prism in such a position that the light from the collimator falls upon its face *more obliquely* than it should for minimum deviation, and focus the telescope upon the slit; it will be more accurately adjusted for parallel light than the collimator is. Without moving the telescope, rotate the prism, and when the

light is striking it *more directly* than for minimum deviation, an image of the slit will once more be formed in the field of the telescope. This time the light from the collimator, after passing through the prism, will be less parallel than that leaving the collimator, and the image of the slit will be out of focus. But if this time the collimator is focussed (the telescope not being touched) until the slit is once more distinct, the light leaving the collimator, as we have seen, will be more nearly parallel than that entering the telescope; so that now the collimator is in better adjustment than the telescope. By again turning the prism so that the light from the collimator falls upon it obliquely, and this time focussing the telescope, its adjustment will be still further improved. In this way, by alternately focussing collimator and telescope, they may be adjusted as accurately as is required. (If the adjustments are made in the wrong order, the telescope when the prism is in the position for the adjustment of the collimator, and *vice versâ*, the definition will so rapidly become worse that the mistake cannot fail to be recognised.) In the final adjustment of the telescope, to see that the slit is really focussed in the same plane as that of the cross-wire, make sure that there is no relative displacement of slit and cross-wire as the eye is moved to and fro across the field of the eye-piece.

94. **To adjust the Prism so that the Refracting Edge may be Parallel to the Axis of the Instrument.**—Let BAC be the prism and A the angle to be determined.

Place it upon the levelling table with one face of the prism, such as AC, perpendicular to the line joining a pair of the levelling screws, S_1, S_2. Place the table on the spectrometer resting in the "hole, slot, and plane," with the edge A opposite the collimator, and the faces AB and AC making as nearly as possible equal angles with the axis of the collimator, so that the light issuing from the collimator may be divided into two approximately equal beams, one reflected from each face of the prism. Turn the telescope to receive one of these reflected beams. If there is any difficulty in finding the image of the slit formed by this beam, turn the telescope aside and look in the face AC, as in a mirror, when an image of the slit will be visible along the line PQ (this

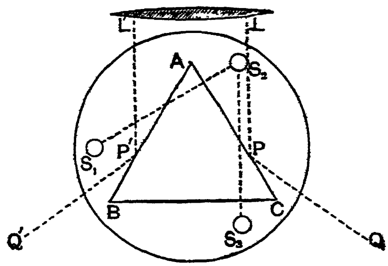


FIG. 12c.—Angle of a Prism.

image will generally seem to be indistinct; it is formed by parallel light, and therefore will only be clear to an eye focussed for parallel rays, and as the observer knows the slit is actually at a short distance he finds this difficult to accomplish).

The image of the end of the telescope can always be clearly seen, and from that, the image of the slit may generally be easily found. Then, without moving the head, swing the telescope round until it is in a line with the eye, and the slit should be in its field of view.

If the face of the prism AC is truly perpendicular to the plane of the instrument, the image of the slit will be formed in the eye-piece at the same height as if viewed directly. If this is not the case, one of the screws must be rotated until it is at the same height. AC, the face which was set perpendicular to the line S_1S_2, is the face which must be adjusted first.

Now turn the telescope to receive the beam reflected from AB and adjust the screw S_3, until the image is the right height. It will be seen that the screw S_3 will rotate the face AC in its own plane, and therefore not interfere with its adjustment. It is well to test this by once more turning the telescope to receive the light from AC. If this is not quite correct and a screw has to be moved, the face AB must be adjusted once more with the screw S_3.

Measurements.

95. **To measure the Angle of the Prism.** *1st Method.*—Turn the telescope to receive the light reflected from the face AB, and read its position with the vernier. Then turn the telescope to receive the light refracted from the face AC, and again read its position. As LP is parallel to L′P′, and the angles of incidence and reflection are equal, it is easily seen from Fig. 120 that the angle between PQ and P′Q′ is exactly double the angle A; thus the angle of the prism is half the difference between the readings of the vernier when the telescope is at PQ and P′Q′ respectively.

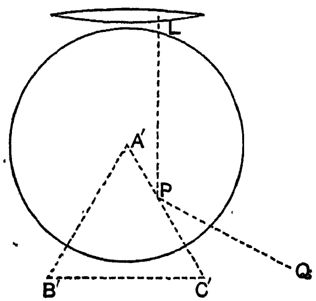


FIG. 121.—Angle of a Prism.

2nd Method.—Instead of reading the angle through which the telescope is moved, the telescope may be kept in the position PQ and the prism rotated until the light is reflected from the face AB in the same direction as PQ. This time the angle of the prism will be 180° minus the angle the prism has been turned through. To obtain equally powerful beams for the two reflections, the prism must be drawn back until the edge A is over the centre of the table, as indicated by the dotted lines A′B′C′ (Fig. 121).

As in these experiments the reflections from AC and AB use different bundles of rays from the collimator, it is obvious that unless the light from the collimator is truly parallel the results obtained will be incorrect; therefore, not only must the collimator be focussed for parallel light, but its lens must be practically free from spherical aberration. The same thing is true of the telescope.

96. **Determination of the Deviation produced by a Prism, and hence of its Refractive Index.** *1st Method.*—It is shown in books on Geometrical Optics that the deviation of a ray of light in passing through a prism is least when it passes through the prism symmetrically.

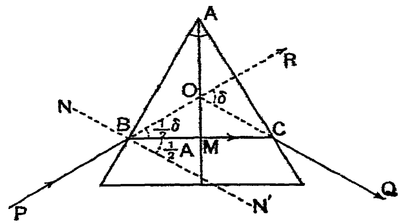


FIG. 122.—Refractive Index of Prism.

Let PBCQ be such a ray passing through the prism ABC, and AOM a line bisecting the angle A. Then BC is perpendicular to AM. Produce BO to R; the angle ROQ is the deviation δ, and, therefore, OBC$=\frac{1}{2}\delta$. If NN′ be the normal at B, the angle N′BM is half the angle A of the prism, for it equals the angle MAB.

Now

$$\mu=\frac{\sin i}{\sin r}$$
$$=\frac{\sin \text{PBN}}{\sin \text{MBN}'}$$
$$=\frac{\sin \text{OBN}'}{\sin \text{MBN}'}$$
$$=\frac{\sin \frac{1}{2}(\delta+\text{A})}{\sin \frac{1}{2}\text{A}}.$$

As we have already determined the angle of the prism, A, if we can find δ, we shall be able to obtain the refractive index from this formula,

To obtain the minimum deviation, δ, produced by the prism, set up the prism on the instrument as directed in Experiment § 94 with the edge A vertical, and place it so that the light from the collimator falls upon the face AB in some such direction as PB, then the end of the collimator tube will be visible on looking through the prism along the line QO.

Using this as a guide, seek for the slit, the telescope being turned aside. Notice in what direction it appears to lie (for instance, that it appears to coincide with some object on the wall of the room). Turn the telescope until it is pointing in this direction; on looking through, the slit ought to be visible. The deviation produced by some of the heavy glass prisms (which have generally a yellowish hue) is very considerable, it is often, therefore, larger than the student expects.

On rotating the prism the image will appear to move across the field. If the rotation is continued and the image followed, it will be found that as the prism is rotated past the position of minimum deviation, the slit moves up to a certain point and then comes back again. This point should be found carefully and the cross-wires placed upon it. The prism will now be in minimum deviation and the position of the telescope must be read upon the divided circle.

Remove the prism, point the telescope directly at the collimator, set the slit upon the cross-wire, and again read its position. The difference between these readings is the deviation produced by the prism. The refractive index is found by substituting in the above formula.

Another Method, suitable only for Prisms of Small Angle.—As an exercise it is useful to find the refractive index when one face of the prism is placed perpendicular to the axis of the collimator. As the light from the collimator will strike the prism directly, it may be looked upon as one half of the prism ABC (Fig. 122), namely, such as a prism AMC,—the entrant light being represented by the line BMC, and the emergent ray by CQ. The deviation and the angle of the prism will be half that of the prism ABC. Thus, if A′ and δ' be the deviation and angle of this prism, the refractive index is given by

$$\frac{\sin(\delta'+A')}{\sin A'}.$$

To perform the experiment, point the telescope directly at the collimator and take the readings. Turn it through 90°, using the divided circle, and clamp it. Set the prism up, and reflect the light from the collimator, by the face, AM, down the telescope, and adjust the image of the slit on the cross-wire. Turn the prism through 45°, using the divided circle, so that the face, AM, shall be perpendicular to the collimator. Now set the telescope to receive the light that has traversed the prism, and so obtain the deviation. The angle of the prism must be found as in the previous case.

97. **To find the Refractive Index of a Liquid.**—*Apparatus.*—For this we shall require a hollow prism.

The hollow prism should have the faces movable, so that it can be easily cleaned. It usually consists of a triangular block of glass, through which a cylindrical hole is drilled. On each face a piece of worked glass is held in place by a pair of rubber bands. It is important that these faces shall be of *parallel* worked glass. To see if this is the case, set up the empty prism and look directly through it, and adjust the telescope in a line with the collimator. Remove the prism and see if there is any displacement of the slit. If not, the faces are parallel. If the faces are not parallel, the deviation must be reckoned from the position of the slit as seen through the prism, and the angle of the prism taken to be that of the central block of glass. If the surfaces of this block are sufficiently well-polished to use them as the reflecting surfaces, it is best to determine the angle as described above, using the reflections from these surfaces. If not, the plates must be put into position, and two images will be seen on each side, one due to the front, and one due to the back surface of the plate. If the prism can be clamped sufficiently securely for the plate to be reversed, the image due to the front surface of the plate will have shifted, and can thus be distinguished from that due to the back surface. Otherwise the prism may be partly filled with water or any other similar liquid, when the image from the back surface will be rendered fainter; or with

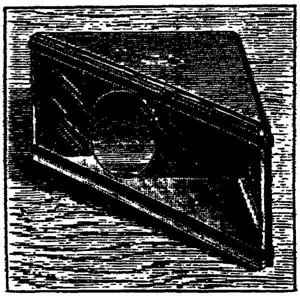


FIG. 123.—Hollow Prism for a Liquid.

mercury which will make it brighter, it can thus be distinguished from that formed by the front surface. It is the image of the back surface whose position must be determined in finding the angle of the prism. If the plates are squares, and the two surfaces, which we are supposing to be not parallel to one another, meet in a line parallel to one edge of the plate, this edge should be made horizontal as the want of parallelism will not then affect the result. It is easy to see in which direction the surfaces will meet together, by observing the images of a distant flame viewed very obliquely, as already described on p. 36. The line of images must be vertical in the prism as put together.

A very fair home-made prism can be constructed by cutting a piece of stout glass tube and grinding with turpentine and coarse emery cloth to an angle of about 60°. Use for the faces two pieces of patent plate selected by the method on p. 36, as nearly parallel as possible. If these plates are not quite parallel, turn them until the line of images they form of a distant flame is perpendicular to the refracting edge of the prism. A small hole for introducing the liquid into the prism should be bored through the top of the tube with an ordinary watchmaker's drill, using plenty of turpentine. The prism must be set up, and its refracting edge adjusted parallel to the axis of the instrument as already described for the ordinary prism.

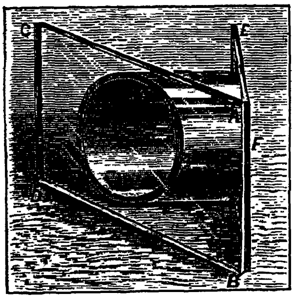


FIG. 124.—Home-made Prism for a Liquid.

Except that the deviation must be measured from the position of the image of the slit with the empty prism inserted, and that the inner faces of the glass plates are to be used in determining the angle of the prism, the measurements are similar to those already described in Experiment, § 96.

98. **To map the Spectrum formed by a Prism.**—By this is understood the determination of the positions in the spectrum of the lines of known wave-length, a curve then being drawn on squared paper, plotting as abscissæ the readings of the instrument, and as ordinates the wave-lengths of the corresponding lines.

The best lines to select will depend upon the source of light

available. The lines of sodium, lithium, thallium, potassium can be obtained with the ordinary Bunsen burner. If an induction coil and hydrogen tube are available, it will give three, in the red, blue, and violet, respectively, which are exceedingly useful. Calcium, barium, and strontium give so many lines that they are practically useless to a beginner, as he is unable to identify them. It is very difficult, indeed almost impossible, for a single student to observe more than the sodium, lithium, thallium, and hydrogen lines. For the potassium ones, two persons are required, one to feed the salt into the flame, and the other to make the observations. The sodium line (or lines if the resolution of the prism is sufficient to separate them) having been determined with the prism at minimum deviation, the next easiest will be the lithium lines. A small quantity of lithium chloride is to be placed in the flame, either by heating a piece of platinum wire, and dipping it whilst still hot into the salt, when sufficient will adhere to the wire for the purpose, or the salt may be placed upon a piece of gauze in the same way as the sodium is usually supplied. A red and an orange line are usually visible. The Bunsen flame must be supplied with a large amount of air so that it may give a high temperature, or the salt will not be volatilised. Potassium gives two lines, a red and a violet one; neither of these can be seen at all easily. The best results can be obtained with nitre. A piece of fine straight wire is heated and dipped into powdered nitre, then held in the edge of the flame. The nitre will melt to a kind of bead, and then suddenly burst into flame. It is only while this flame lasts that the lines are bright enough to be seen. The violet line is so far in the extreme violet that students often do not move the telescope far enough to get it in the field. It is only after a large number of these flares has been produced that the readings can generally be obtained. The lines are very valuable to a beginner, being quite unmistakable and giving the positions of the extreme ends of the spectrum. For the other metals mentioned, the heated wire is dipped into its chloride and held in the flame. Thallium gives a very useful line in the green, and strontium in the blue.

If an arc lamp is available, it should be focussed upon the slit with a lens (its own condenser will do). The carbons must be set up vertically and separated considerably so that an image is formed of the space between the carbons. The salts may then be fed with a spare carbon. Lithium in the arc gives a line practically identical with the blue strontium line. Magnesium gives three

nearly identical lines in the green. Neither of these is visible with the ordinary Bunsen, as the temperature is not high enough.

For the hydrogen lines, the hydrogen tube must be connected to an induction coil, and mounted vertically so that its narrow part may be opposite the slit. On working the coil, the hydrogen lines will be very easily found, and their positions can be read, then the curve may be drawn through the lines obtained.

Enter the results thus:

Line.	Reading.	Wave-length.	Square of Wave Number.	Source of Light.
Na, D_1 . . .	—	5889	$2883 \cdot 4 \cdot 10^5$	Bunsen
D_2 . . .	—	5896	2876·8 ,,	,,
Li, red . . .	—	6708	2222·2 ,,	,,
orange .	—	6102	2685·6 ,,	Arc
blue . .	—	4603	4719·6 ,,	,,
K, α red . .	—	7680	1695·4 ,,	Bunsen
γ violet .	—	4045	6109·3 ,,	,,
H, α red . .	—	6563	2321·6 ,,	Vacuum tube
β green .	—	4860	4231·2 ,,	,, ,,
γ blue . .	—	4340	5309·3 ,,	,, ,,
Tl, green .	—	5350	3492·8 ,,	Bunsen
Mg, green .	—	5183	3722·5 ,,	Arc
,, .	—	5172	3738·3 ,,	,,
,, .	—	5167	3745·5 ,,	,,
Sr, blue . .	—	4607	4710·5 ,,	Bunsen
Ca, blue . .	—	4227	5597·0 ,,	,,

On a piece of squared paper plot a curve with the readings of the instrument along the horizontal line and the wave-length vertically; also plot a curve on a second piece of squared paper giving the readings horizontally, and the squares of the wave numbers vertically. This curve will be found almost a straight line, and therefore is much more easily drawn accurately. Any reading that is not very approximately on the line should be repeated, as it indicates an experimental error. With the aid of either of these curves, preferably the latter, the wave-length of any unknown line can be determined, and therefore the line can be identified: if, for instance, strontium or barium be put in the flame, each of which gives a large number of lines, the readings for some of these lines may be taken and the lines identified by the table at the end of the book.

99. **The Constants of Cauchy's Formula.**—Adjust the prism to be examined on the table of the spectrometer, set it to minimum deviation with the sodium flame, and take the reading of one or both of the lines. Now replace this by a hydrogen tube, and taking the red hydrogen line readjust the prism for minimum deviation, and set the cross-wire on the line.

Observe the deviation. Repeat this for the blue hydrogen line.

Measure the angle of the prism and calculate the refractive index for each line. Then we have

$$\left.\begin{aligned}\mu_C &= a + b\frac{1}{\lambda_C^2}\\ \mu_F &= a + b\frac{1}{\lambda_F^2}\end{aligned}\right\}$$

$$\therefore\ b = \frac{\mu_F - \mu_C}{\dfrac{1}{\lambda_F^2} - \dfrac{1}{\lambda_C^2}}.$$

From which, on substituting the values of $\frac{1}{\lambda_C^2}$ and $\frac{1}{\lambda_F^2}$ from the table (page 130), b is easily found.

Substitute the value for b in either of the equations and get a.

Other Methods of Determining the Refractive Index.

100. **The index of refraction of a prism by critical incidence** (Kohlrausch).—Remove the slit from the collimator. Place a sodium flame in its usual position, and place the prism on the table of the spectrometer with one face parallel to the tube of the collimator, its base towards the collimator. Bring the telescope round to look through the prism and receive the light which is refracted through it. It will be found that in a certain position half the field appears bright, the other half dark, the dividing line being vertical as in Fig. 125. Adjust the telescope on this line and take the reading.

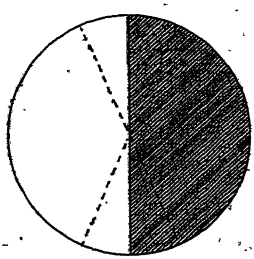


Fig. 125.—Field Half-shadow.

If the collimator can be rotated, bring it round together with the flame *without moving the prism* until the light falls at grazing incidence on the other face of the prism. Again place the telescope to receive the light refracted from the prism, but of course this time emerging from its first surface, and adjust the cross-wire upon the line dividing the field into dark and light.

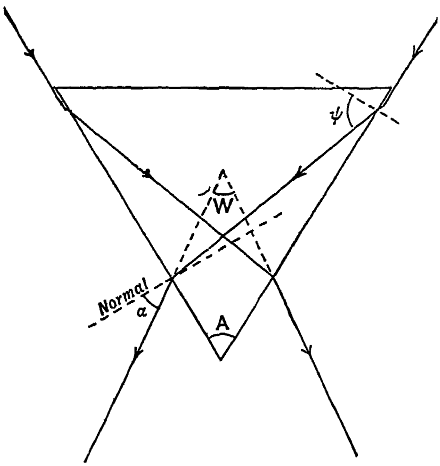


FIG. 126.—Refraction Index of a Prism.

If the angle turned through by the telescope be W, and the angle of the prism A, the angle the telescope made with the normal to either surface, α, is given by

$$\alpha = 90° - \tfrac{1}{2}(W + A).$$

The refractive index is given by

$$\mu^2 - 1 = \left(\frac{\cos A - \sin \alpha}{\sin A}\right)^2 *$$

If the collimator cannot be rotated, it will be necessary to rotate the prism and allow for the amount it was rotated in determining the angle through which the telescope is turned, W. Or the collimator may be removed and a simple flame and lens used, the two being so adjusted as to cause a slightly convergent beam to fall at grazing incidence upon each surface of the prism in turn.

* If ψ is the critical angle, $\sin \psi = \frac{1}{\mu}$. If r is the angle of refraction in the glass at the second surface, $r = \psi - A$. Then, by Snell's law,

$$\sin \alpha = \mu \sin r = \mu (\sin \psi \cos A - \cos \psi \sin A)$$
$$= \cos A - \sin A \sqrt{\mu^2 - 1}$$
$$\sqrt{\mu^2 - 1} = \frac{\cos A - \sin \alpha}{\sin A}.$$

101. **The refractive index of a liquid from the critical angle. Woolaston's Method.**—A drop of the liquid is placed upon the hypotenuse of a right-angled glass prism, of which all three faces are polished. The prism is placed on the table of a spectrometer and the image of the slit formed by internal reflection at that surface is obtained. Remove the eye-piece from the telescope, replacing it by an ocular slit, and rack it in until the slit is at its principal focus, when the whole of the objective of the telescope will appear illuminated. (The collimator slit may be opened rather wide.)

The drop of liquid will be visible as a dark spot on the field, for the light will be transmitted there and will not suffer total reflection. By increasing the angle of incidence until the critical angle from the glass to the liquid is reached, total reflection will occur here also, and the whole field will become equally illuminated. The exact angle at which this occurs must be determined.

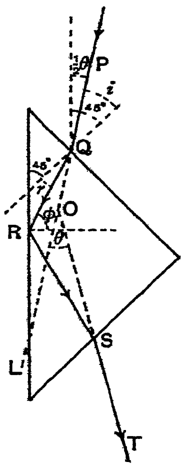


FIG. 127.

Let PQRST be the path of the light through the prism. The angle between PQ produced and ST, that is the angle LOT, is the angle θ between the telescope and the collimator. If the angles at A and B are each 45°, the lines PQ and ST will be equally inclined to the plane AB, and therefore the angle of incidence, i, at Q, is given by $i=45^\circ-\frac{1}{2}\theta$. From this and the refractive index of the prism, μ_g, supposed known (it must be independently determined), the angle of refraction can be calculated from $\mu_g=\frac{\sin i}{\sin r}$. The angle of incidence ϕ at R will be this angle of refraction, plus 45°.

Thus $$\sin(r+45^\circ)=\sin\phi=\frac{\mu_c}{\mu_g},$$

where μ_l and μ_g are the refractive indices of the liquid and glass respectively. The method does not give very good results, as it is difficult to say exactly when the total reflection occurs.

102. **Third Method.**—If the base of the prism of Fig. 126 is polished, attach a piece of tissue paper to it. (If it is ground,

this is not necessary.) Place a sodium flame to illuminate the base, turn the telescope to view the light emerging from one surface of the prism after reflection at the other surface, and adjust the cross-wire upon the dividing line between light and dark as before. The light will be reflected at the critical angle. Take the reading.

Then turn the telescope to receive the light emerging from the second surface, which has been reflected from the first. Apply the formula of § 100.

Let a drop of liquid be placed against the surfaces when the reflections take place at those surfaces respectively. Remove the eye-piece and observe the surface of the prism through an ocular slit placed at the principal focus of the lens. Adjust the telescope until the drop of liquid just disappears in each case. The μ obtained, using the same formula as before, will be the ratio of the refractive index of the prism to that of the liquid.

To find the refractive index of the liquid relative to air, multiply the μ thus obtained by the known refractive index of the prism.

103. **Differential Prism of Hallwachs and Tornoe.**—This consists essentially of a small vessel with parallel glass faces, which is divided by a plate of parallel glass into two parts, as in the figure. The faces of the trough are ground to a matt surface on the left side of the partition, so that only diffused light enters or leaves the trough 1.

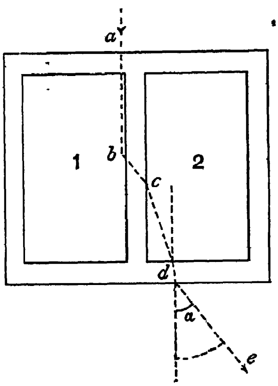


FIG. 128.—Differential Prism.

It is placed on the table of the spectrometer so that light shall enter the back of the vessel normally. Two liquids of slightly different refractive indices, such as, for instance, water and a salt solution, are placed in the two parts. It is obvious that light will come directly through, but there will also be a beam of light refracted from the trough at a small angle. For suppose the liquid 2 to have slightly higher refractive index than 1, then a ray of light *ab* in 1,

which is parallel to the glass plate AB, will be refracted into 2, forming at c the critical angle, and will finally pass out to the air in a direction DE, making an angle α with the normal.

If μ_1 and μ_2 be the refractive indices of the liquids in 1 and 2 respectively, the critical angle at c is given by

$$\sin\phi=\frac{\mu_1}{\mu_2}.$$

As this angle is the complement of the angle at ϕ for the refraction into air, we have

$$\frac{\sin\alpha}{\cos\phi}=\mu_2;$$

therefore

$$\begin{aligned}\sin^2\alpha&=\mu_2^2\,.\cos^2\phi\\&=\mu_2^2\,.\left(1-\frac{\mu_1^2}{\mu_2^2}\right)\\&=\mu_2^2-\mu_1^2\end{aligned}$$

and if the index μ_1 is known this will determine μ_2.

In using the instrument, the collimator is removed from a spectrometer, and its place taken by a simple convex lens some two inches diameter, and say ten inches focus. This is put at

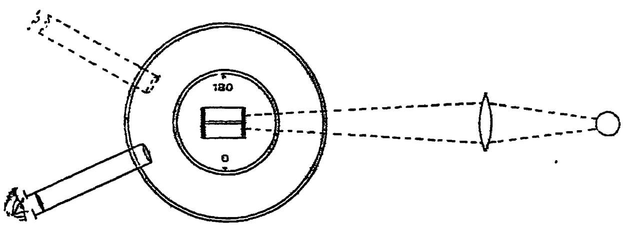


FIG. 129.—Method of using Differential Prism.

such a distance from the spectrometer that the image of a sodium flame beyond it is roughly focussed on the objective of the telescope. The differential prism is placed upon the ordinary prism table, so that the partition is in a line with the centre of the lens. The telescope is turned and so adjusted that the cross-wire of the telescope is upon the vertical line that divides the field of view into light and dark halves. To be sure that this gives the true direction, the illuminating lens is moved to

and fro sideways, or the instrument rotated to and fro as a whole, when the cross-wire should not shift relatively to the line dividing light from dark.

The prism must now be turned through 180°, and the telescope again adjusted until the edge of the shadow coincides with the cross-wire. The angle between the two positions of the telescope will be 2α.

It is very important that there shall be no dirt in the angles between the dividing plates AB and the front and back surfaces of the trough.

104. **Refractive index of a liquid by the method employed in the Abbé Refractometer. Clay's Method.**—The essential part of this instrument is a combination of two similar right-angle prisms of highly refractive glass, which have acute angles of 45°; a drop of the liquid to be examined is placed on the hypotenuse of one of the prisms, and the hypotenuse of the other prism is then placed upon that of the first, so that the two prisms together form a rectangular block of glass.

If a beam of parallel light be supposed incident upon one end of this block, and the block be slowly turned so as to vary the angle of incidence, as long as the incidence upon the interface takes place, at an angle within the critical angle, the beam will be transmitted and emerge from the other end of the block. But if the incidence is made more oblique, at a certain angle (if the liquid has a lower refractive index than the glass) total reflection will take place, and no light will emerge from the end face.

If therefore a telescope focussed at infinity be arranged to receive the light emergent from the prisms, and the block be adjusted so that the incidence on the interface is just at the critical angle for light originally parallel to the axis of the telescope (and therefore also finally since the end faces of the block are parallel), the field of the telescope will appear half light and half dark, the dividing line being vertical if light from an extended source be used, or if a slightly divergent or convergent beam is used.

It may not at first sight appear obvious that there should be a sharp line of division between the light and the dark.

It must be remembered that while a beam of parallel light parallel to the axis of the telescope converges to the axial point A of the focal

plane of the objective, another beam of parallel light that is oblique to the axis, such as P_1O', Q_1O, R_1O'', will converge to a point A_1, on the

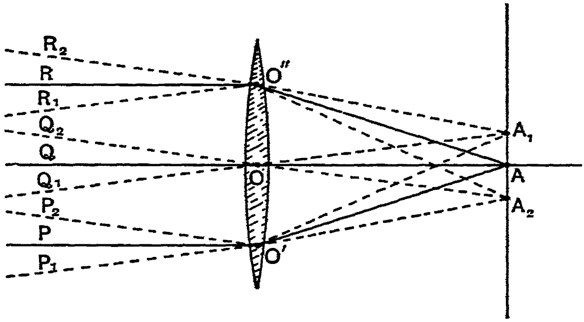


FIG. 130.—Half-shadow of Field.

focal plane on one side of A, and a third beam P_2O', Q_2O, R_2O'' inclined to the opposite way to the axis will converge to a point A_2, on the other side of A.

If, therefore, the first beam just strikes the interface at the critical angle, one of the others—for instance that converging to A_1—will be incident at such an angle that it is transmitted, and A_1 will be bright. The point A_2 would have been the focus of the beam that is totally reflected, which therefore does not emerge from the glass block nor fall upon the telescope. The point A_2 will thus be dark. Thus the whole field on one side of A will be dark, and that on the other side of A will be bright.

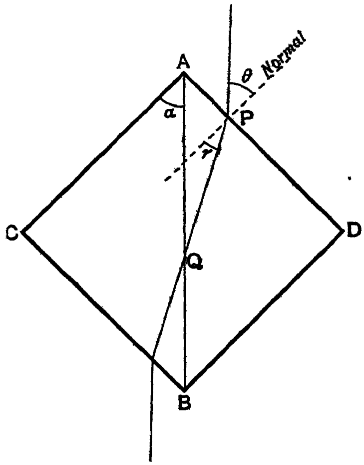


FIG. 131.—Refraction through Pair of Right-angle Prisms.

Let θ be the angle of incident at P, r the angle of refraction at P. Let α be the inclinative of the surface AB to AD. Then the angle AQP is $\alpha - r$. This is the complement of the critical angle, when PQ is the limiting ray. Let μ_g be the refractive index of the glass (supposed known) and μ the refractive index of the liquid between the adjacent

hypotenuses AB of the prisms ACB and ADB (which it is required to find).

Then $$\sin Cr = \cos(\alpha - r) = \frac{\mu}{\mu_g}.$$

Also $$\frac{\sin\theta}{\sin r} = \mu_g;$$

$$\therefore \quad \mu = \mu_g \cos(\alpha - r)$$

$$= \mu_g \left\{ \cos\alpha \sqrt{1 - \frac{1}{\mu_g^2}\sin^2\theta} + \sin\alpha \frac{1}{\mu_g}\sin\theta \right\}$$

$$= \cos\alpha\sqrt{\mu_g^2 - \sin^2\theta} + \sin\alpha\sin\theta.$$

If $\alpha = 45°$, $\mu = \frac{1}{\sqrt{2}}\{\sqrt{\mu_g^2 - \sin^2\theta} + \sin\theta\}$.

Thus as α and μ_g can be determined once for all, if θ is measured for any given liquid μ can be calculated.

105. *Apparatus.*—The ordinary spectrometer may be used with a pair of similar right-angle 45″ prisms; the prisms must be polished on all three faces. The liquid must have a lower refractive index than that of the glass. A sodium flame, and a convex lens two or three inches in diameter will be wanted.

Clamp the prism ACB to the spectrometer table by a clamp at the right angle C. Holding the table and prism in the hand so that AB is horizontal, place a little liquid (water for instance) on the face AB. Now put the hypotenuse of the other prism ADC on AB, so that the liquid is squeezed out and covers the whole interface AB. Place the table carefully on the spectrometer. Place blocks against the faces AC and BD to cut off stray light. With the aid of the convex lens illuminate the prism by a slightly convergent beam from the sodium flame, falling about normally upon AC. Place the telescope, focussed for parallel light, to receive the light emergent from BD. Rotate the table with the prisms, and find the position for which the vertical dividing line between light and dark is on the cross-wire of the telescope. Read the position of the table.

Now remove the blocks and place them against BC and AD; rotate the table counter-clockwise, so that the light may fall upon AD and emerge from CD. Adjust until the dividing line of the light and dark fields is upon the cross-wire, and again read the position of the table.

From the formula a table can be constructed for the pair of prisms used, which gives the refractive index corresponding to each value of ϕ, the angle between the two positions of the prisms. For instance, if the prisms are made from Schott's glass, No. 41, O.4113, of which $\mu_g = 1 \cdot 7172$, the following is the relation:

ϕ	+24	+18	+12	+6	0	−6
θ	33	36	39	42	45	48
μ	1·5367	1·5566	1·5747	1·5924	1·6064	1·6202
ϕ	−12	−18	−24	−30	−36	−42
θ	51	54	57	60	63	66
μ	1·6323	1·6430	1·6525	1·6608	1·6679	1·6742

From the calculated values, a curve can be plotted giving the corresponding values of ϕ and μ.

106. By this method very accurate determinations of the refractive index can be obtained by the use of quite rough apparatus as below.

On a board about 7″ × 15″ is mounted a small fixed telescope T furnished with a vertical cross-wire. A short metal rod is fixed

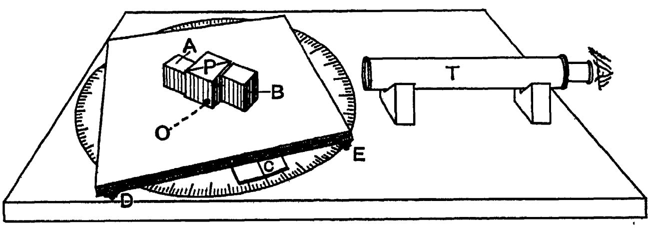


FIG. 132.—Simple form of Clay's Refractometer.

in the board at O, so that it is in line with the axis of the telescope. A paper scale, divided to degrees, is attached to the board with drawing pins with its centre at O. A thin board about 4″ square (depending on the size of the paper scale) rotates rather tightly on O to form the prism table. It can be kept clear of the paper scale by three bone buttons attached to it at the corners D, E, F. The rotation of the board is read by two small pieces of celluloid,

C (cut from a photographic "film" negative), which are attached to the under side of the table, and which have a scratch on their lower side. These pieces of celluloid can be cut with a pair of scissors or a penknife, and can be attached to the table with screws or tacks. Or the celluloid can be inserted in a dovetail groove cut in the underside of the table. The celluloid should press lightly on the paper scale where the scratch is. The position can be read to a fifth of a degree. By using two such celluloid pointers at opposite ends of a diameter of the scale, errors of centering will be eliminated.

The apparatus is pointed at a sodium flame. The prism is placed on the table with its diagonal in a line with the telescope, and the nearest angle almost on the axis of rotation of the table. Place blocks A and B (match boxes will do) against two opposite faces, and rotate the table until the edge of the shadow coincides with the cross-wire. Take the reading by both pointers. Change the blocks and repeat with the beam through the other pair of faces.

If the layer of liquid between the two halves of the glass cube is not very thin, or if the refractive index of the liquid is not perfectly uniform, the edge of the shadow will not be sharp.

The method is very sensitive. Thus the refractive index in the example given changes from 1·6323 to 1·6430 for a rotation of 6°, or 1° corresponds to a change of ·0018, and as a fifth of a degree can be estimated, when the shadow ends sharply the refractive index can be found to ·0004.

It is obvious from the figure that the angle ϕ that the table has been turned through, is given by

$$\phi = 90 - 2\theta$$

or

$$\theta = 45 - \frac{\phi}{2}.$$

(NOTE.—If ϕ is greater than 90°, θ is negative, and therefore, $\sin\theta$ in the formula is negative.)

Then if μ_g is the refractive index of the glass, that of the liquid is given by

$$\mu = \frac{1}{\sqrt{2}}\{\sqrt{\mu_g^2 - \sin^2\theta} + \sin\theta\}.$$

If the spectrometer table has no scale, the telescope may be moved instead, but of course the flame and converging lens will have to be moved also.

If the refractive index of the liquid is high compared with that of the prisms, the table may have to be rotated *clockwise* to obtain the second reading. In this case ϕ is of course negative.

107. **Refractive index of a liquid by the method employed in the Pulfricht Refractometer.**—This instrument, like the Abbé's refractometer, makes use of the critical angle at the interface between the liquid and a known block of glass—a cube in this case—to find the

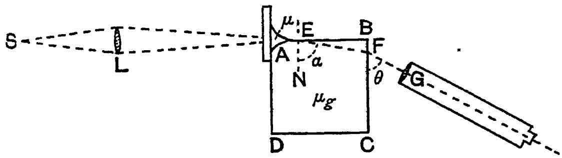


FIG. 133.—Pulfricht Refractometer.

refractive index of the liquid. The glass must again have a refractive index higher than that of the liquid. Thus the essential part of the apparatus is a glass cube (preferably of dense glass of high refractive index) of which two adjacent surfaces AB, BC must be optically flat and at right angles to one another. The corner A can be slightly rounded. A piece of glass is cemented on AD projecting a little beyond A as shown. A few drops of the liquid of which the refractive index is required are placed in the corner A.

Light from a sodium flame S is rendered slightly convergent by a lens L and falls on the corner A approximately in the direction of the face EB. It enters the liquid, and some is refracted into the block. The line EF, which makes with the normal EN an angle equal to the critical angle from liquid to glass, is the limiting direction of the light refracted into the block. Thus if a telescope focussed at infinity is directed along FG—the emergent ray corresponding to EF—half the field of the telescope will appear bright and half dark.

Let α be the critical angle NEF from the liquid to the glass.

Then $$\sin \alpha = \frac{\mu}{\mu_g},$$

where μ and μ_g are the refractive indices of the liquid and the glass respectively.

Let θ be the angle the emergent ray, FG, makes with the surface BC. Then

$$\cos\theta = \mu_g \cos a$$

$$= \mu_g \sqrt{1 - \frac{\mu^2}{\mu_g^2}},$$

$$\mu = \sqrt{\mu_g^2 - \cos^2\theta}. \quad \text{.........................(i)}$$

If no liquid is put in the corner A, the field will obviously be half in shadow for a value of θ' given by

$$\cos\theta' = \mu_g \sqrt{1 - \frac{1}{\mu_g^2}} = \sqrt{\mu_g^2 - 1}$$

or

$$\mu_g^2 = 1 + \cos^2\theta'. \quad \text{.................................(ii)}$$

Thus μ_g itself can be determined.

108. *Apparatus.*—The spectrometer; glass block as above described; sodium flame; convex lens of say 8 or 10 inches focus, and support for same; liquid of which refractive index is required.

The experiment can be performed on the spectrometer. Focus the telescope and collimator for infinity.

Set them in a line, and take the reading of the telescope. By means of the scale turn the latter through some known angle—say 90°. Now place the block on the table with the surface BC about the centre of the table, and adjust it so that the image of the slit shall be on the cross-wire.

The angle GFC which the telescope now makes with the face BC is of course half the angle between the illuminator and telescope—say 45°. If, therefore, the block is clamped in this position, the angle which the telescope makes with BC in any further measurements can easily be found.

Set up the sodium flame in a line with BA; at a distance from the flame a little greater than its focal length, place the convex lens so as to throw a slightly convergent beam on the corner A.

Move the telescope until the field is half in shadow, with the dividing line between light and dark on the cross-wire. Take the reading, and (knowing the reading when the angle CFG was 45°) calculate θ', so determining μ_g from the equation

$$\mu_g = 1 + \cos^2\theta'.$$

Carefully insert a few drops of the liquid in the angle A without disturbing the position of the prism, and again adjust the telescope, until the line dividing light and dark is on the cross-wire, and so find θ.

Then $$\mu = \sqrt{\mu_g^2 - \cos^2\theta}.$$

The refractive index of a crystal or other transparent solid can be found if it has a plane surface. Place this surface on the face BA with a drop of cedar oil or monobromonaphthaline between them, and find the position of the dividing line as before. The light must be focussed on the interface BA between the block and the crystal. There may appear only a band of light; if so, care must be taken to use the right dividing line between bright and dark. To make sure that the right one is being observed, slightly move the lens or the light; the dividing line should not move.

109. **Refractive Index of a Liquid by change of Focal Length of Lens.**—*Apparatus.*—Biconvex lens about 6″ focal length, small plane mirror, a stand carrying a horizontal needle, of which the height above the table may be varied; water, and a little aniline or other liquid.

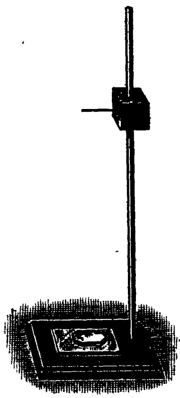


FIG. 134.—Refractive Index of a Liquid.

Lie the mirror on the table on its back and place the lens upon it. By the method of parallax adjust the needle to coincide with its image as seen through the lens by reflection in the mirror.[1] Measure the distance of the needle from the lens, d_1.

Now insert a little water between the lens and mirror.

Again adjust the needle to coincide with its image and measure its distance from the lens, d_2.

Lastly, dry the lens and mirror, insert a little aniline, and find the new distance of the needle from the lens, d_3, at which it coincides with its image.

[1] The experiment may be made with the optical bench as in Experiment § 43, the mirror being attached to the lens by a pair of rubber bands, the power of the lens dry and with each liquid in turn being found as there directed.

Then it can easily be proved, that the insertion of a liquid between the lens and the mirror decreases the power of the lens by an amount $\frac{\mu - 1}{r}$, where μ is the refractive index of the liquid and r the radius of curvature of the lower surface of the lens.

Thus in this case, if μ_a and μ_w are the refractive indices of aniline and water respectively, we have

$$\frac{\mu_a - 1}{\mu_w - 1} = \frac{\frac{1}{d_1} - \frac{1}{d_3}}{\frac{1}{d_1} - \frac{1}{d_2}}.$$

110. **Determination of the Refractive Index of a Glass Prism by the angle of Polarisation by Reflection.**—Place the prism upon the table of the spectrometer, and using a sodium flame observe the image of the slit formed by reflection from one of its surfaces through a double image prism placed in front of the eye-piece of the telescope. There will be two white images of the slit; the double image prism must be so placed that these two images are on the same horizontal level. Then one will be formed of light polarised in a horizontal plane, and the other of light polarised in a vertical plane. The light reflected from the glass will be partially polarised in a horizontal plane by the reflection. If the table be now slowly rotated and the images followed by the telescope, one of the images formed by the double image prism will become fainter and at one position of the prism will vanish altogether, the whole light being now concentrated in the other image. When this is the case, the polarisation by reflection must be complete, and the light is falling on the prism at the polarising angle. The angle between the axes of the collimator and telescope is therefore double this polarising angle. Read the position of the telescope.[1] Several independent readings should be taken, both the prism and the telescope being disturbed

[1] There should be no ambiguity in this, as the double image prism will form two images of the cross-wire as well as of the slit, and if either image is placed upon its *corresponding* cross-wire, the other image will also be found to coincide with its cross-wire. To make sure that there has been no mistake, remove the double-image prism for a moment, and see if the image of the slit is upon the cross-wire.

between each reading. Now point the telescope directly at the slit, the angle it has to be moved through will be 180° minus twice the polarising angle, which latter can therefore be calculated.

The refractive index is given by Brewster's law,

$$\mu = \tan i.$$

ADDITIONAL EXERCISES ON REFRACTION

1. Stick a narrow strip of white paper on a black card. Hold a prism close to the eye, parallel to this strip, and look at the strip through it. Make a drawing of the appearance, indicating which edge of the strip appears red, and which blue. Draw a diagram of the course of rays through the prism to explain the colour.

2. Draw a thick black line on a white card, and examine it with a prism held parallel to the line. Make a drawing showing which edge of the line appears red, and which blue. Draw a diagram to explain this appearance.

3. Look at a distant window through a prism, and slowly rotate the latter. The window sometimes appears coloured and more or less indistinct: at other times, perfectly distinct, and free from colour. In both cases, the image moves as the prism is rotated. Note, and explain, the direction of motion in each case. Has it anything to do with minimum deviation?

4. Replace the screen in Experiment, § 88, by an eye-piece, or a very short focus lens. The spectrum, as seen through either of these, will be found exceedingly brilliant.

5. Replace the screen by a plane mirror, or, better, by a concave mirror, of a radius of curvature equal to the focal length of the second lens. The light forming the spectrum will be reflected back. Turn the mirror until the reflected light passes once more through the system. It will be focussed on the screen; the image will practically coincide with the original slit, and should be free from colour. Rotate the mirror—it will not displace this image.

Between the slit and the first lens interpose a silvered mirror at an angle of 45° to receive about half the beam. This part of the beam will now be reflected off at right angles, and may either be received on a screen or examined in an eye-piece. It will be practically white, although, owing to the imperfection of the lenses, the edges will be coloured.

Pass a knitting needle slowly across the spectrum near the mirror. It will cut off the colour in a part of the spectrum only; the image of the slit will become beautifully tinted, being formed by those colours which are left.

6. Replace the screen of Experiment, § 88, by a convex lens, of large enough diameter to include the whole of the spectrum; place the screen at a little more than its focal length distant from this lens, and adjust it at the conjugate focus of the prism. In this position, a rectangular patch of light will be formed upon the screen, which should be white and evenly illuminated. (If only the *vertical* edges are coloured, the colour can generally be removed by slightly inclining the lens.)

Pass a knitting needle over the surface of the lens, the colour of the patch will change as the knitting needle obstructs different parts of the spectrum. Should sufficient light not be obtained to see the patch on the screen, an eye-piece may be substituted.

7. Place a piece of coloured gelatine in the path of the light through the spectrometer. A dark band will be formed in the spectrum, whose position and extent will depend upon the colour of the gelatine. Set the cross-wire of the telescope, successively, upon the places where this band begins and ends, and take the scale readings. Estimate as nearly as possible the diminution of brightness caused by the gelatine along the spectrum. (This is the most easily done if the gelatine is placed close in front of the slit and covering only its lower half.) Plot the scale readings horizontally, and let the vertical height represent the apparent brightness of the spectrum.

The Dispersive Power of a Prism.

111. *Apparatus.*—Spectrometer; prism; Bunsen flame; salt; lithium chloride; nitre; thallium chloride; magnesium chloride; strontium chloride; an arc light if available; a hydrogen tube, and an induction coil to work it.

Definitions.—The *dispersion* produced by a prism is the difference between the deviations of two known colours.

The dispersive power of a given variety of glass is the ratio of the dispersion produced by a prism of small angle to the deviation it produces.

The reciprocal of the diversive power is more useful and is denoted by the letter ν. It is often called the "**ν number**"; it may be termed its "**aspersivity.**"

It is obvious that the dispersion, and therefore also the dispersive power, depend upon the colours selected. For most purposes yellow and blue are selected; the sodium line and the blue hydrogen or strontium line may therefore be used.

Experiment.—Adjust the prism for minimum deviation as in Experiment, § 96. Determine the deviations of the following lines

as on page 129 (the results of that experiment may of course be used): Sodium; lithium, red and blue (or strontium blue); Hydrogen, red and green.

Subtract the deviation of the red from that of the blue line, and divide it into the deviation of the yellow line; thus obtain roughly the aspersivity from red to blue. To obtain it accurately, calculate the several refractive indices, then $\nu = \frac{\mu_y - 1}{\mu_b - \mu_r}$.

In the same way obtain the aspersivity from red to green and green to blue, each time using in the numerator the refractive index of the yellow.

If possible repeat with other varieties of glass: especially use ordinary crown and flint glasses.

Reversal.

Absorption by sodium vapour of the D lines in a continuous spectrum.

112. *Apparatus.*—Arc light and condenser; spectrometer, the telescope, furnished either with an eye-piece or with a lens to project the spectrum on a screen; ground glass screen; Bunsen flame; wire gauze and metallic sodium; a square of thin glass.

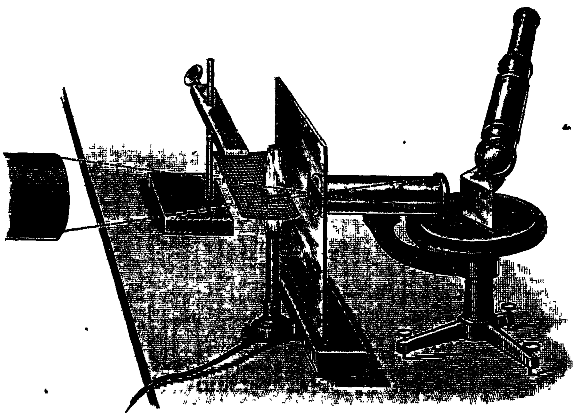


FIG. 135.—Reversal of the Sodium Lines.

Experiment.—Project the image of the crater of the arc upon the slit of the spectrometer with the condenser, arranging the

distances so that the angle subtended at the slit by the condenser may equal that subtended there by the lens of the collimator; then the light from the condenser will just fill the collimator lens. The collimator must be adjusted as usual for parallel light, and the screen properly focussed. A continuous spectrum is produced. Put a piece of metallic sodium about the size of a pea on the edge of the wire gauze and put it in the flame of the Bunsen burner, placed just beyond the slit of the collimator between the slit and the arc. (A sheet of glass must be interposed between the flame and the slit to save the latter from the fumes of the sodium which are very injurious to the metal work.) A dark band will appear in the yellow of the spectrum, which sometimes extends a considerable distance beyond the D lines owing to the large amount of the sodium vapour present. But as the flame dies away, the D lines alone appear as two clear black lines on the bright spectrum.

Interpose a card between the arc and the sodium flame, see that the black lines are suddenly exchanged for two bright lines and occupy exactly the same position. Or arrange the comparison prism to reflect the light from a sodium flame through the upper half of the slit and see that the dark lines in the arc spectrum are exactly collinear with the bright ones of the sodium flame.

It is said to be possible to perform this experiment by surrounding the arc itself by a vessel containing the metallic sodium, so that the sodium flame immediately surrounds the arc; but it is not easy to do it this way, as the sodium interferes with the arc.

Spectrum Photography.

113. *Apparatus.*—Source of light, preferably, an arc lamp; the spectrometer may be used, but better results can be obtained with the boxes described on page 108.

The eye-piece of the telescope is removed and a small camera substituted. The distance from the objective to the plate must be capable of being adjusted and also the inclination of the plate to the axis of the tube, for by setting the plate at an angle with the axis it is generally possible to improve the definition, as the

objective supplied with the instrument is not usually corrected for the violet end of the spectrum. If a photograph of the continuous spectrum is required, the crater of the arc, or the lime if an oxy-hydrogen jet is used, is focussed upon the slit.

The distances of the arc and lens should be so adjusted that the angle L_1SL_2 subtended at the slit by the lens (Fig. 136) is equal to the angle C_1SC_2 subtended by the lens of the collimator. If this is the case the whole surface of the collimating lens will be lighted up, and at the same time the internal reflection from the

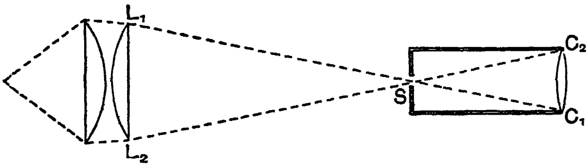


FIG. 136.—Illumination of Collimator by an Arc.

tube of the collimator will be diminished. It is therefore necessary, as a rule, to remove the arc to some distance from the collimator. To focus the spectrum it is best to obtain a bright line spectrum by separating the carbons, the plate being adjusted until the lines are sharp throughout the length of the spectrum. The time of exposure will depend upon the width of the slit and the magnification. It will probably be from 5 to 30 seconds. To obtain the bright line spectrum, the carbons should be vertical and separated. This time, with the ordinary tube spectroscopes, great difficulty will be caused by the internal reflections in the tube of the collimator from the arc crater. To diminish this, a stop should be placed over the slit to reduce its vertical height, so that the light from the crater may not enter the tube at all.

With the ordinary spectrometer it is impossible to obtain a good spectrum photograph. There is always a very large amount of reflected light. This is because the tubes of both collimator and telescope are made only a little larger than the diameters of their lenses. For good work these should be very large indeed, and the best form of collimator for photographic purposes is the cubical box, blackened inside with the slit and lens in the

centre of opposite faces, as described on p. 108. Even then the slit must not be mounted in a narrow tube. If it is wished to be able to focus it, it should be mounted in a tube 2″ or 3″ in diameter. With such a box, internal reflection is reduced to a minimum. In the same way the telescope objective and the photographic plate should be at opposite ends of a large box. When these boxes have been adjusted for minimum deviation, they may be screwed down to the table with small angle pieces.

During the exposure, the space between the collimator and telescope lenses must be covered in to exclude the scattered light in the room.

It is useful to have some fixed reference mark to enable the spectra to be readily compared with one another, and their position easily identified. A needle point attached to the fixed frame that carries the dark slide at the position of the D line is very useful for this purpose.

Hydroquinine forms a clean and convenient developer for this work. Pyro-soda may also be used. The spectra should be taken on orthochromatic plates, and a yellow solution will be required to equalise the exposures required for the violet and for the other parts of the spectrum. A solution of auramine in a glass cell will do all that is required, and its strength can be adjusted to anything desired. It should be inserted between the arc and the slit. Photographs should be taken of:

1. An ordinary spectrum.
2. A bright line spectrum.
3. An absorption spectra side by side with a bright line spectrum, or with the interference spectrum referred to on page 300, using the comparison prism for the absorption one.

Ordinary dry plates may be rendered orthochromatic by bathing them in a weak solution of erythrosin and ammonia, or better by a new dye called Homocol (Bayer & Co.).

Prepare a solution of:

Water - - - - - - - -	100	parts.
Homocol (1 in 1000 alcoholic solution) -	1·5	„
Ammonia (·880) - - - - - -	·2	„

The bath must be in a *perfectly* clean dish. The plate, after being dusted, must be immersed in the solution for two minutes, placed in running water for two or three minutes, and then dried as quickly as possible in the dark. It will keep for some weeks. The solution does not keep.

Orthochromatic plates must be developed in a very dim light. A tank lamp containing a solution of about equal parts of naphthol yellow and Titan scarlet, and then a sheet of double flashed ruby glass will give a safe light.

Anomalous Dispersion.

114. The colours of smaller wave-lengths are as a general rule deviated to a greater extent on passing through a prism than those of longer wave-length. The extent to which different substances deviate the colours vary, and even the proportions in which colours of different wave-lengths are deviated vary. With some substances the variation becomes very marked, and bright dyes which absorb any portion of the spectrum strongly have a refractive index which varies very rapidly as it approaches the colours absorbed. It was shown by Sellmeier that if colours of certain wave-lengths, $\lambda_1, \lambda_2, \ldots$, are entirely absorbed by a medium (*e.g.* incandescent sodium vapour absorbs the light from the crater of an arc in the neighbourhood of each of the D lines), the refractive index is given by

$$\mu^2 = 1 + n_1 k_1 \frac{\lambda^2}{\lambda^2 - \lambda_1^2} + n_2 k_2 \frac{\lambda^2}{\lambda^2 - \lambda_2^2} + \cdots,$$

where $n_1 n_2 \ldots$, $k_1 k_2 \ldots$ are constants. (See Edser, page 376.)

Thus on either side of an absorption band there is an abnormal change of refrangibility of such a kind that the refraction is increased on the one side of the absorption band, and diminished on the other side. The result is that when light passes through a prism of such a material, the colours on the opposite sides of the absorption band are relatively displaced in the spectrum. This will be most noticeable to the eye if the absorption occurs somewhere near the yellow. This is the case with fuchsine, an aniline dye which has a marked absorption in the green, with the result that a fuchsine prism deviates the red, orange, and yellow to a greater degree than the blue and violet, the green of course being

missing. The order is thus violet, red, orange, yellow, the violet being least refracted. There is a small dark space between the violet and the red.

By making a very strong solution of fuchsine in alcohol, and placing a drop between two glass plates inclined at an angle of about 10° (the drop being kept in position by capillary attraction), and looking through the prism at a distant bright slit, the order of the colours can easily be seen.

Kundt has shown that anomalous dispersion occurs with all dyes which have a "surface colour," *i.e.* which reflect a colour different from the one they transmit. The light reflected from fuchsine is green, whilst the light it transmits is red.

115. Wood (*Phil. Mag.*, 1898, and June, 1901) describes some cyanine prisms which he made by melting the solid dye. He cut two pieces of 3 cms. from plate glass, five to seven millimetres thick. Having cleaned them, he pasted a strip cut from a visiting card along one edge of one of them. Along the opposite edge he strewed a train of crystals about 2 mm. wide. They should be of uniform depth and pushed into a straight line with a glass plate. Both plates are then laid on asbestos over a Bunsen burner and heated until the cyanine fuses. Just before fusion, which occurs at 175° C., the surface colour will change from a brilliant green to a plum colour. As soon as no solid particles remain, an edge of the other plate is dipped in the liquid and the plate lowered until its other edge rests on the paper strip. Use plenty of crystals. The plates are now lifted up together and put in a vice. The exact pressure is important, as if too little pressure is used it will not be transparent, and if too much the surfaces will be curved. All the prism except a narrow strip near the thin edge should be covered with black paper, or it may be mounted on a card with a narrow rectangular aperture. For illustration he advises the glass plates to be left on. For measurement strike the edge of the plate with a block of wood, when it will be split apart, generally leaving the whole prism on the one plate.

The angle of the prism can be found by reflecting the light first from the surface of the glass and then from the prism, the back of the glass plate being greased to avoid confusion. Wood used an angle of about 1°. The refractive index which he measured is shown in the accompanying diagram.

In the later paper he says he succeeded with a new specimen

of cyanine obtained from Grübler of Leipzig, in obtaining transparent prisms with an angle of more than one degree. The new specimen was in the form of lumps of quite small crystals. The old cyanine had the appearance of long shaped crystals not caked together.

The vice is to be applied close to the refracting edge only, as shown in the figure, and he finds that a clamp of one of Gurtner's laboratory supports gives the best results. I find an ordinary vice convenient, the glass being held between two pieces of cardboard.

116. By crossing the prism with a diffraction grating of 2,000 or 3,000 lines to the inch, and looking at an arc lamp through the combination, the diffraction spectra are seen deviated by the prism, the red ends being turned up and the blue ends down. An ordinary narrow angle prism will do nearly as well as the grating. The arc must be at some distance, say, fifteen feet. A rather large pinhole may be put in front of the arc, and then it need not be so distant.

117. But the best way to exhibit anomalous dispersion is one (also due to Wood) which makes use of a very curious property of sodium vapour. If a pellet of sodium is heated in an exhausted glass tube, the sodium melts, and then vaporises; but the vapour does not fill the tube as an ordinary vapour would; on the contrary, it forms a dense cloud over the sodium. The density also is not uniform, but decreases upwards towards the edge of the cloud. Thus if a beam of light is caused to pass through it, the amount of sodium it will encounter increases from the margin of the clouds at the top to the centre, and the cloud behaves as a prism of which the axis is horizontal and the edge uppermost.

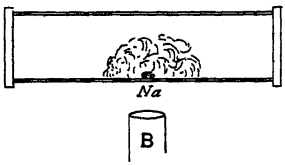


FIG. 137.—Sodium Vapour in a Vacuum.

The experiment consists essentially in combining a spectrometer in which a number of sodium clouds (*i.e.* a train of sodium prisms edge uppermost) disperse the light in a vertical plane, with an ordinary prism or grating spectroscope dispersing the light in a horizontal plane.

The source of light may be an arc M, or a Nernst lamp; the latter may be put close to F without the lens L. The collimator CF for the first spectroscope has a horizontal slit; it consists merely of a tube with an achromatic lens at one end, of about 6 or 8 inches focal length, and a slit at the other. The slit must as usual be at the principal focus of the lens. The tube may be supported in a retort stand. The sodium is best contained in a steel bicycle tube AB, about $1\frac{1}{4}$ or $1\frac{1}{2}$ inches diameter. The tube must have a small side tube D, by which it is connected to a Fleusz pump to exhaust the air from it. The tube should be about a yard long, and furnished with worked glass ends (plate glass will do), which can be fixed on with sealing wax. In attaching the ends both the tube and the

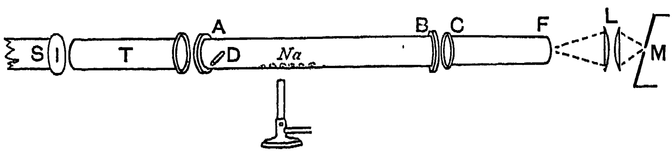


FIG. 138.—Anomalous Dispersion of Sodium Vapour.

glass should be warmed, and the wax can be put on with the aid of an ordinary soldering iron. If cracks come as it cools, they can be removed with the iron. The sodium must of course be inserted before the second end is attached. About a quarter of an ounce of sodium, cut into small cubes about $\frac{3}{8}$ in. square, is used; the cubes are to be arranged in a row about the middle foot of the tube. The tube is then supported horizontally, the end is put on, and the tube exhausted. It is to be adjusted exactly in line with the collimator.

The tube is followed in the same line by a telescope T, with the eye-piece removed, *i.e.* by an achromatic lens mounted at the end of a tube a little shorter than the focal length of the lens.

Still in the same line follows the collimator S of the second spectroscope, which should have a high dispersive power. The slit of this spectroscope must be exactly in the focal plane of the telescope T, which will then focus the spectrum produced by the sodium prisms upon it. It is as well to mount the telescope T on an extension of the spectroscope.

When all is in adjustment, before heating the sodium a sharp image of the first horizontal slit F should be thrown on the middle of vertical slit of the ordinary spectroscope S. To see

that all the tubes are in line, hold a white card in succession in front of the first collimator lens C, the ends of the sodium tube A, the second collimator lens and the prism, and see that each time there is a full beam of light. Some black velvet should be put over the tubes at S, A, and B, to exclude extraneous light.

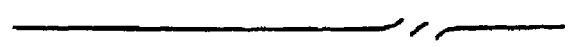


FIG. 138.—Anomalous Dispersion.

Now, place a Bunsen burner under the sodium tube, so that it may heat one or two pellets of sodium. As the cloud forms, the two black sodium lines will appear; at the same time the spectrum, which was originally a fine straight horizontal line of colour, will become bent, being curled up as it approaches each dark line from the red end, and curled down as it approaches each line from the violet side. As the cloud becomes more dense the absorption broadens, and the little curl between the two sodium lines fades away; but on removing the burner this will presently reappear. In repeating the experiment, the flame should be applied to another of the sodium pellets. If it is desired to see the sodium cloud form, a glass tube must be used in place of the steel one; but a glass tube nearly always cracks if it is heated a second time, and is therefore not so suitable for the experiment itself.

CHAPTER VI

ILLUSTRATIONS OF METHODS OF DETERMINING THE VELOCITY OF LIGHT

Velocity of Light.—It is impossible, of course, to determine the velocity of light without very special apparatus: but the principle of the various methods can be much better understood if the optical parts are adjusted in each case, and this can be done without any special apparatus. Especially in the case of Foucault's method, the ordinary student usually finds that the adjustments of the apparatus are by no means so easy as they appear from cursory reading in the text-book, and that actual experimenting with the rotating mirror greatly assists a proper comprehension of the theory.

Fizeau's Method for determining the Velocity of Light.

118. *Apparatus.*—The source of light may be the screen with the small hole and the burner used with the optical bench; a lens of 6 or 8 inches focus; a piece of very thin plane parallel glass about half an inch broad (or a flat micro cover glass selected as described on p. 108); a pair of lenses to form the collimator and telescope of about 10 or 12 inches focus; a positive low-power eye-piece, or failing that, an ordinary small convex lens of about 2 inches focus; a plane mirror; a cardboard disc with a large number of teeth equal in breadth to the spaces between them—mounted on an axis that may be easily rotated, or better, it may be turned by a small electro-motor.

Adjustments.—Measure the focal lengths of the lenses that are to be used for the collimator and telescope respectively. Mount them some distance apart on a table facing one another. Place the rotating disc exactly at one of the focal points of the lens L_1

(Fig. 140). A little behind it place the unsilvered mirror at an angle of 45°. With the 6-inch lens form an image of the hole in the screen A, illuminated from behind, upon a tooth of the disc at *a*. Turn the sector until a space allows the light to pass through to the lens L_1. It will emerge from L_1 in a parallel beam, since A was at the principal focus of L_1, and so reach L_2. The lens L_2 will form an image of the source of light at its principal focus. Hold a piece of paper here to find its position,

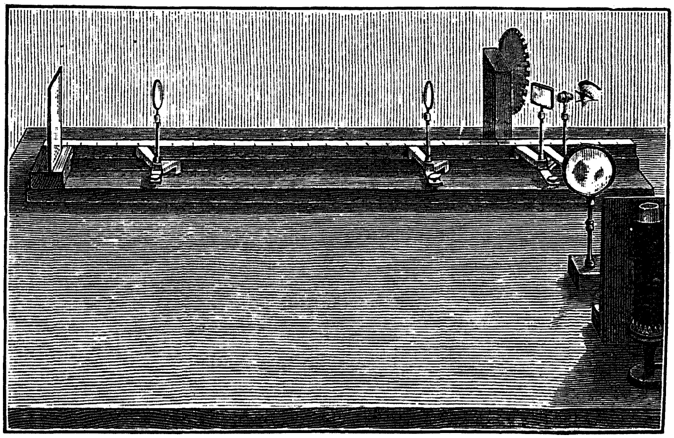


FIG. 139.—Arrangement of Apparatus to illustrate Fizeau's Method of determining the Velocity of Light.

and then place the mirror M in the position occupied by the paper, so that the light may be reflected back upon its course. If the mirror is normal to the common axis of the lenses, the light will again pass through the lenses L_2 and L_1, and be converged to A. From here a part of it will be reflected by the parallel mirror back to the source, but part will go through and may be received in an eye-piece, E. (The eye-piece must be of sufficiently low power to allow room between it and the disc for the unsilvered reflector.)

On rotating the disc, the light will alternately be cut off or allowed to pass as the teeth and spaces pass A.

When the speed is increased, persistence of vision will cause

the light to appear uniform. If the speed of the disc could be sufficiently increased for a cog to have time to occupy the position of a space while the light travels from A to the mirror M and back, the light which passes through each space would be cut off by the succeeding cog, and darkness would result. It is, of course, quite impossible to reach this speed. In the actual

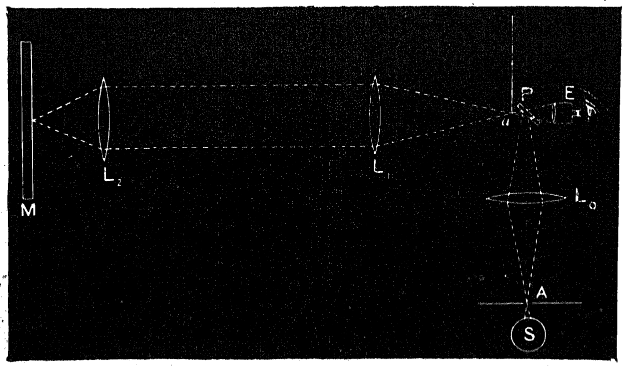


FIG. 140.—Path of Rays through the Apparatus illustrating Fizeau's Method of determining the Velocity of Light.

experiment, the distance from A to M was rather over 17,000 metres, and then with 720 teeth on the disc, the speed was able to be made sufficiently high.

The time from A to M will be $\frac{d}{V}$, where V is the velocity of light, and d this distance, so that in the time $\frac{2d}{V}$, the tooth must move into the position occupied by a space. If there be m teeth on the wheel, this means $\frac{1}{2m}$ of a revolution, and if the wheel rotates n times per second, it will occupy $\frac{1}{2mn}$ of a second for this movement.

Therefore, when darkness occurs:

$$\frac{1}{2mn}=\frac{2d}{V}, \text{ or } V=4mn \, . \, d.$$

If the speed of the disc be still further increased, the light will appear again, and will reach its maximum brightness when a space has time to take the place of a space while the light goes from A to M,

and back, and V is given by $2mn'd$, n' being the number of revolutions a second.

Now as the speed is increased, the brightness dies away and grows again gradually. If teeth and spaces are all equal in width, the diagram of brightness is made of straight lines as in Fig. 141.

FIG. 141.—Curve of brightness.

Fizeau, in 1849, with this apparatus found the velocity 315,364 kilometres a second in air.

Cornu, by employing a small wheel of only 2 or 3 cms. diameter, was able to give it a much higher speed. He also placed the telescope at a greater distance, so that he was able to obtain extinctions of a higher order, so that the light which went through a space returned upon the successive teeth up to the 21st. His collimator and telescope were also of a very large diameter, 15 cms. for the telescope and 37 cms. for the collimator lens.

Messrs. Young and Forbes used two telescopes at distances in the ratio of 12 and 13, and observed when the light that was fading in the one was equally bright with that growing in the other, as at P and Q on the curve of brightness (Fig. 141), they were able to determine with more accuracy the speed at which this occurred, than the speed at which the light was just extinguished.

Foucault's Method for Determination of Velocity of Light.

119. *Apparatus.*—For the rotating mirror, the mirror supplied for a manometric flame apparatus will do very well. Failing this, a plane mirror about 4 inches square can be mounted roughly on a vertical axle. A source of light; a screen with a hole in it; an unsilvered reflector; an eye-piece; a convex lens of about 10 or 12 inches focal length; and also a concave mirror of long focal length—2 or 3 feet if possible.

Adjustments.—Measure the focal length of the mirror. Set up the screen A and adjust the lens to form an image of A at a distance of 5 or 6 feet on a screen S. At a distance from S equal to the radius of curvature of the concave mirror, place the centre of the rotating mirror M. Place the concave mirror as in Fig. 142, so that its centre of curvature coincides with the axis of rotation of the revolving mirror. Adjust the lens to

focus the light from A upon the surface of the concave mirror, after reflection in the rotating mirror when this mirror is turned into the right direction. Then, it ought to return upon its own path, after reflection in the concave mirror, striking the rotating mirror again at the same angle, and forming an image of A coincident with itself.

Place the unsilvered reflector in the path of the rays between A and L, and some of this light will now form an image at B. Adjust the eye-piece behind B to view this image. Rocking the rotating mirror ought now to produce no displacement of the image B; and if the mirror is rotated, an image of A ought to be visible once in each revolution, during the time that the

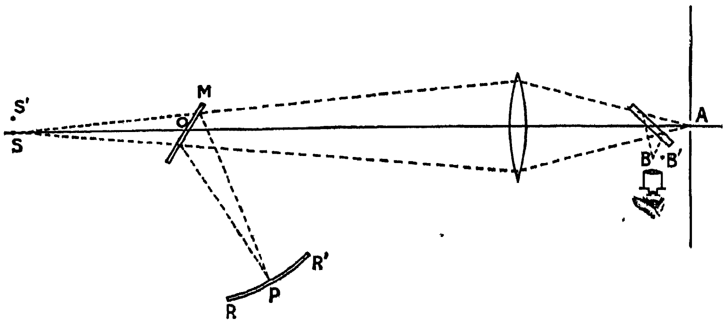


FIG. 142.—Path of Rays through Apparatus illustrating Foucault's Method of determining the Velocity of Light.

light falls upon the concave mirror RR′; if the mirror be rotated sufficiently rapidly, a faint image should be visible at B the whole time.

In whatever position M may be, if the light falls anywhere upon RR′, the image formed on RR′ will always be the image in the mirror M of that at S, so that the apparent position of the image in R is always at S. But if the speed of rotation of M could be made sufficient, the light which forms the image P in R which is reflected from the mirror while in one position, would, after travelling from M to P and back, find the mirror had moved, and therefore the image of P in the mirror would appear at a point, S′, a little to one side of S, and the conjugate image B would have moved to B′. As the light will be deflected through twice the angle θ, the angle SOS_1 will be 2θ.

Let the distance MP be d, then

$$SS_1 = 2\theta d. \quad \text{(i)}$$

If t be the time taken by the light to pass from M to P and back,

$$t = \frac{2d}{V}. \quad \text{(ii)}$$

If n is the number of revolutions per second of the rotating mirror at this moment, the angle described in one second will be $2\pi n$, and therefore, in the time t, it will be given by

$$\theta = 2\pi nt. \quad \text{(iii)}$$

Let $AL = a$, $LM = b$; then as BB′ and SS′ are conjugate with respect to the lens L,

$$\begin{aligned} BB' &= SS_1 \left\{\frac{a}{b+d}\right\} \\ &= 2d\theta \cdot \frac{a}{b+d} \text{ from (i)} \\ &= 2d \cdot 2\pi nt \cdot \frac{a}{b+d} \text{ from (iii)} \\ &= \frac{8\pi nad^2}{V \cdot (b+d)} \text{ from (ii),} \end{aligned}$$

from which V can be found.

Foucault made the distance d about 20 metres, and used a turbine to rotate his mirror, the latter being very accurately balanced on its axis of rotation that it might rotate without shake. He was unable to place the concave mirror at any great distance from the rotating one, owing to the loss of light, for the light will only be reflected while the mirror M moves through an angle equal to $\frac{1}{2}$ROR′, and thus, as RR′ is placed at a greater distance, the light becomes fainter and fainter.

Michelson's Method for Determination of Velocity of Light.

Michelson, in 1879, used a slight modification of Foucault's method, replacing the concave mirror by a plane mirror and placing a convex lens between the two mirrors.

120. *Apparatus.*—To imitate this experiment we shall require to replace the concave mirror by a plane mirror and a convex lens of as long a focal length as possible.

Measure the focal length of the lens, and place it so that it may be this distance from the axis of rotation of the rotating

mirror M, so that light which diverged from this axis would emerge from the lens as a parallel beam, and would fall normally upon the plane mirror if this is placed perpendicular to the axis of the lens. Place the screen A at a short distance from the revolving mirror. Then, as the distance AMC is greater than the focal length of the lens, a real image of A will be formed on the other side of the lens. The reflecting surface of the mirror RR′ must be made to coincide with this image. To do this put a screen to receive the light emerging from L, and

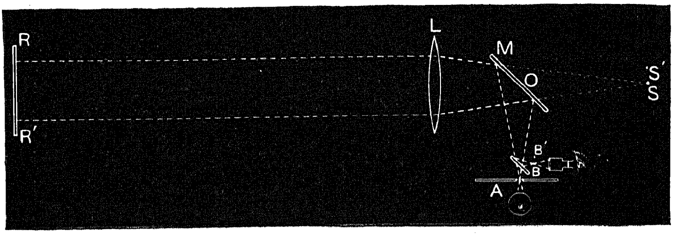


FIG. 143.—Path of Rays through Apparatus illustrating Michelson's Method of determining the Velocity of Light.

adjust it until the image of A is clearly focussed. Then replace the screen by the mirror RR′. See that the light from RR′ passes again through the lens L. Insert a plane unsilvered mirror to reflect the light to B, and examine the image with an eye-piece as before.

If AM $=a$, MC $=b$, and RL $=d$.

The time taken from M to R and back will be

$$\frac{2(b+d)}{V}=t.$$

During this time the mirror will move through an angle θ, given by

$$\theta=2\pi nt,$$

n being the number of revolutions per second; and therefore AA_1 is the displacement of the image

$$AA_1=2\theta\,.\,a=4\pi nt\,.\,a=\frac{8\pi na(b+d)}{V}.$$

As this image would be visible during the whole time that the light from M falls upon the lens L, the distance RR′, which depends merely

upon the distance from A to M, can be increased indefinitely without appreciable loss of light.

Michelson made this distance about 2000 feet, and using an air turbine to drive M, he was able to obtain a displacement of the image AA′ of more than 13 mms.

ADDITIONAL EXERCISES ON CHAPTER VI

1. Measure the distances apart of the pieces of apparatus of Experiment, § 118. Count the number of teeth, *m*, on the wheel: assume the velocity of the light given in the text, and calculate the number of revolutions of the disc that would be required to exactly extinguish the light.

2. Measure the distances *a*, *b*, *d*, in Experiment, § 119, and calculate the number of revolutions of the mirror per second that would be required to produce a displacement, BB′, of 1 mm. in the position of the image.

3. Measure the distances *a*, *b*, *d*, in Experiment, § 120, and calculate the number of revolutions of the mirror per second that would be required to produce a displacement, AA′, of 1 mm. in the position of the image.

CHAPTER VII

FURTHER EXPERIMENTS WITH THE OPTICAL BENCH

The Refractive Index of a Concave Lens.

Apparatus.—Optical bench; bi-concave lens of about 12 inches focus.

121. (*a*) Find the focal length by one of the methods already given.

(*b*) Find the radius of curvature of each surface treated as a concave mirror by reflecting the light back to form an image of the ruled glass on the white paper round it. The distance from the surface to the screen is the radius of curvature.

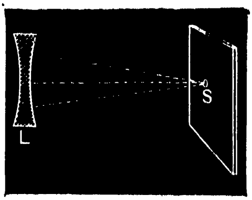


FIG. 144.—Curvature of the Surface of a Concave Lens, L. The light is reflected back and focussed on the screen, S.

Find μ, the refractive index, by substituting in the formula:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right),$$

where f, r, s are measured towards the light, and r is the surface upon which the light falls.

The Refractive Index of a Bi-convex Lens.

Apparatus.—Optical bench; bi-convex lens of about 10 inches focus.

122. (*a*) Find the focal length. This is most easily and accurately done by placing a plate-glass mirror behind the lens,

and reflecting the light back to form an image of the ruled screen on the paper round it. The distance of the lens from the screen is the focal length, f (with of course a negative sign).

(*b*) Move the lens forward and (without a mirror behind) obtain an image by the light reflected from the back surface of the lens. It will be much fainter than the one obtained with the mirror, but is easily seen. Let the distance of the lens from the screen be u_1.

(*c*) Turn the lens round and again obtain an image by reflection from the back surface of the lens. Measure the distance of the lens from the screen. Let it be u_2.

Then, $$\mu = 1 - \frac{1}{2 + f\left(\frac{1}{u_1} + \frac{1}{u_2}\right)}.$$

Remember that f is negative.

In the case (*b*) the light is returning practically along its path, and therefore on reflection at the back surface, the incident and reflected rays must coincide, and must therefore strike the surface normally. So that the rays *in the glass* are radii of the surface; and if they emerged at the back of the lens,—as most of the light does emerge,—they will proceed without bending, since the incidence is normal; and will seem to come from the centre of curvature of that surface. But the point from which the emergent light appears to come is the "image." Thus, in this case the image of the screen is at the centre of curvature of the back surface of the lens. As the focal length is known, we can easily calculate the position of this image by the formula

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f},$$

and the v_1 so found is the radius of curvature of the back surface of the lens. Call it s_1. Then

$$\frac{1}{s_1} = \frac{1}{f} + \frac{1}{u_1}.$$

On turning the lens round, the curvature of the other surface is found in the same way from the formula

$$\frac{1}{s_2} = \frac{1}{f} + \frac{1}{u_2}.$$

In the ordinary formula for a lens, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$, the curvatures

are to be measured towards the light, so that the curvature of the surface s_2 is to be considered negative. Substitute $r=-s_2$, $s=+s_1$,

$$\frac{1}{f}=(\mu-1)\left\{-\frac{1}{f}-\frac{1}{u_2}-\frac{1}{f}-\frac{1}{u_1}\right\}$$

$$=-(\mu-1)\left\{\frac{2}{f}+\frac{1}{u_2}+\frac{1}{u_1}\right\},$$

or

$$\mu=1-\frac{1}{2+f\left(\frac{1}{u_1}+\frac{1}{u_2}\right)}.$$

(Of course, f is a negative quantity as this is a convex lens.)

The Dispersive Power of a Convex Lens.

DEFINITION.—The dispersive power, ϖ, is the ratio of the dispersion, Δ, to the deviation, δ; therefore $\varpi=\frac{\Delta}{\delta}$.

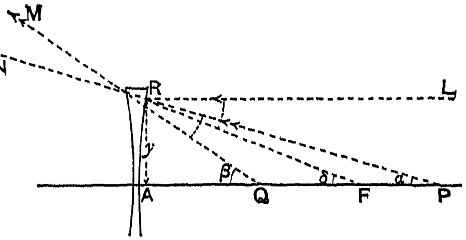


FIG. 145.

123. Let P and Q be conjugate foci of a thin lens, and F its principal focus. Then

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

becomes

$$\frac{1}{AQ}-\frac{1}{AP}=\frac{1}{AF}.$$

Call $AR=y$, and multiply this equation all through by y.

$$\frac{y}{AQ}-\frac{y}{AP}=\frac{y}{AF},$$

or

$$\beta-\alpha=\delta;$$

where α, β, δ are the angles made with the axis by PR, QR, and FR respectively; for, since the angles are small, we may use their circular measure instead of their sines, and write

$$\beta=\sin\beta=\frac{y}{RQ}=\frac{y}{AQ}\text{ (nearly).}$$

But $\beta-\alpha$ is the angle ORP, *i.e.* the deviation. So that the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ expresses the fact that the deviation of the ray from any point P, incident at a given point on the lens R, is constant, and equal to that of a parallel ray incident at the same point, and this deviation is $\frac{y}{f}$.

This deviation varies with the colour, and we have

$$\delta_v=\frac{y}{f_v}, \quad \delta_r=\frac{y}{f_r},$$

where v and r refer to the violet and red light respectively. Thus

$$\varpi=\frac{\Delta}{\delta}=\frac{\delta_v-\delta_r}{\delta}=\frac{\frac{y}{f_v}-\frac{y}{f_r}}{\frac{y}{f}};$$

or, dividing by y,

$$\varpi=\frac{\frac{1}{f_v}-\frac{1}{f_r}}{\frac{1}{f}}.$$

The f in the denominator is supposed to be the focal length for yellow light.

Suppose the ruled glass, lens, and screen to be adjusted to give conjugate foci. The screen will have to be moved slightly to get the image clear for the different colours. If the lens and ruled glass are not moved, u will remain constant, and we shall have

$$\varpi=\frac{\left(\frac{1}{v_v}-\frac{1}{u}\right)-\left(\frac{1}{v_r}-\frac{1}{u}\right)}{\frac{1}{v}-\frac{1}{u}}$$

$$=\frac{\frac{1}{v_v}-\frac{1}{v_r}}{\frac{1}{v}-\frac{1}{u}}=\frac{\frac{v_r-v_v}{v_v v_r}}{\frac{u-v}{uv}};$$

or

$$\varpi=\frac{v_r-v_v}{u-v}\cdot\frac{u}{v}$$

(nearly, for v_v, v_r, and v may be considered equal for purposes of ratio).

Thus, we have to find the difference between v_v and v_r with the same accuracy as the other quantities involved.

124. *Apparatus.*—Optical bench: coloured glasses or solutions;[1] positive eye-piece with cross-wires on stand, white light and a sodium flame.

(*a*) Adjust the cross-wires, lens, and focussing screen, and using a sodium flame find u and v as usual (§ 41) (correcting for index errors).

(*b*) Replace the focussing screen by the eye-piece. (See that the cross-wires are clearly in focus.) Look through the eye-piece, and the wires will appear very brilliant and highly coloured.

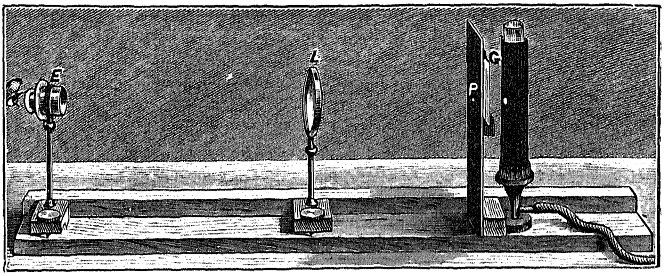


FIG. 146.—Achromatism of a Lens.

Place the red gelatine, G, close in front of the cross-wire screen, P (Fig. 146), and set the eye-piece so that the lines seem quite clear.

To obtain the exact position the method of parallax must be used. Adjust the eye-piece and the lens stands, until the cross-wire of the eye-piece is exactly on the image of the wires of the screen, and move the eye to and fro to see if they shift over one another, altering the distance of the eye-piece until there is no perceptible movement.

Then take the reading on the scale.

(*c*) Change the red gelatine for a deep blue one, and repeat the observation.

The difference between the readings with the red and the blue gelatines is $v_r - v_v$.

(*d*) Substitute in the formula:

$$\varpi = \frac{v_r - v_u}{u - v} \cdot \frac{u}{v}.$$

(Note that v is *negative.*)

[1] See p. 446.

The above determination does not give any definite value for ϖ, as we do not know for what wave-length we had adjusted the eye-piece. Much more accurate results can be obtained by using monochromatic lights.

A sodium flame should be used for the determination of u and v. The wave-length is 5896. A hydrogen tube with a red glass and a blue glass may be used to find $v_r - v_v$. The lines given by the hydrogen tube are: a bright red line, C, of wave-length 6563; a greenish blue line, F, of wave-length 4863; and a bluish violet line, H, of wave-length 4341. (This last is very close to the Fraunhofer line G.) The red potassium line, K, 7677, gives another line in the extreme red, and by using it, the partial dispersion from A to D can be found if wished. (A is very near the potassium line.) The dispersion generally used is from C to F. As we shall be observing the image directly with an eye-piece, the small amount of light given by the hydrogen tube or the potassium flame will not matter; there will be sufficient for the purpose.

Focal Lines by Oblique Reflection at a Concave Mirror.

125. *Discussion.*—If light from a point P falls obliquely on a concave mirror ASR, of which O is the centre of curvature, the light in the

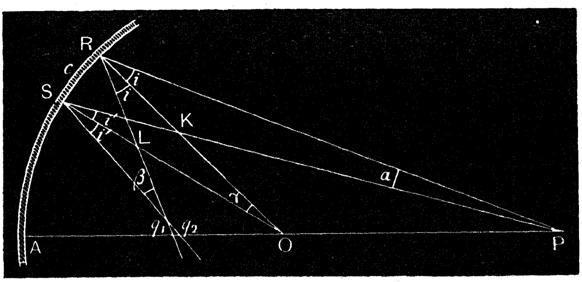


FIG. 147.—Focal Lines.

plane of the paper that is incident on the mirror between the points S and R passes approximately through the point q_1 (which is in the limit, the point of contact of Rq_1 with the caustic. Every ray cuts the axis (somewhere near q_2). If the whole figure be imagined to be rotated about the axis POA of the mirror, the point q_1 will

describe a circle of which the centre lies on the axis OA, the part of the axis through which the rays pass is not moved by the rotation. Thus the whole of the light from the zone of the sphere which RS is developed into by this rotation, passes firstly through the circular ring or annulus resulting from the relation of q_1, and secondly through a short length of the axis near q_2. If the figure be rotated only through a small angle A (instead of a complete revolution), RS will sweep out a small portion of a sphere only, viz., $R_1R_2S_2S_1$, of which the centre of curvature is still O, and the point q_1 will describe only a very short portion of the arc of the circle, which may be considered

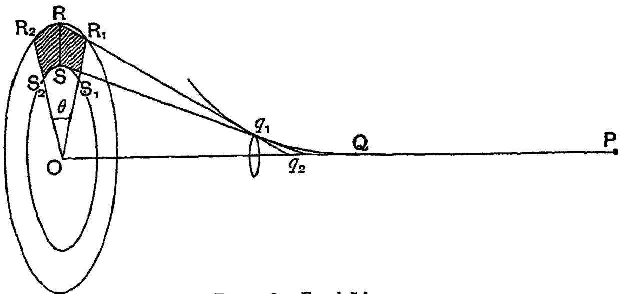


FIG. 148.—Focal Lines.

to be a short straight line normal to the plane of the paper and passing through q_1. This line is called the primary focal line, and is one image of the point P formed by oblique reflection at the mirror $R_1S_2S_1R_2$. Its position is given by

$$\frac{1}{v_1}+\frac{1}{u}=\frac{2}{r\cos i}, \qquad \text{(i)*}$$

when
$$PR=u,$$
$$q_1R=v_1,$$
$$\angle PRO=i.$$

* To prove these, we have

$$\left.\begin{aligned}OKP&=i+\alpha\\ =RKS&=i'+\gamma\end{aligned}\right\}; \quad \therefore\ i'-i=\alpha-\gamma$$

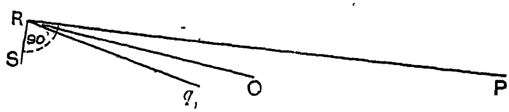


FIG. 149.

and
$$\left.\begin{aligned}OLq_1&=i+\gamma\\ =RLS&=i'+\beta\end{aligned}\right\}; \quad \therefore\ i'-i=\gamma-\beta;$$
$$\therefore\ \alpha+\beta=2\gamma,$$

The second focal line is at q_2, and if the screen upon which the light is to be received is about normal to the line Rq_2, the line will lie in the plane of the paper. If the outline of the mirror is circular (instead of bound by the two arcs R_1R_2, S_1S_2 and the two radial lines R_1S_1 and R_2S_2), the shape will be a figure of eight, but for a mirror of the outline shown in Fig. 148, this is two narrow triangles with a common vertex.

The position of q_2 is given by

$$\frac{1}{v_1}+\frac{1}{u}=\frac{2\cos i}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)*}$$

126. Apparatus as above, with the addition of a protractor.

Set up the mirror and let the $\angle$PRO be known (say 30°), focus the vertical wire and find v_1, and the horizontal wire and find v_2. Repeat for two or three values of u. Compare the values with those given by the equations.

Tabulate results thus:

r	i	u	By experiment.		Calculated.	
			v_1	v_2	v_1	v_2
(found previously)						

The Focal Lines by Oblique Refraction at a single Spherical Surface are not important.

Focal Lines produced by Oblique Refraction through a Lens.

127. *Theory.*—Let ASR be the first surface of the lens. Let P be the bright point and q_1 the *primary focal line.*

---

i.e.
$$\frac{RS.\cos i}{RP}+\frac{RS.\cos i}{Rq_1}=\frac{2RS}{RO},$$

or
$$\frac{1}{u}+\frac{1}{v_1}=\frac{2}{r.\cos i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Again, the area of the triangle PRq_2=sum of areas of PRO and ORq_2.

Thus $\frac{1}{2}uv_2\sin 2i=\frac{1}{2}ur\sin i+\frac{1}{2}rv_2\sin i,$

or, dividing through by $\frac{1}{2}urv_2\sin i$,

$$\frac{2\cos i}{r}=\frac{1}{v_2}+\frac{1}{u} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

From the figure we get

$$\left.\begin{aligned} \mathrm{OKP} &= i+\alpha \\ =\mathrm{SKR} &= i+\delta i+\gamma \end{aligned}\right\}; \quad \therefore \quad \delta i=\alpha-\gamma.$$

$$\left.\begin{aligned} q_1\mathrm{OL} &= r+\beta \\ =\mathrm{RLS} &= r+\delta r+\gamma \end{aligned}\right\}; \quad \therefore \quad \delta r=\beta-\gamma.$$

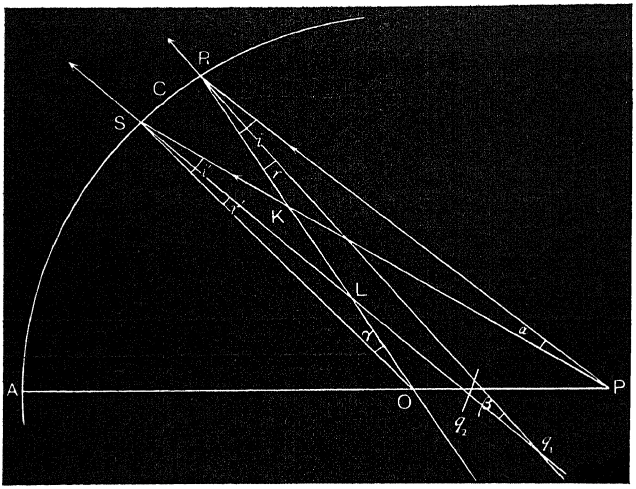


FIG. 150.—Focal Lines by Refraction.

But
$$\sin i=\mu \sin r,$$
$$\cos i \,.\, \delta i=\mu \cos r \,.\, \delta r\,;$$
$$\therefore \quad (\alpha+\gamma)\cos i=(\beta-\gamma)\cos r.$$

Substitute and transpose, and we get

$$\frac{\mu \cos^2 r}{v_1'}-\frac{\cos^2 i}{u}=\frac{\mu \cos r-\cos i}{r}.$$

If v_1 be the primary focal line on emerging from the lens, we have for the second surface (radius s)

$$\frac{\frac{1}{\mu}\,.\cos^2 i}{v_1}-\frac{\cos^2 r}{v_1'}=\frac{\frac{1}{\mu}\,.\cos i-\cos r}{s}.$$

Eliminate v_1', and we get

$$\frac{1}{v_1}-\frac{1}{u}=\frac{\mu \cos r-\cos i}{\cos^2 i}\left\{\frac{1}{r}-\frac{1}{s}\right\} \quad \text{......................(iii)}$$
$$=\frac{\mu \cos r-\cos i}{\cos^2 i(\mu-1)}\cdot\frac{1}{f}.$$

For the secondary focal line, consider the triangles ORq_2 and q_2RP.

$$\triangle ORP = \triangle ORq_2 + \triangle q_2RP,$$

$$ru \sin i = rv_2' \sin r + v_2u \sin (i-r),$$

or (dividing by $urv_2' \sin r$),

$$\frac{\mu}{v_2'} - \frac{1}{u} = \frac{1}{r}\{\mu \cos r - \cos i\}.$$

If v_2 be the corresponding secondary line on emergence from the second surface,

$$\frac{\frac{1}{\mu}}{v_2} - \frac{1}{v_2'} = \frac{1}{s}\left\{\frac{1}{\mu}. \cos i - \cos r\right\}.$$

Eliminate v_2',

$$\frac{1}{v_2} - \frac{1}{u} = (\mu \cos r - \cos i)\left(\frac{1}{r} - \frac{1}{s}\right). \qquad \text{(iv)}$$

128. Apparatus as in § 36, with the addition of a protractor.

Set up apparatus as in § 41, but set the lens at a known angle to the beam of light, and focus the vertical and horizontal wires separately, measuring the distance from the lens to the screen in each case. Or obtain a focal line (say the vertical one) with the lens and screen in a certain position; then, without altering the distance, incline the lens the opposite way until the same focal line is again sharply focussed. Half the angle the lens had to be turned through to pass from the one position to the other must be the obliquity of the light (the i of the formulae). Then if μ be supposed known (see § 122), the values of v_1 and v_2 can be calculated from the formulae (iii) and (iv).

Tabulate the results as before.

ADDITIONAL EXERCISES ON CHAPTER VII

1. Measure the focal lengths and the dispersive powers of each of the lenses of an achromatic combination.

Calculate $\frac{\varpi_1}{f_1}$ and $\frac{\varpi_2}{f_2}$. They should be numerically equal; for the condition for achromatism is $\frac{\varpi_1}{f_1} = \frac{\varpi_2}{f_2} = 0$.

2. Repeat the determination of the dispersive powers with a different lens distance, and compare the results.

3. Examine the achromatic combination with the eye-piece, adjusted as in Experiment, § 124, and compare the appearance of the image with that formed by an ordinary simple lens.

4. Measure the curvatures of the surfaces of each of the lenses of the achromatic combination of Exercise 3, and calculate their refractive indices.

5. Find the ratio of the dispersive power of the glass, as found in the first two exercises, to the refractive index, in the case of each lens.

6. Find the refractive index, and the dispersive power of another lens. See if they are the same as those of either of the lenses of the achromatic combination.

7. Find, by the methods of § 122, the curvatures of the surfaces of one of the lenses of a lantern condenser. Check the measurements with the optical lever, § 80, or with a spherometer.

8. Measure the thickness t of the same lens, also measure the apparent thickness in the middle, viewed from each side, by the method of § 24. Apply the formula $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$ in each case, and so find μ.

9. Find, by the method of § 43, the positions of the focal points of the same lens.

10. Calculate the focal length of the same lens, by substituting in the formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right)-\frac{(\mu-1)^2 t}{\mu r s}.$$

CHAPTER VIII

COMPOUND LENS

Focal Lengths and Gauss Points of Thick Lens and Combination of Lenses.

129. **Definitions and Formulae.**—We will suppose first that the initial and final media are different.

Focal Points.—A bundle of parallel rays parallel to the axis of the combination incident on one face of the combination after passing through the combination will converge to or diverge from

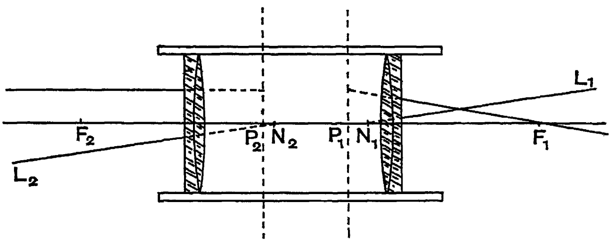


FIG. 151.—Gauss Points of a Lens.

a point. This point is one of the *principal foci.* There will be another point for the parallel rays traversing the lens in the opposite direction. (Let them be F_1 and F_2 in the figure.)

Principal Points and Planes.—The principal points are two conjugate foci, such that an object placed at one forms an image at the other, *of the same size as the object* and *erect.* The principal planes are planes through them normal to the axis. (Let them be P_1 and P_2 in the figure.)

Focal Lengths.—These are the distances P_1F_1 and P_2F_2 respectively, and they are in the ratio of the refractive indices of the media.[1] They become equal when the initial and final media are the same. They are of opposite signs.

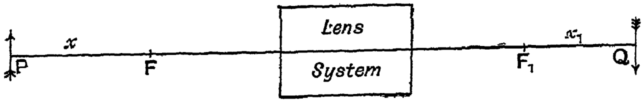


Fig. 152.—Conjugate Foci.

Also, if P and Q are any pair of conjugate points distant x and x' from F and F′ respectively, $xx' = ff'$, where f and f' are the two focal lengths, and x and x' are measured towards the light.

Nodal Points.—These are defined to be a pair of conjugate foci such that a ray L_1N_1 incident at one emerges in a parallel direction N_2L_2 from the other.

Drawing Images.—Let AQ_2 be the object.

i. Draw a ray AB, parallel to the axis, meeting the principal plane P_2 at B.

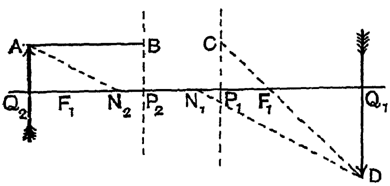


Fig. 153.—Images.

ii. In the plane P_1 take a point C, such that $P_1C = P_2B$. From C draw CF_1D through the focal point F_1.

iii. Draw a second ray from A to N_2, and through N_1 draw a parallel ray N_1D, meeting the ray CF_1 in D.

Then Q_1D will be the image of Q_2A.

For, as the image of P_2B is erect and the same size as P_1, it must be P_1C. Thus any ray incident through B will emerge through C. Also as AB was parallel to the axis, its emergent ray must pass through F_1.

Produce AN_2 to meet BP_2 at H_2 (Fig. 154). Since, by the definition of the principal planes, any ray meeting one at a distance P_2H_2 from the axis must issue through the other at the same distance, $P_1H_1 = P_2H_2$. Thus, as N_2H_2 is parallel to N_1H_1, the triangles are equal in all respects, and $P_2N_2 = P_1N_1$.

From F_1 draw F_1L_1 parallel to N_1D. This will emerge from L_2 parallel to the axis, where $P_2L_2 = P_1L_1$. Also it must meet AH_2 in the focal plane, since L_1F_1 and N_1H_1 are parallel. Then it is obvious

[1] Proved in Heath's *Optics*.

that the triangles EF_2N_2 and $L_1P_1F_1$ are equal in all respects, and $F_2N_2 = P_1F_1$. So it could be proved that $N_1F_1 = P_2F_2$.

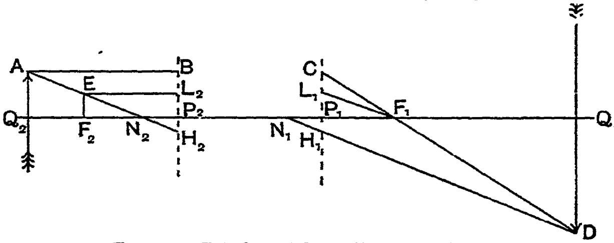


FIG. 154.—Relation of Gauss Points and Images.

Thus the distance from one focal point to its nodal point is equal to the distance from the other principal point to its focal point.

When the initial and final media are the same P_1F_1 and P_2F_2 are equal, and are the focal length of the combination. Thus in this case N_1 and P_1 coincide, as do also N_2 and P_2.

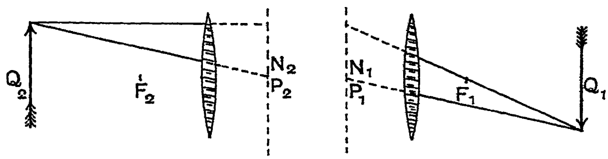


FIG. 155.—Initial and Final Media the same.

If u and v are the distances measured from the principal points to Q_1 and Q_2, reckoned positive when measured towards the incident light (as on page 55),

$$\frac{1}{v} - \frac{1}{u} = -\frac{\mu_1}{f_1} = \frac{\mu_2}{f_2} = \frac{1}{f},$$

also $$f_1f_2 = x_1x_2,$$

where x_1, x_2 are the distances of the object and image from the focal points.

Divergent Combination.—If the combination is equivalent to a concave lens the focal points will be on the other side of their principal and nodal points. The figure is given.

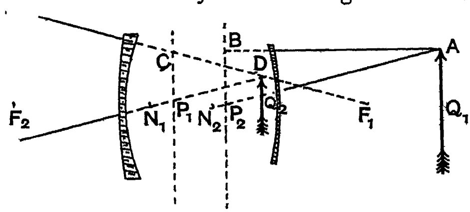


FIG. 156.—Divergent Combination.

We still have $f_1f_2 = x_1x_2$.

Simple Equivalent Lens.—In the thin lens the two principal points coincide, and as $F_1N_1 = F_2P_2$ and $F_1P_1 = F_2N_2$;

i.e. $$F_1P_1 + P_1N_1 = F_2P_2 = F_2N_2 + N_2P_2,$$

we have $$P_1N_1 = F_2N_2,$$

so that the nodal points also coincide. It will be seen that the emergent rays would coincide with those of the combination if they were shifted along the axis P_1P_2. Thus the thin lens, of which the focal lengths are the same as those of the combination, is called its "Simple Equivalent Lens."

If the initial and final media are different, N_1, N_2 are not in the centre of the lens.

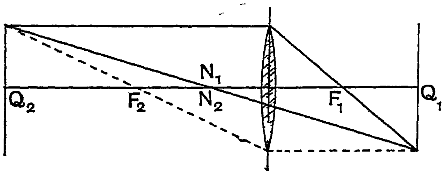


FIG. 157.—Equivalent Lens.

Initial and Final Media the same.—In this case $x_1x_2 = -f^2$.

The figures for the combination (Fig. 155) and the thin lens are given. The focal lengths being numerically equal, f_2 is taken as the focal length of the lens.

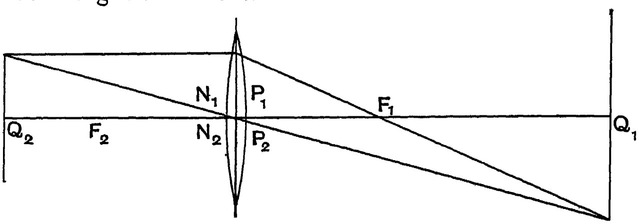


FIG. 158.—Equivalent Lens.

Experimental Determinations of Gauss Points.

130. **Focal Points. First Method.**—If the combination be used to produce the image of a distant object on a screen, the point where the axis meets the screen will be a focal point when the image is sharply focussed. By reversing the combination the other point can be found. The distance from the brass work of the mount to the screen should be taken (Fig. 159).

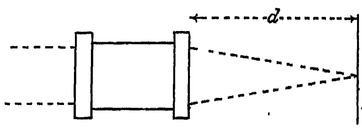


FIG. 159.—Focal Point.

Second Method.—The light from a cross-wire screen can be parallelised by an auxiliary lens, and then focussed on a screen by the combination.

Third Method.—But the best method is to place a plate-glass mirror behind the combination to reflect the light back again through the combination, and to form an image of the cross-wire by the side of the cross-wire itself, the latter being surrounded by a white screen for this purpose. Both foci should be found, the distance from the *mount* to the screen being taken as before. For as the light is returned nearly along its course it must strike the mirror perpendicularly, and the emergent pencil must be a parallel one; the incident light must therefore be coming from the principal focus on that side.

131. Nodal Points and Focal Length.

Theory.—Suppose a ray BC to be incident through N_1 from a very distant object. It will emerge parallel through N_2, and cut the screen at A. A will be the image of the point of the object from which BC

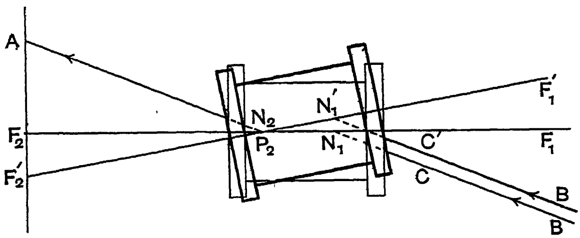


FIG. 160.—Nodal Points.

came. As the object is supposed to be at a great distance its image will be formed in the focal plane.

Now suppose the combination rotated about N_2. As BC is coming from a great distance, the ray from the same point of the object that is incident through N_1 will still be practically parallel to BC. Thus the ray that emerges from N_2 has not moved, and the image at A will remain at rest. Any movement of N_2 will, however, cause A to move also.

Thus to find N_2 we have to find the point about which a rotation of the combination produces no movement of the image of a distant object.

Apparatus.—To do this conveniently, the combination is mounted on a "Nodal Slide," that is on a carriage sliding on rails, which rails rotate round a vertical axis. By adjusting the carriage on its rails, any point of the axis of the combination can be brought over this vertical axis, and thus the rotation effected about that point. The V's upon which the combination rests may be varied as to distance apart and height, thus the axis

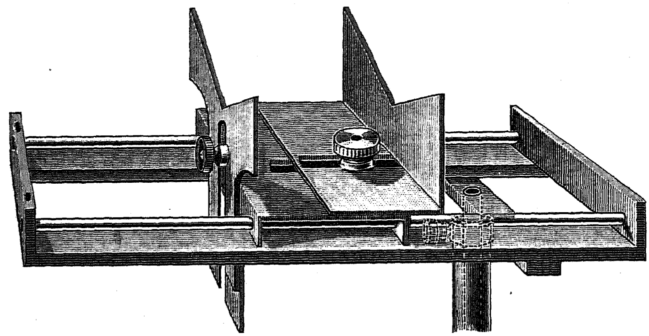


FIG. 161.—Nodal Slide.

of the combination may always be made horizontal, and a large range of lenses may be measured on the same stand. The position of N_2 will be found when the image of a distant object is not displaced by a rotation of the combination about the vertical axis.

First Method.—Put the nodal slide and screen at one end of the bench and the cross-wire at the other (or better, on an independent stand as far away as possible). Focus the cross-wire on

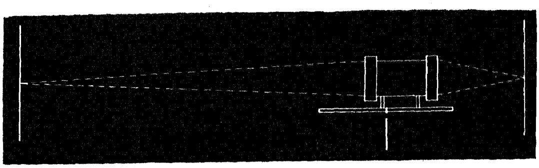


FIG. 162.

the screen. Now rotate the combination. The image will probably shift. If N_2 be in front of the axis of rotation, the point about which the rotation takes place, then a small rotation will carry N_2 to N_2' and therefore A to A'. If on the contrary

N_2 is behind the axis, a similar rotation will take it to N_2'' and A to A″. Thus, by observing the direction in which the image moves, we can tell which way to slide the combination on its rails to get N_2 to coincide with O. The distance from the axis

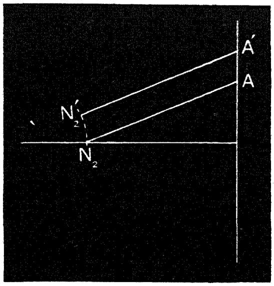


FIG. 163.

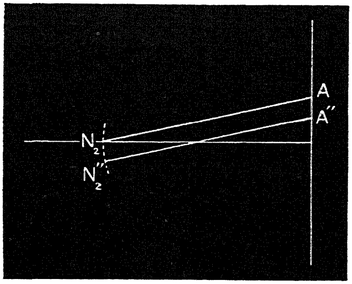


FIG. 164.

O to the front of the brasswork is measured and fixes the position of N_2 (call it n_2). By reversing the combination the position of N_1 can be found.

Second Method. Syn-focal Method.—As the object is not at an infinite distance in the above, the positions found will not be absolutely correct, though the error is almost negligible even with a 6″ lens.

The absolute position can, however, be found very easily. Put the nodal slide at about its correct position; stand up a plane mirror behind it, and adjust the lens and mirror to form an image of the cross-wire by its side in its own plane. The light falling on the mirror and reflected back by it will be parallel light. Thus if the combination be so placed on its rails that the image remains stationary during a small rotation of the lens, the nodal point will be on the axis of rotation. Moreover, in this case **the distance from the nodal point to the screen is the focal length of the lens.** If the combination be turned through 180°, and moved along the rails till it is again in focus (the axis of

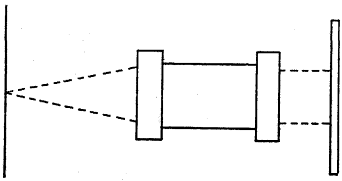


FIG. 165.—Clay's Syn-focal Method.

rotation not being moved), the other nodal point ought to be found at once.

Also the distance the carriage is moved on its rails will be the distance from one nodal point to the other.

These points and distances ought to be found easily to $\frac{1}{5}$ mm. in the case of a good lens. This is the most accurate way of finding the focal length.

Third Method. Collimator Method.—Instead of obtaining parallel light by the use of a plane mirror and reflecting the beam back through the lens, a virtual image at an infinite distance may be obtained by the use of a good collimator. The collimator may be furnished with a ruled glass plate, as described in § 84, in fact that collimator may be used. The axis of the lens and the collimator must coincide. To see if this is so, look along the axis towards the light, reduce the entrant beam to a pinhole, and observe the series of images formed by the reflections from the surfaces of the lenses; they should all lie in a line on the common axis of the lenses.

132. **Conjugate Foci. First Case, Focal Points known.**—In this case the distances from the focal points to the object and image can be found. The formula then is $x_1x_2=f^2$.

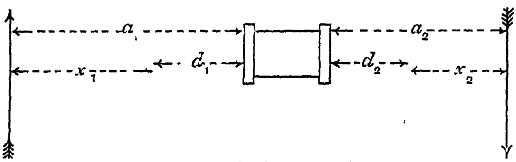


FIG. 166.—Conjugate Foci.

Set up the cross-wire, combination and screen, and focus the cross-wire on the screen. For each position of the lens the screen should be adjusted some five or ten times to form a sharp image and the position read. The mean of the readings is then taken as the true position. The distances a_1, a_2 from the brasswork to the cross-wire and screen are read. From these readings the previously found distances of the focal points from the brasswork, d_1 and d_2, must be subtracted. The remainders will be the x_1, x_2 in the formula $f^2=x_1x_2$. In this paragraph x_1 and x_2 are used for the distances irrespective of sign.

Second Case. Nodal Points known.—As these coincide with the principal points, we may measure from them to the image and screen respectively, and use the formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

In this case

$$u=a_1+n_1$$
$$-v=a_2+n_2.$$

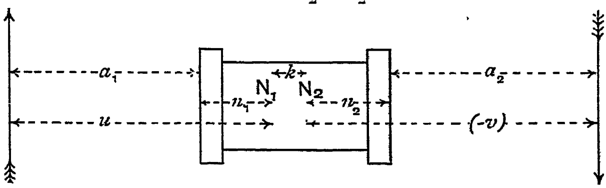


FIG. 167.

Third Case. Distance between Nodal Points known.—Let it be "k." Place the cross-wire and screen a distance L apart. Find the two positions for the combination which produce a clear image on the screen,

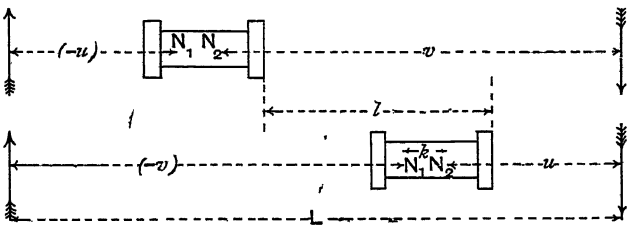


FIG. 168.

and let the distance the lens is moved from one of these positions to the other be l. Then (remembering that v is a negative quantity),

$$\left.\begin{array}{l} L=u+(-v)+k \\ l=(-v)-u \end{array}\right\};$$

and therefore

$$u=\frac{L-k-l}{2},$$

$$v=\frac{k-L-l}{2},$$

and

$$\frac{1}{f}=\frac{2}{k-L-l}-\frac{2}{L-k-l}$$
$$=-\frac{4(L-k)}{(L-k)^2-l^2}.$$

Focal Length by Magnification Methods.

133. *Apparatus.*—Compound lens; two glass scales on stands, to serve as image and object respectively; a gas flame and a sheet of ground glass supported in a vertical plane; the stand for the lens, preferably a stand on a divided optical branch, so that the amount of its movement can be accurately determined.

First Method.—Set up the lens and scales in one line, put the sheet of ground glass behind the scale that is to serve as an object and the gas beyond that. So adjust the distances of the scales from the lens that an image of the one scale enlarged about twice shall be projected upon the other scale. Focus carefully (by parallax preferably). Obtain the exact magnification m_1. To do this read on the scale the positions of say ten of the divisions of the image; subtract the reading of the first from the sixth, the second from the seventh, and so on; obtain the mean of the result, and divide by the actual distance apart on the original scale of the first and sixth. The result is the magnification; it is a negative quantity, as the image is inverted.

Without disturbing anything else move the lens along until a reduced image of the first scale is formed on the second. (It may be more convenient to put the ground glass and the light at the opposite end and so enable the *enlarged* image of the second scale to be observed instead of the *reduced* image of the first scale.) Focus again carefully, of course by moving the lens only. Again take the magnification m_2.[1]

Then if l is the distance between the two positions of the lens read off on the scale of the optical bench—the focal length is given by

$$f=\frac{+l}{m_1-m_2}.$$

For let N_1, N_2 (Fig. 169) be the nodal points, let u_1 v_1 be the actual distances from N_1 and N_2 to the scales A and B respectively in the first position of the lens. Then as the image is inverted

$$m_1=-\frac{v_1}{u_1},$$

and the conjugate foci are connected by

$$\frac{1}{-v_1}-\frac{1}{u_1}=\frac{1}{f}.$$

[1] If correctly taken m_2 should obviously be the reciprocal of m_1.

In the second position of the lens, v_1 becomes the distance from N_1 to A, and u_1 the distance from N_2 to B; therefore

$$m_2 = -\frac{u_1}{v_1},$$

while u_1 and v_1 are still connected by the same formula, for now

$$-\frac{1}{u_1} - \frac{1}{v_1} = \frac{1}{f}.$$

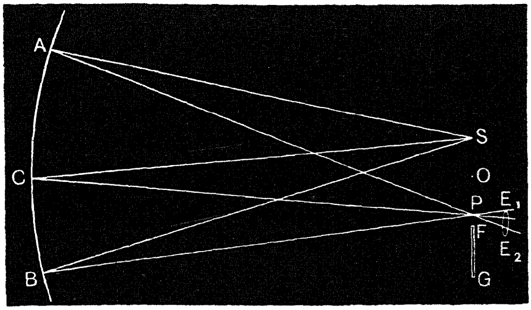


FIG. 169.

Thus
$$m_1 - m_2 = -\frac{v_1^2 - u_1^2}{u_1 v_1}$$
$$= -(v_1 - u_1)\left(\frac{1}{v_1} + \frac{1}{u_1}\right)$$
$$= \frac{l}{f}.$$

Second Method, Abbé's.—Commence as before, but make the first magnification unity or less.

Then keep the *lens* fixed, move B away from the lens some known distance *d*, adjust A until it is perfectly imaged once more on B, and obtain the new magnification m_2; m_2 is obviously numerically greater than m_1, and both are negative quantities.

The focal length is given by

$$f = \frac{d}{m_2 - m_1}. \qquad \text{(See § 45.)}$$

Note that m_2 and m_1 are *negative* quantities.

Third Method.—When the distance between the nodal points can be neglected or can be allowed for, the following formulae due to Nelson are useful.

We have
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}; \quad m = \frac{v}{u}.$$

Thus $$f=\frac{uv}{u+v}=\frac{\frac{v^2}{m}}{\frac{v}{m}+v}=\frac{v}{m+1}; \quad u=f\left(1+\frac{1}{m}\right).$$

If $$u+v=\mathrm{L}=\frac{(m+1)v}{v}$$
$$=(m+1)u,$$
$$f=\frac{m\mathrm{L}}{(m+1)^2},$$

where L is the distance from the cross-wire to the screen arranged as in the first method, less the distance between the nodal points, and m is the magnification obtained as there described.

Fourth Method, Blakesley's.—Let a scale be fixed at the principal focus of a lens normal to its axis, and illuminated from behind; let a second lens be placed coaxial with the first, this will form an image of the scale in its principal focal plane; the magnification will obviously be equal to the ratio of the focal length of the lenses. Thus, if the focal length of the first lens is known once for all, the focal length of other lenses can easily be found. The method is particularly useful for lenses of very short focal length, as it can be used with a microscope which has a micrometer eye-piece. The scale is first observed through the microscope direct, and then the image of the scale formed by the two lenses in succession; the ratio of the sizes of the images as read on the micrometer scale gives the magnification required. The focal length of the first lens can be found by the first method above described. The scale and lens are mounted in a short tube; the scale may be adjusted in the focal plane by an auto-collimation method.

Focal Lengths of a Lens separating different Media.

134. *Apparatus.*—Convex lens about 2″ diameter and 6″ focal length; lamp chimney of stout glass; or better, a brass tube 8″ long, $2\frac{1}{2}$″ diameter; plane glass $2\frac{1}{2}$″ diameter; two convex lenses, each 8″ focus; cementium.

The lens and end of the chimney may be ground together with coarse emery and turpentine until they exhibit a ground appear-

ance all round the circumference when dry (care being taken, of course, to avoid scratching the central portion of the lens), and then cemented together with Canada balsam. Nearly close the opposite end of the tube with a flat glass plate attached in the same way. When dry this makes a good cell that can be put aside, and is always ready for use. Instead of grinding the two together and cementing them with balsam, a very good cell can be made by cementing the lens to the chimney with sealing wax. To do this, warm the chimney carefully until it will melt the wax, and coat the one end with a continuous line of wax. Warm the lens slightly and press it and the still hot chimney together. If the wax cracks in cooling, use a thick copper wire filed to a point to melt it, keeping the wire hot by holding it all the time in the flame of a spirit lamp

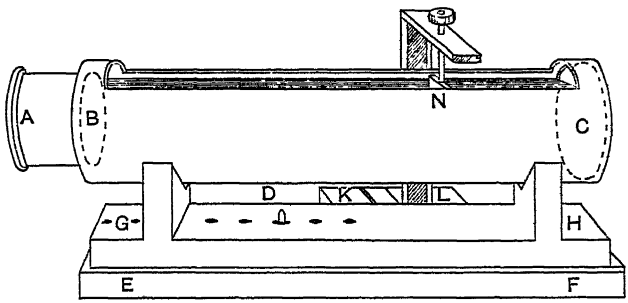


FIG. 170.—Model Eye.

or Bunsen burner. Or they may be cemented with cementium or fortafix in a brass collar. Mount a small piece of plane mirror about an inch across, and a piece of opal glass (or white card) of the same size, on the ends of brass rods with cementium.

For the nodal point experiments (vii. below) a more convenient cell can be made of a zinc or brass tube $2\frac{1}{2}''$ diameter and $8''$ long, closed at one end C with a piece of plane glass. A longitudinal opening about an inch wide is made all along one side of the tube (Fig. 170). The lens may be, first, a single one inserted in the end of a short tube AB which fits the main tube; second, a pair of lenses, each about $8''$ focus, mounted at opposite ends of a tube about $2''$ long, which also fits the main tube. With the latter lens the two nodal points will be separated, and thus the most general case will be represented. This cell will, of course, be used horizontally. It may be mounted on V's on a stand GH, which rotates about a pin D in a lower stand EF. The pin D

may be inserted in any one of a series of holes drilled at intervals along the centre line of the stand GH.

Also the following will be required: a white screen, with a hole and a cross-wire; a small photographic objective, or a collimator; a plane mirror; a gas flame; meter scale, and supports. The screen and the mirror may be mounted as shown at N (Fig. 170) on separate stands KL. For the glass cell, these must be mounted at right angles at the end of a wire, by which they can be inserted in the cell through the opening left at the plane end.

i. **Obtain the Focal Length of the Lens in Air.**—Set up the cell horizontally in a retort stand, put no water in it. Support the mirror N inside the cell, with its plane vertical. Set up the screen, with the cross-wire on the axis of the lens, and let the light from the gas flame pass through it along the axis of the cell to the mirror N. Adjust the distance from the screen to the cell until the rays in the cell are parallel, which will be shown by the formation of a clear image of the cross-wire on the screen S, by the light that has been reflected back by the mirror N. Measure the distance f from the screen to the lens.

ii. **Obtain the Focal Length of the Lens in Air for Rays that are parallel in the Water.**—Set up the apparatus as in i.; fill it with water and repeat exactly. The distance f_1 from the lens to the screen is the focal length f_1. The power ϕ_1 is $\frac{1}{f_1}$.

iii. **Obtain the Focal Length in the Water for Rays that are parallel in the Air.**—Set up the apparatus as above, but insert between the cell and the screen S a photographic objective, and so adjust its distance from S that the emergent light shall be parallel (this position can be easily found by placing a plane mirror temporarily against the lens to reflect the light back again, as in § 43). Or use the collimator. Receive the light after refraction into the water on the small white opal screen. Then the distance from the lens to the opal screen is the focal length required, f_2. The power ϕ_2 is $\frac{1}{f_2}$. Or pass the light through the plane glass end of the cell, then through the small cross-wire screen, out through the lens, on to a plane mirror, by which it is to be reflected back into the cell, and an image formed on the cross-wire screen at the side of the cross-wire. f_1 and f_2 will be of opposite signs.

iv. The refractive index of the water should be the ratio of the focal lengths in the water and the air respectively.

Thus $$-\mu=\frac{f_2}{f_1}=\frac{\phi_1}{\phi_2}.$$

Reduced Power.—If we define the "Reduced Power" of a lens as the product of the power in either bounding medium into the refractive index of that medium, we have

$$\Phi=-\phi_1=\mu\phi_2=-\frac{1}{f_1}=\frac{\mu}{f_2}.$$

For if the radii of the surface of a lens are r and s for a lens in air,

$$\frac{1}{v_1}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right)=\frac{1}{f}=\phi.$$

If the space B be imagined filled with a medium of refractive index μ_2, the refraction into it can be obtained by supposing the ray which was emerging into air to be refracted at an air-medium surface of radius s.

$$\frac{\mu_2}{v}-\frac{1}{v_1}=\frac{\mu_2-1}{s},$$

adding $$\frac{\mu_2}{v}-\frac{1}{u}=\frac{1}{f}+\frac{\mu_2-1}{s}=\Phi \text{ (say)}.$$

Making u infinite, v becomes f_2, and

$$\frac{\mu_2}{f_2}\equiv\mu_2\phi_2=\Phi.$$

Making v infinite, u becomes f_1, and

$$-\frac{1}{f_1}=-\phi_1=\Phi.$$

v. Thus the gain of reduced power of the lens

$$=\Phi-\phi=\frac{\mu_2-1}{s}=\frac{-\frac{f_2}{f_1}-1}{s},$$

or $$s=\frac{-\frac{f_2}{f_1}-1}{\Phi-\phi}=\frac{-\frac{f_2}{f_1}-1}{-\frac{1}{f_1}-\frac{1}{f}}=\frac{f_2+f_1}{f+f_1}\cdot f,$$

and if s has been determined this can be verified.

vi. The **principal points** coincide with the lens if the lens is thin, as any object placed close against one side of it will appear when

viewed from the other side of the lens (*i.e.* when looking at it through the lens) to be the same size as it actually is, and in the same plane.

vii. But the **nodal points** will not coincide with the lens. On the contrary, they are together, and on the tube on its axis a distance from the lens equal to the difference between the two focal lengths.

That this is so can be easily verified roughly. Unclamp the tube in either ii. or iii. above, and, holding it between the thumb and finger across a horizontal diameter at this distance from the lens, gently rock it about this line as axis. The image on the screen or the opal, as the case may be, will remain practically stationary. If, however, it is rocked about either a horizontal axis through the lens itself, or about any other horizontal axis nearer to or farther from the lens than the one above described, there will be an obvious movement of the image, which is greater the farther the axis of rotation is from its correct position.

By placing the cell in the nodal slide (Fig. 161), or in the wooden cradle, turning on a pin in the base as shown in Fig. 170, the exact position of the nodal points can be found, and the true value of the focal lengths determined in the case when the lens is not the simple thin lens, but a compound one. The method is obvious.

The Focal Length of a Micro-Objective.

135. **Abbé's Method.**—*Apparatus.*—A microscope will be required which is furnished with a draw tube. A stage micrometer and a micrometer eye-piece to determine the magnification will also be wanted.

Experiment.—The experiment consists merely in determining the magnification with two different extensions of the draw tube, the distance between the two positions of the draw tube being also observed. If the draw tube is furnished with a centimetre scale, as is the case in many microscopes, this distance can be read directly upon it.

Then $f=\dfrac{d}{m_2-m_1}$, where d is the extension and m_1 and m_2 are the two magnifications.

Let L represent the objective, and N_1 and N_2 be its nodal points. Let A, B be the object and image respectively in the first position of the draw tube; then

$$\frac{1}{AN_1}+\frac{1}{BN_2}=\frac{1}{f},$$

and also $$\frac{BN_2}{AN_1}=m_1, \text{ the magnification };$$

and therefore $$\frac{m_1+1}{BN_2}=\frac{1}{f}.$$

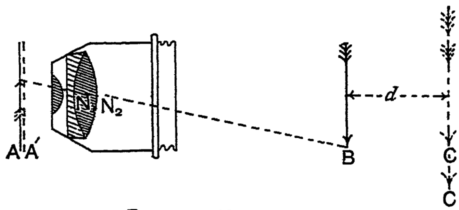


FIG. 171.—Abbé's Method.

If A^1 and C be the object and image in the new position of the draw tube, so that BC is the extension, we have in the same way,

$$\frac{m_2+1}{CN_2}=\frac{1}{f};$$

and therefore $$BC=CN_2-BN_2=f(m_2-m_1),$$

or $$f=\frac{d}{m_2-m_1},$$

where d is the extension.

It is obvious that as the actual magnification is required, an ordinary negative eye-piece with an arbitrary scale will be useless for our purpose. The eye-piece must be a positive one, and the scale must be divided in millimetres. It will be observed also that this method is identical with the one given for finding the focal length of a convex lens on page 63.

136. **Collimator Method.**—*Apparatus.*—Microscope with lens to be tested; a glass scale with a mm. divided into tenths and hundredths for object; collimator with scale, as described below; pocket magnifying lens; lamp.

For the collimator fix a good lens in the lower end of a tube which itself fits into the body of the microscope; it should be an achromatic lens of about five inches focal length; the exact focal length must be found by one of the methods above

described. Exactly in its focal plane insert a glass scale divided into millimetres; the scale side should be very slightly ground to a matt surface, and should be downwards. The scale may be adjusted in the focal plane either by measurement or by focussing the moon upon it, or by focussing the slit of a collimator upon it which has already been adjusted for parallel light.

The determination of the focal length of the objective is then a simple matter. Place the divided millimetre on the stage of the microscope, screw the lens of which the focal length is required on the nose-piece, insert the collimator, and focus the one scale on the other; use a hand lens to get the exact focus. Note the magnification, *i.e.* see how many divisions on the collimator scale correspond to the image of the millimetre (or a part of it) on the stage. Let it be n, so that n is the magnification. Let f be the focal length of the collimator lens; then the focal length of the objective is $\frac{f}{n}$.

The Focal Length of an Eye-Piece.

137. It was pointed out by Mr. F. Cheshire that the Abbé method might easily be adopted for the determination of the focal length

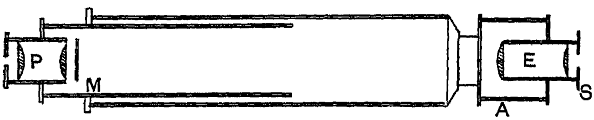


FIG. 172.—Focal Length of Eye-piece.

of an eye-piece. All eye-pieces, whether "positive" or "negative," are equivalent to convex lenses, and can be made to give a real image of a near object, even though they may not do so if the object is distant. If, therefore, an adapter be made at one end to fit the screw in the nose-piece which usually carries the objective, and at the other end to fit the eye-piece whose focal length is required, the eye-piece may be used as an objective. As its focal length will generally be too great to form an image of the micrometer scale, if this were placed upon the stage, it will probably be necessary to attach it to the sub-stage with an

elastic band. Then the focal length may be determined as described above, § 135, using the same formula.

138. **Case where one of the focal points lies without the combination, e.g. Huygen's eye-piece.**—In addition to the method just described, as one focal point is outside, the focal length can be found if the corresponding nodal point is determined with the nodal slide as above, § 131.

139. **Case where the principal focus is within the combination.**—Form an image of the cross-wire A (Fig. 173) by a good lens of short focus, B. Suppose this image at F. Then if F be one of the principal foci of the combination, the light will emerge parallel. Reflect it back by a plate glass mirror; it will retrace its path and will

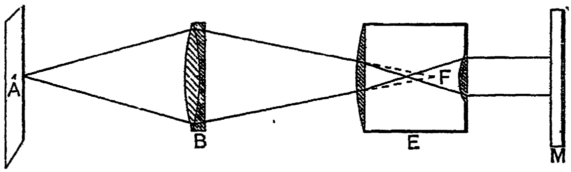


FIG. 173.—Focal Length of Negative Eye-piece.

produce an image of the cross-wire superimposed on the cross-wire itself at A. By a slight tilt of the mirror the image can be shifted to the side of A, and can be accurately focussed. The exact position of the combination is observed. It is then removed and the position of F found by putting a screen there. Thus F can be found relative to the combination.

The other focal point can be determined in the same way.

Then knowing F and F_1 the focal length can be found by taking conjugate foci as in § 132. Or, by mounting the combination to rotate round a vertical axis in the above experiment, and adjusting this axis until the reflected image adjoining A is not shifted by rotating the combination, its nodal point can be found. The distance from the point to F (found as above described) will of course be the focal length.

140. **The focal length can also be very conveniently found by Blakesley's method (§ 133) or an ordinary magnification method (§ 133).**

Graphical Determination of the Optical Constants of a pair of separated Lenses, for instance, the Telephoto Lens. Two Thin Lenses.

141. All photographic workers are aware that the power of a telephoto lens can be varied by altering the distance apart of the convex and the concave systems of which it is composed.

The following simple graphical construction will enable one very easily, not only to find the focal length of a combination of two lenses, but also to determine the positions of the principal focal points and the principal planes, and to follow the way in which all these vary as the lenses are approached or separated.

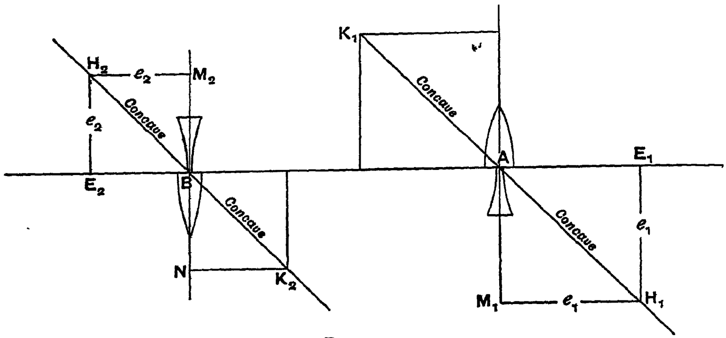


FIG. 174.

Fig. 174. Suppose AB to be the axis of the combination, and that one lens is at A and the other at B. For simplicity I will assume the lenses to be thin lenses. Let E_2 be the principal focus of the lens B for light parallel outside the lenses. If this is a *concave* lens, measure a distance BM_2 equal to the focal length of B, *upwards*, and complete the square $BM_2H_2E_2$. The fourth corner H_2 will be the point made use of in the construction presently to be described. If the lens is a convex one, BN must be measured downwards, and the corresponding point will be at K_2. It will be noticed that H_2BK_2 is a straight line bisecting the angle E_2BM_2.

Proceed in the same way with the lens A, except that the measurements must be made in the opposite direction, so that if A is *concave* the point H_1 will be on the *lower right-hand* side

A telephoto lens consists of a concave lens B and a convex lens A. Thus, in such a combination, the points to be dealt with would be H_2 and K_1.

142. **Principal points.**—Fig. 175 shows how to use these points to find the principal points of a combination of a concave and a convex lens, of which the convex is the stronger. In the figure the distance between the lenses is less than the focal length of the lens A, but this is immaterial.

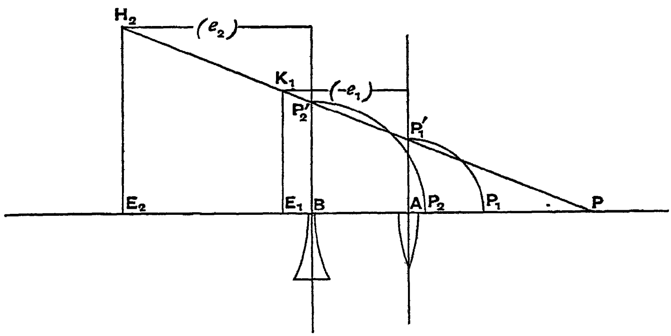


FIG. 175.—Principal Points of Telephoto Lens.

The construction simply consists in joining H_2K_1 by a straight line which is produced to cut the vertical lines through B and A in P'_2 and P'_1. Then, with B as centre, and radius BP'_2, a part of a circle is drawn *clockwise* to project P'_2 on to the axis, cutting the axis at P_2. In the same way with A as centre and AP'_1 as radius, a part of a circle is drawn, also clockwise, to project P'_1 on to the axis at P_1. The points P_2 and P_1 will be the *principal points* of the combination. The point P, where H_2K_1 cuts the axis AB, is the image of P_1 formed by A, and also is the image of P_2 formed by B.

143. **Principal Foci.**—Fig. 176 shows how to find the principal foci of the same system. This time the points H_2 and E_1 are joined, cutting the vertical line through B at F'_2; and E_2 and K_1 are joined, cutting the vertical line through A at F'_1. As before, these points are to be projected *clockwise* on to the axis. Thus with B

as centre and BF'_2 as radius, part of a circle is drawn *clockwise*, cutting the axis in F_2; as F'_2 is below the line the circle will cut

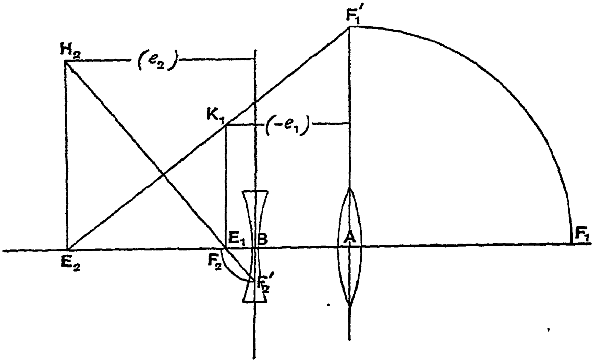


FIG. 176.—Focal Points.

the axis to the left of B, as shown in the figure. So also a circle is drawn with A as centre and AF'_1 as radius, cutting the axis at F_1. The points F_2 and F_1 are the *principal foci* of the combination.

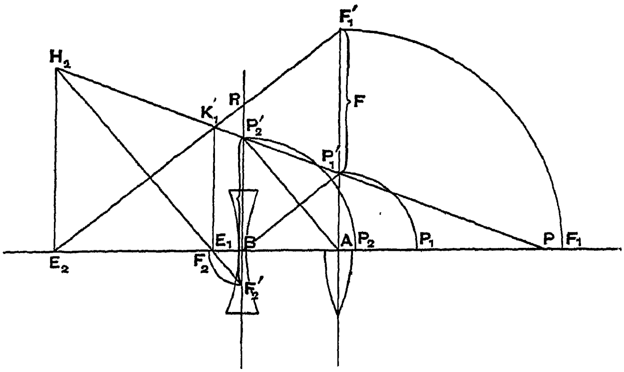


FIG. 177.—Gauss Points, and Focal Length of a Convergent Compound Lens.

144. **Focal Length.**—In Fig. 177 the constructions of Figs. 175 and 176 are included in one figure. It may help to render the construction more easy to remember, if it is pointed out that in

addition to the axis on which two of them lie, and the two vertical lines which can, of course, be drawn through them, the four points, K_1, H_2, E_1, E_2, can be joined in only *three* ways, and that it is the intersection of these three lines with the vertical lines through A and B, which, projected on the axis, give the positions of the principal planes and the focal points. Also that it is the two lines through K_1 which give the positions of the points upon the line through the corresponding lens A, and the two lines through H_2 which give the positions of the principal and focal points on the line through the corresponding lens B. As the projections of F'_1 and P'_1 are the first principal focus and the first principal point, their distance apart, $F'_1P'_1$, is **the Focal Length of the combination.** In the same way $F'_2P'_2$ is also the focal length of the combination.

145. **Formulae.**—The points E_1 and E_2 used in all these constructions are the focal points of the respective lenses, which correspond to *parallel light outside the combination.* For instance, parallel light coming from the right, falling on the lens A, would be converged to E_1. So, parallel light coming from the left and incident on the lens B (that is, parallel light outside the combination), after passing through B, would appear to come from E_2.

Then using k for the distance E_1E_2, a for the distance AB that separates the lenses, and e_1 and e_2 for the focal lengths of the lenses, the following relations easily follow by similar triangles (remembering that in Fig. 177 e_1 and F are negative quantities):

$$F=\frac{e_1e_2}{k}, \text{ or } k=\frac{e_1e_2}{F};$$

$$AP_1=-\frac{ae_1}{k};$$

$$BP_2=\frac{ae_2}{k},$$

for $$\frac{AF'_1}{AE_2}=\frac{E_1K_1}{E_1E_2}; \text{ or } AP'_1-\frac{e_1e_2}{k}=(a+e_2)\frac{-e_1}{k}.$$

Also $$BR=P_1'F_1'=F.$$

It is fairly easy to see that BP'_1 is parallel to $E_2K_1F'_1$, and that AP'_2 is parallel to $F'_2E_1H_2$.

Rule of Signs.—In these equations k is to be considered positive if E_1, E_2 are in the same order as their respective lenses A and B.

The focal length of a concave lens is to be considered positive, that of a convex negative.

The distances AP_1 and BP_2 are to be measured towards the light as usual.

Thus in Fig. 177, as e_2 is positive, a is positive and k positive, the distance BP_2 will be measured towards the light; and as

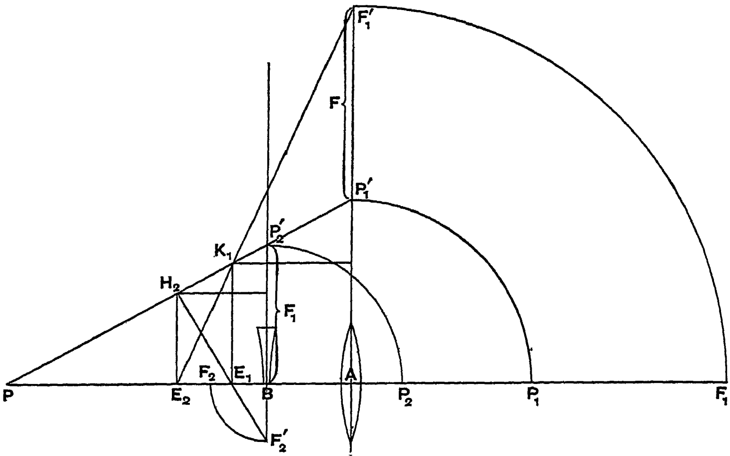


FIG. 178.—Gauss Points and Focal Length of a Convergent Compound Lens.

e_1 is a negative quantity, AP_1 is also measured to the light as in the figure. The focal length is negative as e_1 is negative, *i.e.* the combination is equivalent to a convex lens.

Fig. 178 is the corresponding figure for a convex and a concave lens, in which the concave lens is the stronger. As before, H_2, K_1, E_1, E_2 are joined in the three possible ways, by lines cutting the axis and the vertical lines through A and B; the two lines through K_1 give the positions of F'_1 and P'_1, and those through H_2 give the positions of F'_2 and P'_2 as before, and all the points, F'_1, F'_2, P'_1, P'_2, are projected clockwise on to the axis.

The focal length of the combination and the positions of P_1 and P_2 are given by the same formulae. As e_1 is greater than e_2, if the lenses A and B are at a less distance from one another than the numerical difference of their focal lengths, k becomes negative, and the combination is equivalent to a concave lens. When E_1 and E_2 coincide, the focal length of the combination is infinite; the combination then is, of course, the opera glass.

The effect of the separation of the lenses on their combined focal length is obvious at once from Fig. 177 or Fig. 178, as the case may be. For as the line E_2H_2 is fixed, and the distance of K_1 from the line $AP'_1F'_1$ is a constant, it is obvious as the triangles $E_2K_1H_2$ and $F'_1K_1P'_1$ are similar, that the focal length $P'_1F'_1$ increases rapidly as E_1 is brought nearer the line H_2E_2.

146. **Compound Lenses.**—If the lenses dealt with are not thin lenses, E_1 and E_2 are still the principal focal points corresponding to light which is parallel outside the combination; and A and B are the principal points corresponding to those principal focal points, so that E_1A is, of course, still the equivalent focal length of one combination, and E_2B still the focal length of the other combination. But the distances AP'_1 and AF'_1, instead of merely being projected clockwise on the axis, are now to be measured from the *other principal point* of the A combination; and in the same way the distances BP'_2 and BF'_2 are to be measured from the other principal point of the B combination.

147. **Position of Image.**—In Fig. 179 the principal planes and the corresponding focal points found as above described are used to find geometrically the position of the image of an object placed at Q_1.

For this purpose, from the point of an arrow at Q_1, a line parallel to the axis is drawn to meet at L the principal plane through P_1. As by definition, P_1 and P_2 are the positions of the image and its object which shall be the same size as one another, a ray meeting P_1 at L must emerge from the plane P_2M at the same distance from the axis, that is, from M. Also as Q'_1L is parallel to the axis, the emergent ray must pass through the

principal focus F_2, and must, therefore, emerge along the line MF_2.

As the initial and the final media are assumed to be the same (air, in both cases), P_1 and P_2 are the **nodal points** of the system, as well as the principal points, and any ray incident through P_1

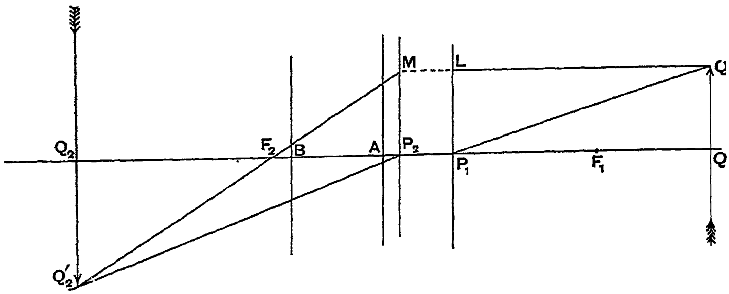


FIG. 179.—Images, using Gauss Points.

will emerge as a parallel ray from P_2. Therefore, join Q'_1P_1 and draw $P_2Q'_2$ parallel to it. The point where this line meets $MF_2Q'_2$ will necessarily be the image of Q'_1.

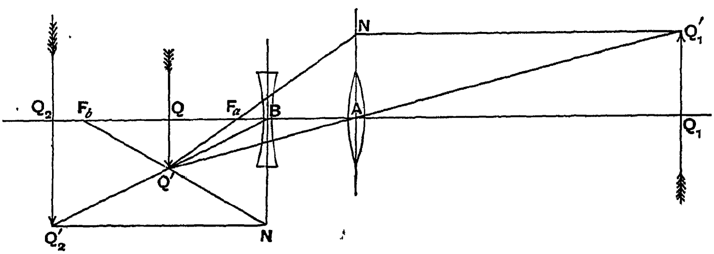


FIG. 180.—Images by Successive Refraction.

In Fig. 180, for the same pair of lenses, the light from the object at Q_1 is traced through the lenses in succession, and it will be seen that the image formed is in the same place as it was in Fig. 179.

The optical constants of a pair of concave lenses or a pair of convex lenses can be found by precisely similar constructions,

the points of intersection being in all cases projected on to the axis in a clockwise direction.

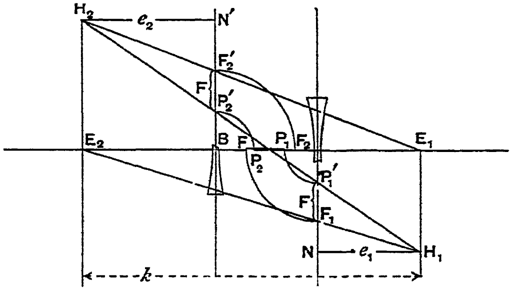


FIG. 181.—Gauss Points and Focal Length of a Divergent Compound Lens.

Fig. 181 gives the construction for a pair of concave when k is positive and Fig. 182 for a pair of convex lenses when k is

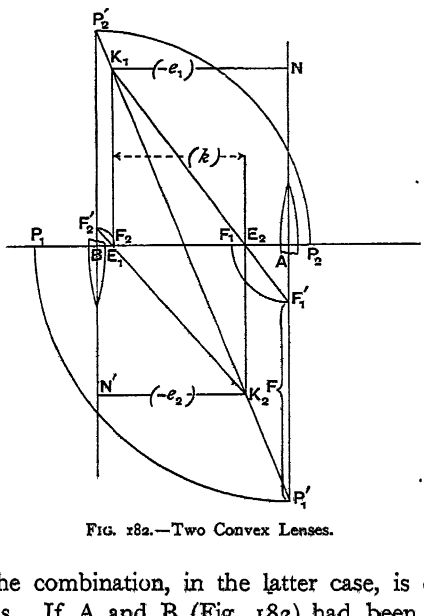


FIG. 182.—Two Convex Lenses.

negative. The combination, in the latter case, is equivalent to a convex lens. If A and B (Fig. 182) had been so far apart

that k was positive, and then the lenses are brought nearer together, so that k grows smaller, vanishes, and then becomes negative, the focal length of the combination would at first be positive, then become greater, infinite, and finally negative,—that is to say, the combination is equivalent to a concave lens when A and B are at a greater distance apart than the sum of their focal length, and becomes equivalent to a convex lens when k is negative.

Graphs for surfaces separating different media.

The graphs for refraction at a single surface separating two media are constructed very similarly to those for refraction through a lens (p. 67), **except that there are two focal lengths, one in each medium.**

148. Let A (Fig. 183) be the pole of a surface separating two media of which the refractive indices are μ_1 and μ respectively. If a beam of axial parallel light in the medium μ_1 falls upon the surface it will there diverge (or converge), and in the medium μ will travel as though it had come from a point which will be the second principal focus of the surface; the distance of this point from the surface is the second focal length e_1; e_1 is then the focal length in the second medium μ for light that was parallel in the first medium μ_1. e_1 is positive for a divergent surface as it is measured towards the incident light.

If the surface is a convergent one the focus will be on the other side of the surface, and e_1 will be a negative quantity. So if light in the medium μ_1 is, after refraction, to form a parallel beam in the medium μ, it must have been converging before refraction to a point behind the refracting surface—the first principal focus. The distance of this point from the surface is the first focal length g_1. g_1 is thus the focal length in the first medium μ_1 corresponding to parallel light in the medium μ. It is negative for a divergent surface.

For any surface one focal length as above defined is necessarily positive and the other negative.

From A as origin measure e_1 and g_1 along AE and AD to E_1 and G_1 respectively. As g_1 is negative the point G_1 will be below A.

Complete the rectangle $AE_1H_1G_1$.

Let u be the distance of some object from the surface A. Measure u along AD to R_1. Join H_1R_1, cutting AE_1 at Q. Then AQ is the distance of the image from A. As AQ is measured to the right (*i.e.* in the positive direction), the image is in front of the surface.

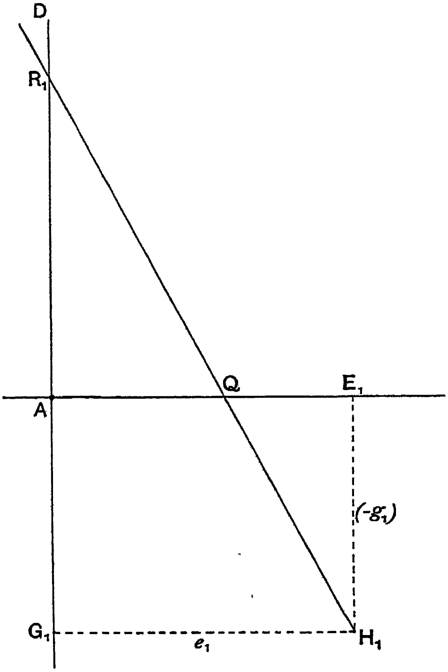


FIG. 183.

If the surface were convergent the point H_1 would be in the upper left-hand quadrant at H'_1, as AE_1 and AG_1 would be reversed in sign.

Then AR_1 and AQ will be the u and v' of the formula for refraction at A.

$$\frac{\mu}{v'}-\frac{\mu_1}{u}=\frac{\mu-\mu_1}{r}=\frac{\mu}{e_1}=-\frac{\mu_1}{g_1}=\phi_A \text{ (say)}.$$

This is easily seen to follow from the similarity of the triangles AR_1Q and $G_1R_1H_1$.

149. **Thick lens.**—Let there be a second surface of which the pole is at B at a distance t from the surface A, separating the medium μ from a medium μ_2.

Let the focal lengths of surface be e_2 in the medium μ for light parallel in the medium μ_2 and g_2 in the medium μ_2 for light that is parallel in the medium μ. If the surface is a divergent one, g_2 is positive and e_2 is negative. Measure AB along the axis from A and draw a vertical line through B.

Measure g_2 and e_2 from B along BG and BA respectively. As e_2 is negative, E_2 is to the left of B. Complete the rectangle

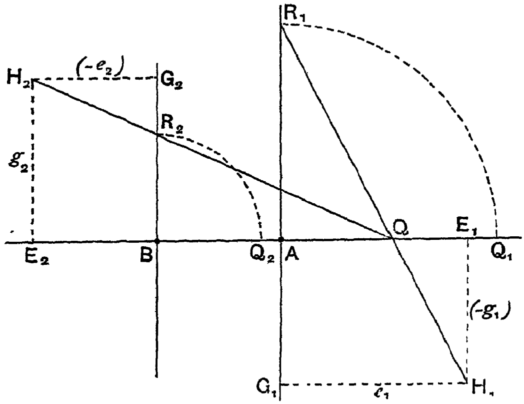


FIG. 184.—Graph for Successive Refractions through Two Spherical Surfaces separating Different Media.

$BG_2H_2E_2$. Then, as AQ is the distance along the axis of the image that is formed by the first lens surface A, and AB has been made equal to the distance of the surface A from the surface B, BQ is the distance u' of the image from the surface B. Join QH_2, cutting the axis BG_2 at R_2. Then BR_2 will be the distance along the axis of the image of Q formed by the surface B, and the v of the formula

$$\frac{\mu_2}{v} - \frac{\mu}{u'} = \frac{\mu_2 - \mu}{s} = \frac{\mu_2}{g_2} = \frac{-\mu}{e_2} = \phi_B \quad \text{(say)},$$

where $$u' = v' + t.$$

150. **Principal Foci.**—Since when R_1 is at infinity, Q coincides with E_1, it is obvious that the second principal focal point is found

by joining E_1H_2, cutting BG_2 at F_2 (Fig. 185). The actual position of the point is found by projecting F_2 on to the axis.

In the same way the first principal point is found by joining E_2H_1, cutting AG_1 at F_1, and projecting F_1 clockwise on to the axis.

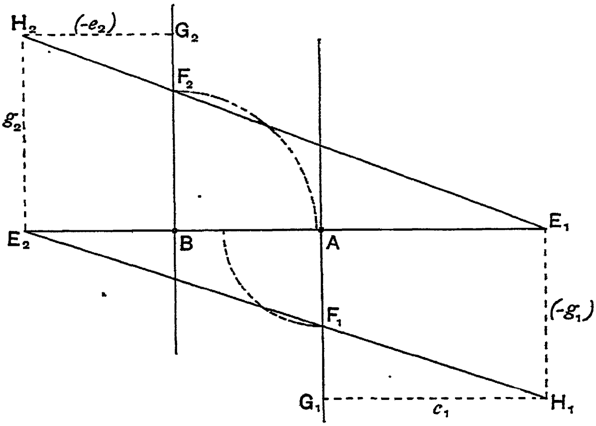


FIG. 185.—Focal Points for two Concave Refracting Surfaces at A and B respectively.

151. i. **Magnification.**—Since a ray passing normally through a refracting surface is not bent, it is obvious that the sizes of the image and object are proportional to their distances from the centre of curvature of the surface; *i.e.* (using the same notation)

$$m_A = \frac{v' - r}{u - r} = \frac{\frac{v'}{\mu}}{\frac{u}{\mu_1}}.$$

So for the surface B,

$$m_B = \frac{v - s}{u' - s} = \frac{\frac{v}{\mu_2}}{\frac{u'}{\mu}}.$$

The final magnification

$$= m = m_A \,.\, m_B.$$

At the principal points this is unity; thus if u_0 and v_0 be the distances of these points from A and B respectively,

$$\frac{\frac{v_0}{\mu_2}}{u'} = \frac{\frac{u_0}{\mu_1}}{v'}.$$

By reference to Fig. 153, it is easy to see that the magnification at the nodal points is $\frac{\mu_1}{\mu_2}$; thus if u_n and v_n be the coordinates of the nodal points,

$$\frac{v_n}{u'}=\frac{u_n}{v'},$$

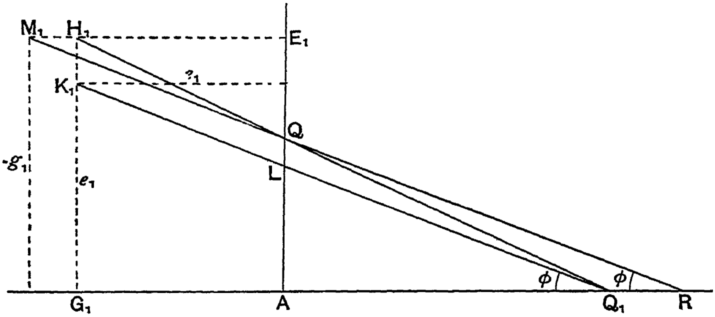


FIG. 186.—Graph for Magnification produced by a Concave Surface.

Graphically the magnification at a single surface is obtained either by making a square AK_1 (Fig. 186), with e_1 as side, and joining K_1Q_1, or by making a square AM_1 with $-g_1$ as side, and drawing M_1QR (Fig. 186); then

$$m_A=\frac{AL}{AQ_1} \text{ or } =\frac{AQ}{AR} \text{ or } =\tan\phi.$$

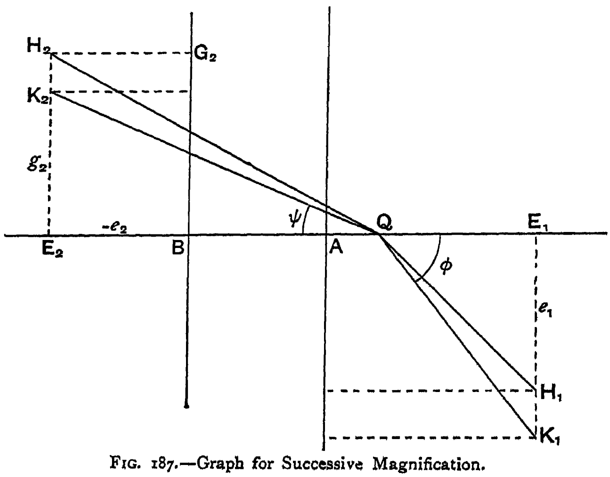


FIG. 187.—Graph for Successive Magnification.

For as $\qquad g_1 = \dfrac{-\mu_1}{\phi_A}$ and $e_1 = \dfrac{\mu}{\phi_A}, \quad \dfrac{-g_1}{e_1} = \dfrac{\mu_1}{\mu};$

and $\qquad \therefore \dfrac{AL}{AQ} = \dfrac{\mu_1}{\mu}$ or $AL = v'\dfrac{\mu_1}{\mu};$

$$\therefore \frac{AL}{AQ_1} = \frac{v'\frac{\mu_1}{\mu}}{u} = \frac{\frac{v'}{\mu}}{\frac{u}{\mu_1}} = m_A = \tan\phi.$$

Complete the squares AK_1, with e_1 as side, and BK_2, with $-e_2$ as side (Fig. 187). Let Q be any intermediate image. Join QK_1, QK_2, making angles ϕ and ψ with the axis; then the magnification is the quotient

$$\frac{\tan\phi}{\tan\psi}.$$

Thus if K_2QK_1 is a straight line, $\phi = \psi$, and Q is the intermediate image of the principal points.

ii. **Principal Points.**—Hence the following construction for these points:

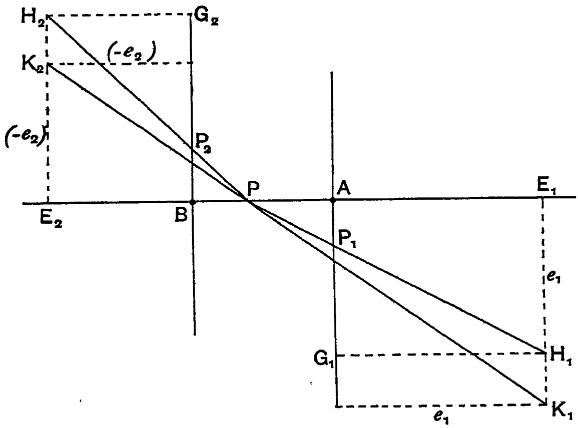


FIG. 188.—Principal Points for two Concave Surfaces.

Complete the squares with the focal lengths of the surfaces in the intermediate medium e_1 and e_2 (corresponding to light, that is parallel in the outer media) as sides. Join K_1K_2, cutting the axis at P. Join PH_1 and PH_2, cutting the perpendiculars through B and A in P_2 and P_1 respectively. Project them clockwise on to the axis.

iii. **Nodal Points.**—Join H_1 and H_2, cutting the axis at N and the planes through A and B at N_1 and N_2. Project the points clockwise on to the axis.

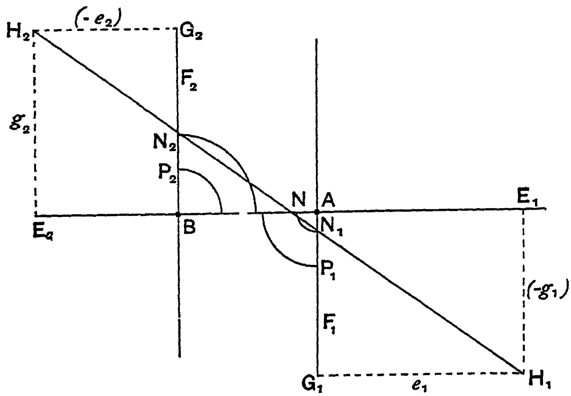


FIG. 189.—Nodal Points for two Concave Surfaces.

For by similar triangles,

$$\frac{BN_2}{BN}=\frac{AN_1}{AN} \text{ or } \frac{v_n}{u^1}=\frac{u_n}{v^1},$$

which is the required condition.

152. **Initial and Final Media the same.**—If $\mu_1=\mu_2$,

$$\frac{e_1}{g_1}=\frac{-\mu_1}{\mu}=\frac{-\mu_2}{\mu}=\frac{e_2}{g_2}.$$

Thus in Fig. 188, the ratio

$$\frac{H_2E_2}{K_2E_2} \text{ would become } =\frac{H_1E_1}{K_1E_1};$$

and therefore H_1PH_2 would be in one straight line. Thus P would coincide with N, and both principal and nodal points are found by joining H_1H_2.

As the focal lengths are measured from the principal points to the focal points, the second focal length is positive in the above figures, and the first focal length negative. The combination is therefore a divergent one.

153. **Formulae.**—Some simple formulae follow easily from the above constructions.

Let the distance E_1E_2 (Fig. 190) be called k, and let k be positive when E_1 and E_2 are in the same order as the surfaces A

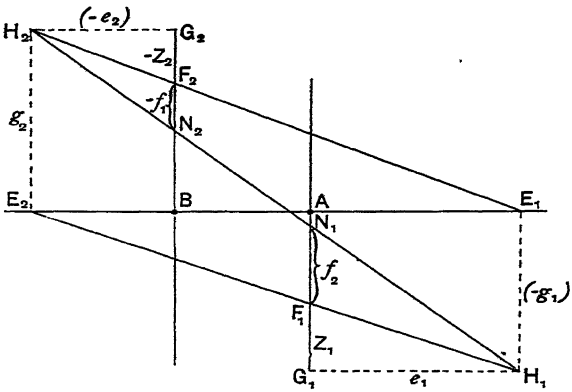


FIG. 190.—Focal Lengths for Combination of two Concave Surfaces at A and B.

and B to which they belong. Note that E_1 and E_2 are the focal points corresponding to light that is parallel outside the combination.

Then, by similar triangles, calling f_1 and f_2 the focal lengths of the combination,

$$\frac{-f_1}{-g_1} = \frac{-e_2}{k}; \text{ or } f_1 = \frac{-g_1e_2}{k}.$$

$$\frac{G_1F_1}{-g_1} = \frac{z_1}{-g_1} = \frac{e_1}{k}; \text{ or } z_1 = \frac{-e_1g_1}{k}.$$

$$\begin{aligned} AF_1 &= \frac{-g_1(t-e_2)}{k} \\ &= (-p_1) + (-f_1) \\ &= -p_1 + \frac{g_1e_2}{k}; \text{ or } p_1 = \frac{g_1t}{k}. \end{aligned}$$

$$\frac{f_2}{g_2} = \frac{e_1}{k}; \text{ or } f_2 = \frac{g_2e_1}{k}.$$

$$\frac{G_2F_2}{g_2} = \frac{-z_2}{g_2} = \frac{-e_2}{k}; \text{ or } z_2 = \frac{e_2g_2}{k}.$$

$$\begin{aligned} BF_2 &= \frac{g_2(t+e_1)}{k} \\ &= p_2 + f_2 \\ &= p_2 + \frac{g_2e_1}{k}; \text{ or } p_2 = \frac{g_2t}{k}. \end{aligned}$$

Where z_1 and z_2 are the distances of the principal foci of the combination from those principal foci of the surfaces which correspond to light parallel within the combination; and p_1, p_2 are the distances of the principal points from the surfaces, all being positive when measured towards the light.

154. **Plano-Concave Lens in Air.**—In this case E_1 is at ∞ on the axis. H_1 is at ∞ on the line AD, inclined at an angle $\tan^{-1}\frac{1}{\mu}$ to the axis. Thus draw H_2F_2 parallel to the axis to find F_2, draw $H_2P_2P_1$ parallel to AD to find P_2 and P_1, draw E_2F_1 parallel to AD to find F_1. It is obvious that P_2 coincides with B.

155. **Thin Lenses.**—The above graphs apply equally to thin lenses separating different media, if e_1g_1, e_2g_2 are still taken to be the focal lengths in the several media.

Gauss Points of Thick Lens.

156. **Experimental.**—*Apparatus.*—The two pairs of an ordinary photographic half-plate lens, mounted in sliding tubes so that their separation can be easily varied. Also a telephoto lens, or a concave lens mounted to combine with one of the lenses above mentioned, in place of one of the components; nodal slide; and other apparatus of § 131. Instead of special tube mounts, the lenses may be supported as in § 39, and their distance apart can then be easily varied. To find their nodal points, the base can stand on a rotating table constructed on the same principle as that shown in Fig. 170. For a telephoto lens, the base will have to be extended for say two or three feet and pivots provided in the extension about which it may rotate.

Find first the focal lengths and focal distances (*i.e.* the distances of the focal points from the lens surfaces) of each of the three lenses. Then insert the two convex ones in the tube at a known distance apart. The accompanying figure shows the graph when they are at a distance apart numerically equal to a quarter of their focal lengths (supposed equal). Find the positions of the focal and principal points experimentally, and compare them with the graph. Repeat with other separations.

Also find the positions of the points for the combination of the concave and convex lenses.

As a further exercise the graph may be drawn for the lens apparatus of § 170.

Measure the focal length of the lantern condenser of p. 174, Nos. 7-10; find the nodal and focal points, and compare with the results found there.

CHAPTER IX

ABERRATIONS OF LENSES AND MIRRORS

The Aberrations of a Photographic Objective.

157. *Explanation and Definitions.*—According to Seidel there are five spherical aberrations in addition to the chromatic aberrations of a lens.[1] These are, first, **axial spherical aberration**, that is, the longitudinal aberration of a pencil of rays which is parallel to the axis. Second, **coma**, that is the want of symmetry of the aberration of an oblique pencil about the axis of that pencil; it produces a nebulous appearance in the image of a point of light not on the axis. Third, **astigmatism**, which is the convergence of an oblique pencil of rays from a point not on the axis to two focal lines instead of to a point. Fourth, **curvature of the surface**, the curvature of the image of a plane surface perpendicular to the axis of the lens, so that the marginal portions of the images are not in the same plane as the central portion. Fifth, **distortion**, the unequal magnification of parts of the image near the centre and the margin of the field respectively.

In addition to the above a lens will possess more or less **chromatic aberration**; firstly, the images formed by different colours in the spectrum may have different positions; secondly, they may be of different sizes, even if in the same position; thirdly, if the images of two colours coincide, the images formed by the other colours in the spectrum may not coincide; fourthly, if the coincidence occurs in the centre of the field, it may not occur near

[1] See S. P. Thompson's translation of Lummer's *Photographic Optics*, Appendix; or Southall's *Geometrical Optics* (Macmillan).

the margins; moreover the five errors of spherical aberration enumerated above might be all eliminated for one colour, and yet not be absent for other colours.

For some purposes it is very important that the lens should be corrected for at least three parts of the spectrum. The lenses intended for ordinary visual work, and indeed almost all ordinary achromatic lenses, are supposed to be corrected only for two colours, say the yellow and blue, but for photographic work this is insufficient. A lens corrected for three parts of the spectrum is called an "**apochromatic.**"

Determination of Aberration Coefficients of a Compound Lens.

Apparatus.—Most of these aberrations may be measured with the apparatus already described (page 180). It will be necessary for some of the experiments to use a compound lens in which the distance between the two components of the lens may be varied.

158. **Spherical aberration of an axial pencil.**—When light passes axially through a lens, that which is incident upon its margin is usually converged to a focus at a shorter distance from the lens than that through its centre. In other words, the power of a lens with spherical surfaces is usually greater at its margin than in its centre. For instance, when the beam from an arc light passes through an ordinary lantern condenser, if the rest of the lantern front be removed so that the beam can be seen in the air beyond the condenser, it will be obvious that it does not consist of a simple cone. The light from the edge of the condenser comes to a focus nearer the lens than that from the central part.

(*a*) **Simple Lens.**—The spherical aberration of such a lens can be found by first cutting away all the central portion of the light through the condenser, by a disc placed over the centre of the lens, which allows only a ring of light round the margin of the lens to get through, and focussing the image of the source of light (which for this purpose may be an ordinary flame illuminating a cross-wire), formed by this hollow cone of rays. Then, cover all the lens except the centre by a card with a hole in it

about 1″ in diameter, and find the focus for the central cone of rays, the cross-wire not having been moved. The distance between the two foci is the longitudinal aberration, s, of that lens for this object distance.

The positions of the foci for central and marginal rays respectively may be conveniently found by covering the lens with a cardboard disc, in which two rings of holes have been pierced, one on a circle nearly as large as the lens-aperture, the other on a small concentric circle. The positions of the points to which the two cones of rays converge can be easily ascertained and distinguished.

The aberration is proportional to the square of the inclination of the rays, and for similar lenses is proportional to their focal lengths. If, when the incident light is parallel, this aberration is put equal to $4sf\phi^2$, f being the focal length of the combination and ϕ the circular measure of the angle which the marginal rays forming the image make with the axis, the quantity s will be independent of the focal length and aperture, and depend only on the design of the lens. It will be constant for a series of lenses of similar design. It may be termed the coefficient of axial spherical aberration, or the "aplanatic coefficient."

(*b*) **Photographic Objective.**—The axial aberration of a photographic objective can be found by the autocolimation method described on page 181. The marginal rays are obtained by opening the diaphragm of the lens to its full extent and cutting off the central pencil by a black disc temporarily attached to the front of the mirror. The axial pencil is easily obtained by stopping down the lens with its own diaphragm. Then, if n is the "f number" of the largest stop, $\phi = \frac{1}{2}\frac{n}{f}$, and therefore the coefficient is given by

$$A = \frac{n^2 s}{f},$$

when A is the longitudinal aberration, *i.e.* the distance between the foci for central and marginal rays respectively.

(*c*) **Micro-objective.**—The aberration is found exactly as that of the photographic objective, using of course the apparatus of § 135.

159. **Astigmatism.**—The astigmatism may be measured very easily by rotating the lens horizontally about the vertical axis of the turn-table through a known angle, and focussing first a vertical wire of a pair of cross-wires of which the image will be formed at the primary focal line; and then the horizontal wire, the image of which will be formed at the secondary focal line. Measure the distance between the two positions, which will be a measure of the astigmatism at that inclination.

The oblique cone of rays after refraction passes approximately through a line perpendicular to the paper at q_3 (Fig. 191), and through a line in the plane of the paper at Q_3, where the refracted ray meets the oblique axis OM. The distance apart of these two lines, t, measures the *astigmatism* of the pencil. The astigmatism depends upon $TF\theta^2$, where θ is the inclination of the axis of the pencil to the axis of the lens; expressed in circular measure, T is a constant, and may be termed the "astigmatism coefficient," for the astigmatism is proportional, according to Petzval, to the square of the inclination of the axis of the incident pencil to the axis of the lens.

Then we may put $$t = TF\theta^2,$$

or $$T = \frac{t}{F\theta^2} = \frac{tf}{r^2} \text{ (nearly)},$$

where r is distance between the points of intersection of the axes of the lens, and of the oblique pencil, respectively, with the focal plane.

160. **Curvature and Distortion.**—Let OA (Fig. 191) be the axis of the lens passing through the centre of the stop D_3, and Pp be the plane of a flat object perpendicular to this axis, and let Q be the conjugate focus to P, so that the plane QM through Qq should be the conjugate focal plane to pP. If the image q of p is not on this plane, the focal surface is curved, the curvature being $\frac{2qM}{QM^2}$. The curvature of the images formed by the primary focal line which is at q_3, and by the secondary focal line at Q_3 will be different from the curvatures of this geometrical image q. In fact, if the curvature of the surface formed by the geometrical foci, the q surface, be $-U$; that of the primary focal line surface, the q_3 surface, will be $-U-3L\theta^2$; that of the secondary surface will b $-U-L\theta^2$.

The curvature of the field may be measured over a small distance with the same apparatus if the mirror is kept close to the lens. The radius of curvature is equal to $\frac{MQ^2}{2qM}$, and the "coefficient of curvature" may therefore be taken to be $\frac{2fqM}{MQ^2}$.

Placing the lens so that its axis is perpendicular to the screen, rotate the mirror so as to form an image at distances of 1, 2, 3, 4

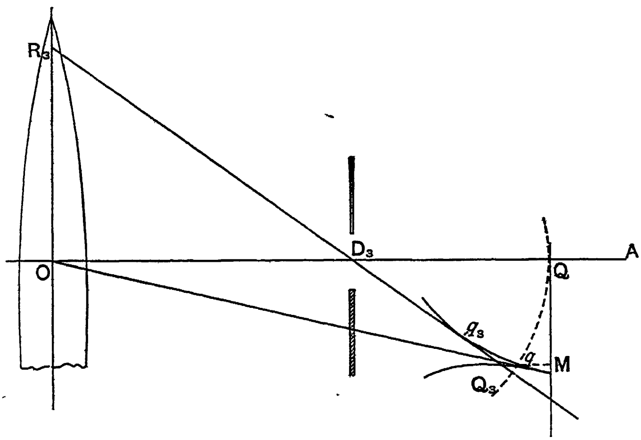


FIG. 191.—Curvature of Field.

cm. from the centre. Keep the lens still, and move the screen forward to obtain a sharp image at each distance, and measure the amount the screen has to be moved. If the curvature is not too great, as this movement forward of the screen throws the rays falling upon the mirror out of parallelism, it will make the apparent curvature of the field only half its actual amount. If, therefore, the readings to right and left of the centres at each distance from the centre be added together, the sum may be used for the value of qM in the formula.

As the flatness of field will only be of importance when copying from other pictures, it will really be fairer to the lens to test it under similar conditions; and therefore both this and

the next error (distortion) are better measured with the following apparatus:

Apparatus.—AB is a screen at right angles to the bed of the optical bench CD. A second similar bed EF is fastened at right angles to CD and parallel to AB. The screen AB is mounted upon a stand so that it can be moved to and fro along CD.

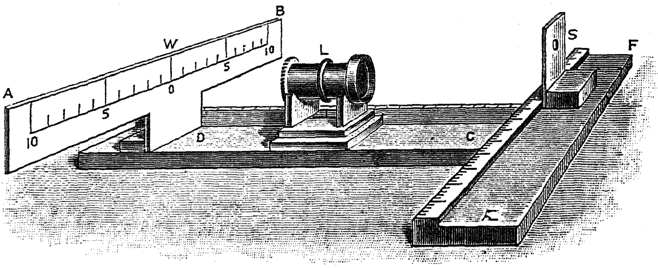


FIG. 192.—Simple Bench for measuring Distortion.

Upon EF is a stand carrying a screen containing a cross-wire facing AB, and a flame to illuminate the cross-wire. The lens L to be tested is placed upon CD with its axis parallel to CD, and the distances of L and AB from the screen S are adjusted so that an image of S is formed on AB the same size as the original; that is, the distances from S to L and L to AB are equal.

A scale is marked in cm. on AB, beginning with the zero opposite the axis of the lens and numbering each way.

Begin with S in the centre of EF so that the image of the cross-wire is formed at the zero on the scale AB. Focus it carefully, then move S until the image is formed at each of a series of equidistant points on AB on each side of the origin. Read the positions of S on the scale on EF, and of the image on the scale on AB, and also the amount the stand AB has to be moved in order to obtain a perfect definition at each position of the image. The lens itself must not be moved, and care must be taken that the stand of S is kept well against the guide on the bench EF so that its movement is made accurately in a plane perpendicular to CD.

Enter the results in columns thus :

Position of S on Scale EF.			Position of Image on Scale AB.	Position of Stand on Scale CD.				
Left.	Right.	Mean $=a$.	$=b$.	Left.	Right.	Sum $=c$.	$\frac{b}{a}-1$.	$\frac{c}{b^2}$.

$\frac{2c}{b^2}$ is the curvature of the focal surface. Therefore, if the mean value of the last column is found and multiplied by $2f$, the coefficient of curvature will be obtained, which is taken to be the focal length of the lens divided by the radius of curvature of the focal surface. The numbers in the last column will be a constant if the surface is spherical; but not if it is irregularly curved.

The **distortion** will be given by $\frac{b}{a}-1$ if the distances of the lens from the scale and screen respectively are equal; for then the ratio of the size of the object to that of the image in the centre of the field will be unity, and for a lens free from distortion will remain unity at all distances from the centre; that is for all sizes of the image.

161. **Coma.**—Let AOB be the axis of a lens, and suppose the lens perfectly corrected for spherical aberration and astigmatism,[1] so that pairs of rays from a point P (not on the axis) which are refracted by the lens at equal distance on opposite sides of the oblique axis POQ are converged to the same points, Q_1, Q_2, etc. It may further be free from curvature of the field, so that Q, Q_1, Q_2 are all on the plane BQ, conjugate to the plane AP.

Yet the points to which these pair of rays converge will not necessarily be Q. Petzval showed that the distance of a point Q_1 from Q is proportional to the square of the distance from O to the points L_1, L_2, at which the pair of rays converging to Q_1 cut the lens. The result of this aberration is to produce a "flare" on the image surface. The image of a bright point P formed by a lens otherwise free from aberration is usually a somewhat egg-shaped patch, much brighter at the small end and

[1] This may occur with some compound lenses.

fading gradually in all directions to the boundary. The diameter of the flare QQ_2 is proportional to the square of the aperture, and to the inclination of the axis and of the pencil (more accurately to its tangent).

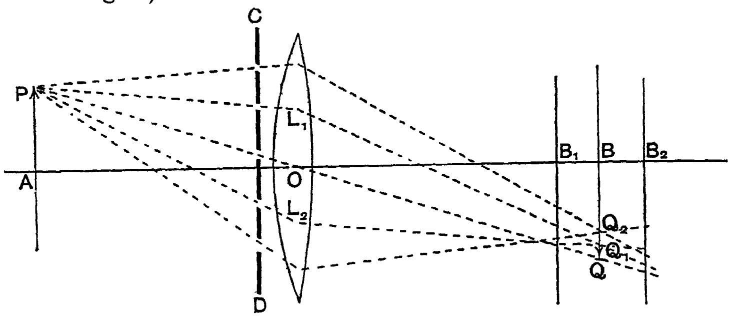


FIG. 193.—Coma.

By placing a screen CD near the lens, in which are cut holes as shown in the figure, photographs may be taken first upon a plate placed at B_1 and then on one placed at B_2 of the intersections of the rays with these planes. From these it is easy to calculate where each ray cuts the image plane BQ. Thus the amount of the coma can be estimated.

Note.—In the best modern lenses the errors above enumerated are corrected to such an approximation that the residual small errors do not obey Seidel's formulae; in other words, the coefficients above defined are not constant for a given lens. In such a case either the *minimum* value of the coefficient, or its value under specified conditions, could be found. The simple methods here given may not, however, be sufficiently delicate to measure the coefficients.

162. **Beck's Testing Bench.**—A very convenient bench, upon which all the measurements of aberration can be made, has been devised by Messrs. Beck, and improved by Chalmers—a modified form is shown in Fig. 194.

The lens to be tested is carried on a turntable C, rotating about a vertical axis O. On this is a slide D, which can be moved to and fro by the milled head M. This slide carries a large ring RS, in which an inner ring fits on ball bearings, so that it can be rotated within the outer ring. The inner ring has a coarse thread, and an adapter a, a_1 is made which fits the thread of the lens to be tested, and screws into the ring. Thus the lens to be tested is attached centrally to the inner ring by its appropriate adapter,

and it can be rotated round its axis to test the alignment of its components; it can also be adjusted by turning M so that one of its nodal points is above O, the axis of the rotation of the turntable.

The turntable is mounted on a carriage on a long bar. On the far end of this bar or on a separate similar bar is a collimator, not shown in the figure, furnished with a good telescope objective. The object, which is of course in the focal plane of the objective,

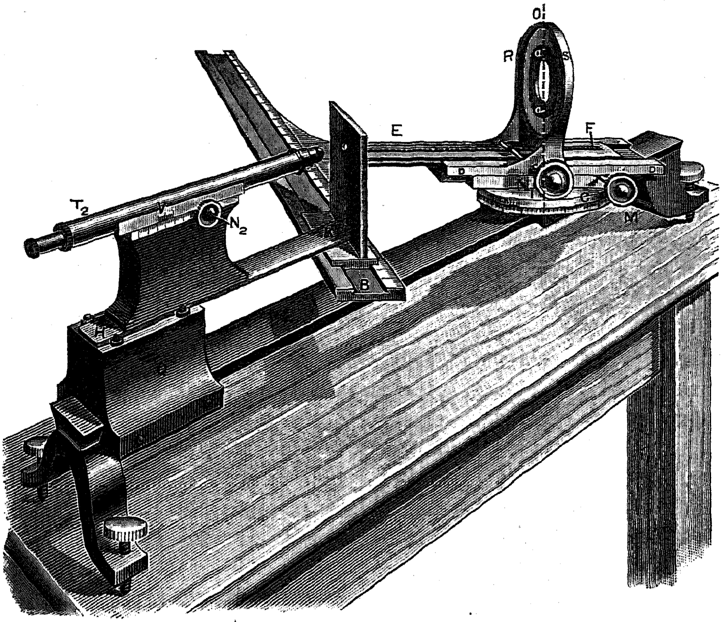


FIG. 194.—Testing Bench for Compound Lenses.

may be either a pin-hole for some tests, or a cross and circle (as in § 33) for others. Any good light may be used. The collimator stand should be furnished with small milled heads to enable i to be adjusted both vertically and horizontally.

At the front end of the bar is a carriage Q, with a sliding bar HK, bearing a microscope T_2, which magnifies some 40 times. It has a long body (about 18 inches), so that it may have a good range of focus, without the observer having to get too close to the end of the bar.

A long straight dovetail groove AB is attached to the turntable ABC by means of a slide EF. The groove is perpendicular to

the slide and parallel to the face of the ring RS, and therefore to the focal plane of the lens to be tested. The slide EF is parallel to the slide D, and like it is fitted with a rack and moved by a pinion N_1, by which the distance of the slot from the plane of RS can be varied from about 3″ up to 18″.

In this groove slides a nicely fitting plate, which is pivoted at K to the sliding bar HK, on the carriage Q. The plate also carries a cross-wire vertically over the pivot K. The milled head N_2 gives a focussing adjustment to the microscope T_2 relatively to this sliding bar through a small range; this movement is furnished with a scale. The focal plane of the microscope is vertically over the pivot K in the zero position of the scale.

Insert a lens in the RS by its adapter a, a_1, set the turntable with the axis of the lens parallel to the bar, and focus the microscope T_2 upon the image of the pin-hole (or cross and circle) by the milled head N_1. If the nodal point is not over the axis of rotation, the image will move sideways when the turntable is rotated; this movement is to be corrected by the milled head M.

If the lens has a flat field, this field will be parallel to the groove AB, and should intersect it in a plane passing through K; thus, if the turntable be rotated, the bar HK will cause the microscope T_2 to be carried back at exactly the rate necessary to keep the image in focus. If the field is not flat, T_2 will have to be adjusted to keep the image in focus, and this is done by the milled head N_2; the amount of the motion, relatively to HK, will measure the error. It is read by a vernier V on N_2. To enable this measurement to be made for definite positions of the turntable, the latter is furnished with a scale of degrees to measure its angular displacement.

If there is astigmatism, the vertical and horizontal lines of the cross must be focussed in succession, and the difference of focus will measure the error.

The amount of coma can be estimated by observing the unsymmetrical flare produced when the pin-hole is used as object at the end of the collimator, and the turntable is rotated.

Measure the curvature of the field, and the astigmatism, of a compound photographic lens, of which the components are mounted in such a way that their distance apart can be varied,

and see how the errors are affected by increasing or decreasing the separation Also investigate the effect of the position of a stop used with a simple lens. Use as lens (1) a plano-convex, and (2) a converging meniscus lens, with the stop in front at various distances, and also behind; also with the convex face of the lens (1) towards and away from the light.

Plot the errors for different angles of rotation, and obtain curves for each variation.

If a lens is to be tested under the conditions in which it is used in enlarging or copying, a fitting carrying a short microscope can be put in the slide AB, and the turntable clamped in its zero position. A glass scale (or a series of clear horizontal and vertical lines on a blackened glass plate) mounted on a stand perpendicular to the length of the bench, and illuminated from behind, may be used as object. The microscope must be capable of being rotated about a vertical axis which should pass through the focal plane of the objective, so that the rotation of the microscope may not displace the image.

Chromatic Errors of Compound Lenses.

163. **Discussion.**—If two colours only are considered, say the yellow and the blue, a lens which is free from all the errors due to spherical aberration may yet suffer in two ways from chromatic errors. (1) The focal length of the lens for the yellow rays may not be the same as that for the blue. This will result in the image of a distant object formed by the yellow rays being of a different size from that formed by the blue rays; it may or may not be in a different position.

(2) The second principal point for yellow light may not coincide with the one for blue light. The focal plane is at a distance equal to the focal length of the lens from the second principal point, and therefore its position depends not only upon the focal length but upon the position of this principal point, and the position of the principal point will depend upon the colour. Thus, even if the lenses are so calculated that the focal lengths for yellow and blue have been made equal, the focal planes may still not coincide, being, as already observed, dependent upon the position of the second principal points.

It might happen, on the other hand, that the focal planes coincided, though the two focal lengths were unequal, if the principal

points for the two colours were so situated that their distance apart was equal to the difference of the focal lengths. If so, the yellow and blue images would coincide in the centre of the field, but they would not be the same size, and would therefore not agree at the edges of the field.

In order that the images formed by the yellow and blue may be identical and identically situated, therefore, not only must the two focal lengths be equal, but the principal points for both colours must be coincident.

For any ordinary achromatic lens this is all that is attempted. For photography, however, this is not sufficient, and those lenses known as "apochromatic lenses" are supposed to be corrected for *three* parts of the spectrum, the red, green and violet, or ultra-violet.

164. **Experimental.**—The equality of the focal lengths can be determined by measuring them with the nodal slide as above described, with each of the coloured lights in turn. The latter can be produced by using the coloured media employed by Maxwell, namely sulpho-cyanate of iron for the red, ammoniated solution of copper for the blue, and copper chloride for the green. The distance from the screen to the axis of rotation should be the same for each colour, this distance being the focal length. If not, the difference between the red and the blue focal lengths divided by the focal length, and also between the green and the blue focal lengths divided by the focal length, will measure the chromatic aberrations for the red and green respectively.

To find the distance between the principal points (or nodal points), the amount the lens has to be moved upon its stand, when adjusting the principal point to coincide with the axis of rotation, must be observed. It may be more useful in practice to know the distance between the focal planes for the three colours instead of between the principal points. This may be calculated from the difference in the focal lengths and the distances between the corresponding principal points, but it may also be observed directly by focussing the image with each of the coloured solutions in turn, by means of the screen only, the lens itself not being disturbed between the three observations.

If desired, the various spherical aberrations might be measured with each of the coloured lights and compared, but the difference

between them will only be of the second order and within the limits of experimental error.

165. **Measurement of Aberration.**—The chromatic aberrations can be *measured* by a modification of the apparatus (p. 180) by which the focal lengths for definite spectrum colours can be found.

The light from an incandescent burner B is reflected by a small right-angle prism R_1 forward to the lens L mounted on a turn-table N. The light then passes through a prism P_1 on to a mirror M; the mirror reflects it back through the prism and lens, and if the lens is at its focal length from the screen S_1 a

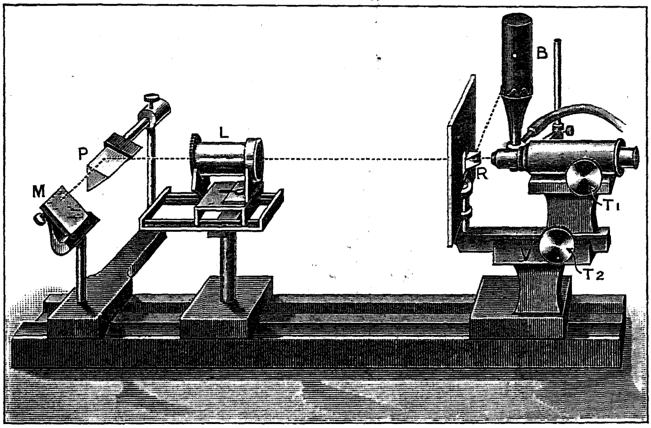


FIG. 195.—Chromatic Aberration of a Compound Lens.

spectrum will be formed on the screen, which has been produced by the dispersion of the prism P. The spectrum is well dispersed since the light has gone twice through the prism P.

To produce a pure spectrum, a horizontal slit would be necessary at S. But it is not necessary to have a pure spectrum, and a narrow slit will cut down the light to too great an extent and make it impossible to say exactly when the lens is in focus. Instead of this use a *vertical* straight edge at S to limit the field on one side only. Adjust the prism and mirror until the image of this edge almost coincides with the edge itself. Then if the straight edge extends above the prism R, and a microscope is mounted to view this edge and its image just above R, a narrow

coloured band will be seen, if properly adjusted. By varying the inclination of the mirror, any colour can be brought into the field of the microscope in turn.

In practice, the microscope is mounted with two focussing screws, T_1 and T_2. T_1 moves the microscope alone in the usual way, and thus enables the straight edge itself to be sharply focussed; T_2 moves the stage carrying the straight edge and the prism R as well as the microscope; thus T_2 enables the distance from S to the lens to be varied without disturbing the focussing of the microscope. The movement produced by T_2 can be read by a vernier, V, on a scale attached to the moving part.

Adjustments.—The lens is put on the turn-table, its axis adjusted in a horizontal line at such a height that this line is level with the top of the prism R. Then a piece of white paper is put against the lens, and the light B placed so that the light reflected by R falls on the paper, and will pass through the lens when the paper is removed. Screens must be put round the light, so that it may only fall on R, and the direct light cannot reach either the lens L or the observer at O; in fact, the more enclosed the light and the darker the room the better.

A mirror is now temporarily put immediately behind the lens, as in the former experiments, and the distance of the lens from the screen S adjusted as before, so that a clear image of the aperture in S is formed by its side on the screen S_1, and that this image is stationary when the lens is rotated.

Next, the prism P is set up with its refracting edge horizontal, as in the figure, to receive the light from the lens; by holding a piece of white paper against the prism, it is easy to see if the light is really falling on it properly.

Then the mirror M is put to receive the light from P (again making use of a piece of paper), and rotated until the light is reflected back through P and L and forms a vertical spectrum on the screen S. (If this screen has been blacked, a piece of white paper, in which a hole has been made for the light to pass through, can be hung on it for a minute while M is adjusted.)

Now rotate P until the light passes through at minimum deviation.

All these adjustments are made without using the microscope.

But having made the spectrum nearly coincide with the aperture in S, remove the paper and insert the straight edge—its edge vertical—and place it so that it appears nearly in the centre of the field of the microscope, and focus the latter upon it with T_1. By rotating M about a horizontal axis get the yellow light into the field, and by rotating either the mirror or the prism about a vertical axis, get the *image* of the slit into the field. Adjust T_2 till the image is as sharp as possible. There should now be a narrow vertical strip of yellow light falling into orange at the bottom and greenish yellow at the top, bounded by the straight edge on one side and the image of the straight edge on the other, *i.e.* it should be nearly dark on each side of the coloured strip.

If the straight edge is not set quite parallel to the length of the spectrum (*i.e.* quite vertical), its image will be bounded by a narrow fringe of green or of pink, according to the direction in which it slopes. It must be turned round in its own plane until this fringe disappears. It will now be found possible to focus the image of the straight edge with T_2 more sharply.

Rotate the lens very slightly—say 10° each side of the axial line—and see if it is correctly placed with its nodal points (for yellow light) over the axis of rotation, by observing if the image of the slit suffers any lateral displacement as the lens rotates. Having if necessary corrected this, and refocussed the image either by moving the lens stand bodily if much is required, or by turning T_2 if the amount required is small, the lens stand should be clamped and all future focussing done with T_2.

The final focussing is best done by getting the slit and its image very close together in the middle of the field of the microscope. First focus the slit with T_1, then the *two together* with T_2. The slit and its image must be so close together that they can be seen at the *same time.* If one is looked at and then the other, the accommodation of the eye can be changed so rapidly that both may appear sharp, although they are not exactly in the same plane, but if the attention is fixed on a point in the centre of the field, midway between the slit and its image, and if they are so close together that they appear separated by a mere line of light, it will be found possible to focus to a high order of accuracy. The accuracy of this focussing can be judged by

taking a series of readings, refocussing both T_1 and T_2 between each reading, and comparing the readings. They should all agree to within one or two tenths of a millimetre with a half inch micro-objective and any ordinary photographic lens.

Readings.—It will probably be found that the achromatism of the photographic lens is better than that of the microscope. As, however, only one colour will be used at a time, this is immaterial if only the microscope is focussed by T_1 on the edge itself each time the colour is altered. Probably there will be a bad flare when the lens is placed exactly axially, caused by the light reflected by the surface of the lens itself. This will be especially noticeable in the red and violet owing to the small luminosity of those colours. It can be almost entirely eliminated and the purity of the spectrum increased by cutting off all the light reflected by R, except that from a small square of about 1 mm. side level with the top of the prism, by a piece of black paper cut this shape.

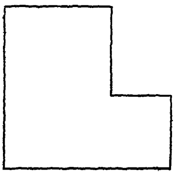


FIG. 196.

By raising or lowering this the amount of light can be varied at will.

It is well to work with known wave lengths. To obtain these interpose a Bunsen burner between B and R, and with sodium, potassium, strontium and thallium in turn set the mirror M so that the yellow, red, violet, blue and green may be seen in the field, screening the light B for the moment. The focussing in each case must be done with B, as the coloured lights given by the salts are not sufficiently bright; their only function is to fix definitely the colours.

166. **Commercial Testing.**—The colour corrections of a lens can be *tested* best by forming an image of a flat sheet of perforated zinc by means of the lens on a ground-glass screen. The zinc must be illuminated from behind through a sheet of opal or ground glass. Then if there are outstanding chromatic errors, the images of the perforations will be drawn out into coloured spectra towards the margins of the field, or there will be coloured margins round their images. The former indicates an inequality in the magnifications for different colours, *i.e.* in the focal lengths for these

colours. The latter shows that the images are not formed in the same plane. By watching the change in the appearance of the image as the distance of the ground glass from the lens is slightly varied, the nature of the error can be estimated. With practice in this method the results become very trustworthy, and it is probably therefore the best to use when adjusting a lens.

Accuracy of Alignment of the Components of a Compound Lens.

167. It is very important in a compound lens that all the surfaces shall really be coaxial. This can be readily tested (and if desired adjustments made) by pointing the lens at a point source of light, and observing the series of images formed by successive reflections in the surfaces of the lenses. The images should all appear to lie on a straight line.

For the point source of light, use the image of a small circular source of light (*e.g.* that from an incandescent gas flame issuing through a small circular aperture) that is formed by a bulb of mercury. This must, of course, itself be accurately on the axis of the lens. The series of bright specks of light are easily seen on looking through the lens towards the light. They should appear to be exactly in one straight line.

Examination of the Surface of a Concave Mirror.

168. **Foucault's Method. Theory.**—When light starts from the centre of curvature of a concave mirror, and is reflected from the mirror, it will be focussed once more at the same point. If it starts from a point a little to one side of the centre of curvature, it will be focussed at a point a little to the other side.

Let ABC be the mirror, and S a bright spot of light close to the centre of curvature; P its image formed on the other side of the centre of curvature. An eye placed at P would receive light emanating from S by reflection from every point of the surface of the mirror AB, and thus the whole of the surface would appear equally bright. No *image*, of course, will be visible. The eye may be withdrawn to a short distance behind P, and still see the whole surface bright if the pupil of the eye is large enough to

contain the whole cone of rays. Thus if E_1E_2 be the pupil of the eye, the angle E_1PE_2 must not be less than the angle APB.

Suppose now that a card with a vertical edge is gradually moved in the direction GF. If the whole of the light from the mirror is concentrated at P, the card will produce no effect until its edge reaches this point P; then it will shade the light from the whole surface of the mirror simultaneously, so that the mirror will grow dim, but will do so uniformly all over. It will not cut the light off instantaneously unless the point is infinitely small. If the card is placed in the path of the rays, either nearer the mirror than P or further away, it will not cut off the

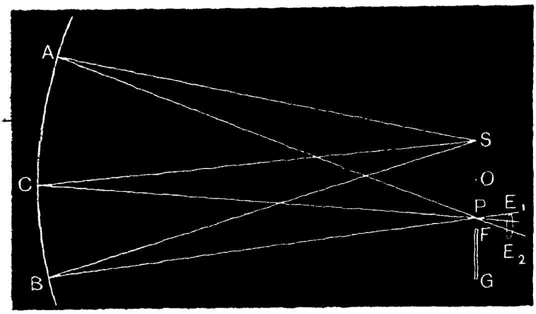


FIG. 197.

light uniformly from all parts of the mirror; for instance, if the card is nearer the mirror than P, it will shade the part of the mirror near B first, and, as the card is moved forward, a shadow will cross the mirror in the direction BA. If the card is further from the mirror than P, it will shade the mirror first on the side A, and the shadow will travel in the opposite direction, namely from A to B.

Suppose next that the curvature of the mirror is not constant over its surface, and that the outer zone of the mirror focusses the image of S at a greater distance from the mirror than the central part of the mirror, and that P_1 is the image of S formed by the central, and P_2 by the marginal part of the mirror AB. Then it is evident that if the card be moved in the direction G_2F_2, to cut the cone of the rays at P_2, it will shade the marginal

parts of the mirror uniformly; but, being further from the mirror than the point P_1, to which the central rays converge, it will cause the centre of the mirror to shade unevenly, and the shadow will travel across the central portion of the mirror in the direction CD. If placed in a position G_1F_1, so that it cuts the cone of rays at P_1, it will shade the centre of the mirror uniformly, and the margin unequally. The shadow on the margin will travel from B to A. At an intermediate point it will shade the one part of the outer zone of the mirror and the opposite part of the central portion of the mirror as in the diagram.

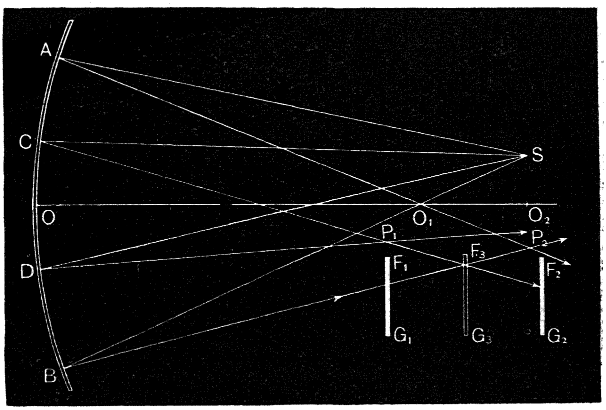


Fig. 198.

If, on the other hand, the marginal part of the mirror is more curved than the central zone, the image P_1 formed by the central zone will be further from the mirror than P_2. In this case the directions in which the shadows appear to travel across the mirror will be reversed, as can easily be seen by drawing the figure.

In the former case the section of the mirror would be elliptical, parabolic or hyperbolic. In the latter case it would be elliptic; the axis being the minor axis of the ellipse, the surface itself would be an oblate spheroid. Whether the mirror be elliptic, parabolic or hyperbolic, can be determined with practice, by the extent of the shadow. It can also be determined by measuring the aberration, that is the distance between P_1 and P_2. For a parabolic

mirror this distance is given by $\frac{R^2}{8f}$, where R is the radius of the surface of the mirror and f its focal length. This aberration will be seen to be very small. For instance, a six inch parabolic mirror of five feet focus would have an aberration less than $\frac{1}{50}''$.

169. *Apparatus.*[1]—The concave mirror to be tested; a point source of light, and a card. The point source of light is most conveniently made by surrounding the glass chimney of a very small lamp (such as is used by a microscopist), with a zinc chimney in which is a hole about $\frac{1}{16}''$ in diameter on a level with the flame.

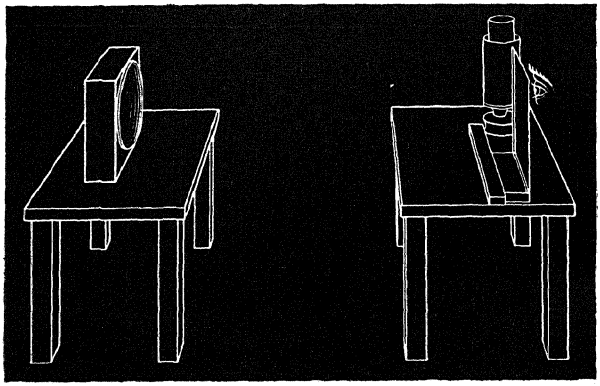


FIG. 199.—Examination of a Concave Mirror.

The card must be attached to a squared block of wood, and a straight edge must be held on the table for this block to slide against, or a sort of small optical bench similar to that described on page 53 may be used.

For accurate work the rough screen above described is not convenient, and one which can be more easily controlled is necessary, as in Fig. 200. In this figure A is a base board which rocks on two rounded legs, G, G′, which stand in a triangular hole (shown separately at L), and a slot K. The third leg is a screw F which rests on a plane. On the base is fixed an upright board B, cut at the top to a portion of a circle of which the centre is at the hinge H. This hinge attaches a narrow board D to the base A, to which the screen C (of card or zinc) is fastened. D is held against B by an elastic band R, and can be forced

[1] For instructions for grinding and polishing a mirror, see p. 494.

away from B to any small distance required by the screw E. A vernier V is attached to the back of D, which reads on a scale marked on the top of the board B, and thus the movement forward of C can be read off.

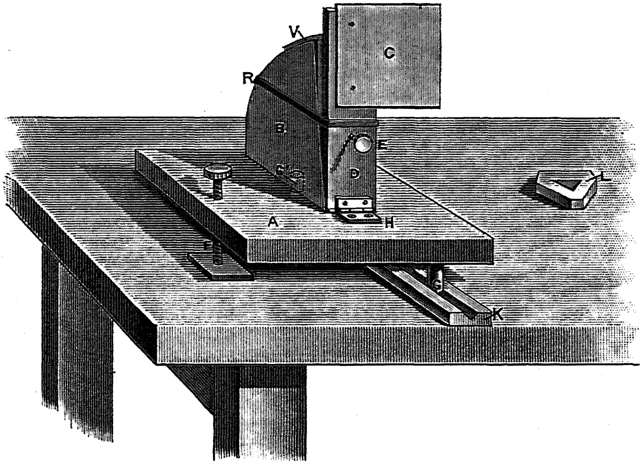


FIG. 200.—Adjustable Screen for Foucault Test.

Experiment.—The mirror is set up with its plane vertical. At a distance equal to twice its focal length, the lamp with its pin hole, and the card, are placed conveniently on a table so that the observer may sit behind them. If the observer prefers to use his right eye, he should have the lamp on the right side of his head, and move the card from left to right; for the left eye these positions should be reversed. If the mirror is a good one, so that its figure is fairly perfect, no reliable observations can be made unless it is treated with very great care. The warmth produced even by touching it with a finger will show for a long time, also the unequal density of the air in the room in the day-time will affect it, so that for good work the observations should be taken at night. The room should be fairly dark unless the mirror is silvered, and then this is not necessary.

The light being adjusted, the eye should be moved about and the position found in which the mirror appears bright. If this

position is not close to the lamp and on a level with it, the mirror must be tilted until this is the case. Or a screen may be placed by the side of the lamp and the mirror adjusted to form an image of the pin hole upon the screen. The pin hole and its image must be at about the same distance from the mirror, but if the mirror is of 5 or 6 feet focal length, they may be 2 or 3 inches apart without injury. For mirrors of short focus a small right-angle prism can be used to reflect the light from the lamp forward on to the mirror. In this case the pin hole should be close to this prism. The eye can then also be put quite close to the prism, and therefore both can be very close to the axis of the mirror. When the eye is placed at the correct position the whole mirror will appear bright. On sliding the card across to cut off the light from the mirror, it will be easy to see if the shadow appears to travel across the mirror in the direction in which the card is moving (in the case of the right eye, from left to right) and if so the card is too close to the mirror. In this way the position of the point P can be easily determined approximately. If the mirror is truly spherical, or even very nearly so, the shadow will be very uniform. The card should then be placed to cut off nearly the whole light, and a sketch made showing the manner in which that which remains is distributed. From this shading the type of surface is deduced. The actual measurement of the aberration is one of some difficulty, and very discordant results will be obtained at first. It is only by taking the mean of a large number of observations of the position of P_1 and P_2 that any reliable numerical determination of aberration can be made.

The speculum should at first appear completely illuminated like the full moon. Then as the screen is passed across from left to right by means of the screw F, if the wall of shadow also passes from left to right, the screen is too near the speculum. As the light fades away irregularities will appear. If there is a patch of especial brightness the glass is too high there; a dark shadow shows a depression. If the curve is spherical, the light will fade equally and at once. If the curve is an oblate spheroid, the edge and the centre will be bright, and the intermediate part dark; if hyperbolic these will be reversed.

For measurements, a series of screens exposing portions of zones must be made of black card. Each card can be made to do for two zones as in the figure, the pair of openings in the zone under test lying right and left of the centre; the card must, of course, be turned through a right angle to measure the other zone. Fig. 201.

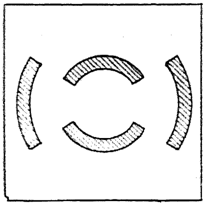


FIG. 201.

For a parabolic mirror

the aberration should be

$$= \frac{(\text{mean radius of zone})^2}{\text{radius of curvature of speculum}}.$$

170. **Retinoscopy** or **scioscopy**, which is one of the best methods of determining the refractive errors of a patient's eye, depends upon the same principle as Foucault's shadow test. To illustrate it, set up a convex lens of any convenient focal length—say 3″—at a little more than its focal length from a white screen. On the

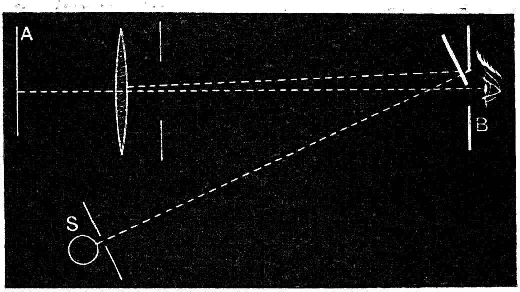


FIG. 202.—Retinoscopy.

other side of the lens put a pin hole about a yard away from the lens, through which the observer is to look at the lens. In front of the pin hole and a little on one side put a plane mirror to reflect the light from a flame S on to the lens. The screen should be shaded from all light but that coming through the lens. Then on the screen will be formed a more or less distinct image of the lamp. By rotating the mirror the image on the screen will move to and fro slightly. If the pin hole B is conjugate to the screen A, the eye will see only a uniform illumination of the lens, the brightness of which depends upon that of the spot of

the screen which is conjugate to the pin hole. Thus, as the mirror is rotated, the lens will suddenly come bright or dark, as the image of the flame falls upon or leaves the spot on A, which is the image of the pin hole.

If now the screen A is moved further from the lens, so that it is no longer conjugate to the eye at O, the lens will light up from one side. The reason for this can easily be seen from Fig. 203. For let DEF be the new position of the screen, and suppose that DE is bright and EF dark; then the lens AB as seen from O will appear dark from A to C and bright from C to B, since the light

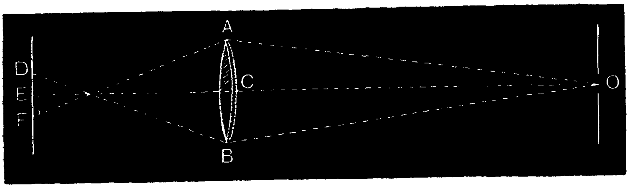


FIG. 203.

from the bright part DE will reach O through the rays which pass through the portion CB of the lens. The direction in which the shadow moves across DE does not depend on the distance of DE from the lens AB, since it is caused by the motion of the image of S (Fig. 202) produced by rotation of the mirror. If, for instance, the mirror is rotated clockwise, the image moves down and the light on A moves up.

If the screen is moved forward, nearer the lens, the latter will light up from the opposite side for any given motion of the mirror. Thus, by observing the way the lens lights up, it is possible to tell when the pin hole is conjugate to the screen.

It is obvious that if the screen were at a fixed distance from the lens, it could be made conjugate to the pin hole by altering the power of the lens, *i.e.* by adding to the lens a suitable convex or concave lens. When this added lens causes the shadow to come over the whole of the lens simultaneously, the screen and pin hole will be conjugate.

Thus if the screen and lens be assumed to represent the retina and lens system of the eye, the lens which placed in front of the

eye causes the shadow to appear all over the pupil at once (and not to move across it from one side or the other as the mirror is rotated), will be the lens which makes the retina conjugate to the observer's pupil. The observer is usually about a metre distant; thus the correction for the patient's sight for infinity is about one diopter less convex (or more concave) than the lens above found.

The Longitudinal Aberration of the Lens of a Lantern Condenser. The Aplanatic Foci.

171. *Apparatus.*—A convergent beam will be required free from spherical aberration, or something that is equivalent to such a beam. For this purpose use a telescope TW, containing a cross-wire at W, but with the eye-piece removed; a candle, S, illuminates the cross-wire. The focal length of the objective of

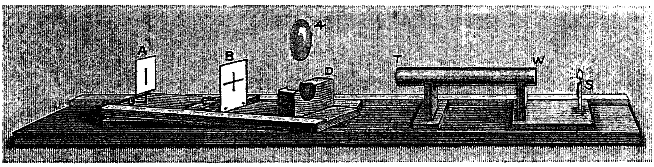


FIG. 204.—Aberration of a Lantern Condenser.

this telescope should not be more than four or five inches. The telescopes supplied with sextants have suitable lenses, but must be able to open to about twelve or thirteen inches. A sextant objective (it can be procured for a few pence) may be temporarily mounted in an ordinary draw tube telescope. The telescope and candle are mounted on a rough carriage, and slide on an optical bench. A short optical bench rotates on a pin at one end O fixed in a heavy block lying on the large optical bench. The lens to be tested is supported on a block D. On still another block C is a small screen B, with a horizontal scale at the level of the telescope, which can be set at any distance from the lens. There is another small screen A, which can be temporarily placed upon the axis O, about which the telescope arm rotates, so that the lens and the screen B may be moved to and fro without disturbing the telescope or the screen A.

Experiment.—Having removed the two blocks, C and D, the cross-wire of the telescope illuminated by the candle is focussed

upon A, so that there emerges from the telescope a beam of rays converging to A.

The lens is now to be replaced, taking care that its centre is at the same height above the table as the axis of the telescope, and that the light from the telescope goes through the centre of the lens; the surface of the lens should be normal to the axis of the telescope, the axes of the lens and telescope being coincident. As the light emerging from the telescope is converged by this lens, it will form an image of the cross-wire somewhere in front of A. Place the screen B to receive this image, and carefully focus it. To assist in focussing the screen, the short bench may be moved a small amount to and fro each way about the axis O, and if the image appears stationary upon B it is correctly focussed. If not, B must be moved nearer or farther from the lens until this is the case. The position of the pointer attached to the base C, on the scale of the bridge, is to be noted.

Now move the bench carrying the lens through a small angle, so that the light passes through the lens at a spot perhaps a quarter of the radius of the lens from its centre.

Move the arm to and fro a very small amount about this position, and observe whether the image of the cross-wire is still stationary; if not, the screen B must be adjusted until it is. The amount it is moved is the *longitudinal aberration* of the lens at this distance from the centre. The aberration is to be considered positive if the movement of B was towards the lens.

Again adjust the arm until the light passes through the lens at a point half way between the centre and the circumference. Find the position of B as before; measure the aberration at this point. Repeat the determinations at a point still nearer the circumference, at the edge itself, and at similar positions on the other side of the centre. The actual distances of A and B from the surface of the lens must be measured when the light is passing through the lens centrally.

Repeat all these readings with the lens at 1″, 2″, 3″, 4″... 10″ from A. It is well to have the telescope as close to the lens as the focal length of its objective will allow.

Enter the result in a table thus:

Distance from A to lens.	Distance from B to lens.	Scale readings at centre, ¼, ½, ¾, edge.	Longitudinal aberrations at ¼, ½, ¾, edge.
1″			
2″			
⋮			
10″			

Plot the aberrations in the last column (for rays which traverse the lens at a point ¾ the distance from the centre) as ordinates, and the distance from the lens to A as abscissae on squared paper. With the ordinary plano-convex condenser lens, the

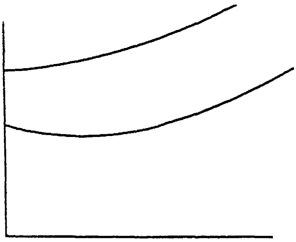


Fig. 205.

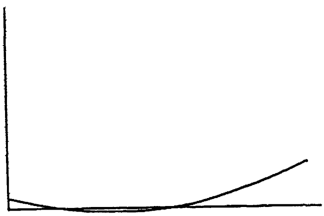


Fig. 206.

curves will be similar to those in the figure. With a meniscus lens, if the curves are sufficiently deep and the lens is fairly thick in the centre, one of the curves may cut the axis, as in Fig. 206. If so at the two points of intersection the axial spherical aberration vanishes. These points are called the *aplanatic foci* of the lens.

The Effect of an Increased Thickness on the Aberration of a Plano-Convex Lens.

172. *Apparatus.*—In addition to that described for the last experiment, we shall require a glass cube of 1 or 2″ side, or a trough with parallel sides about 2″ from front to back, which may be filled with water.

Experiment.—Place the cube against the flat side of the plano-convex lens, and let this side of the lens be towards S (Fig. 204).

Then, with the cube in the centre of the lens, find the position of B for axial rays. By moving the cube so that the rays may still pass through it on their way to B, the aberrations can be determined for the combined lens and cube for nearly all the distances in the preceding table. The aberration will be found to be very much less, and the curve will probably cut the axis. Thus a plano-convex lens of this increased thickness would have aplanatic foci. Hence, the extensive use of nearly hemispherical lenses in micro-objectives and condensers. It is obvious that a water bath will diminish the spherical aberration of an ordinary lantern condenser. If such a water bath could be placed between the condenser and the arc, with one surface close to the light, filling up the whole space between it and the condenser, its performance could be much improved.

It will also be found from the results of the experiment that the aberration (except when the lens has aplanatic foci) is least when the light passes through its peripheral zones at minimum deviation. It is obvious that this should be the case when it is remembered that the aberration is caused by the focal length of the lens for marginal rays being shorter than that for central rays. Thus, anything that reduces the deviation for marginal rays will diminish its spherical aberrations. A good condenser to produce a beam of parallel light with a lantern, would be one formed of two lenses, of which the first had aplanatic foci at A and B in the figure. The light being placed between A and B, this lens by itself would have a negative aberration. Such a lens will be a concave-convex meniscus, with its concave surface towards the light, or a plano-convex lens of very great thickness (but the latter would be certain to crack on account of the heat). The second lens can then be a plano-convex lens with positive aberration, arithmetically equal to the negative aberration of the meniscus. To produce a convergent beam, this may be combined with another similar pair of lenses, or with a plano-convex lens and a water bath.

173. Let ARB (Fig. 207) be a sphere of glass of refractive index μ, of which O is the centre. Let P be a bright point, from which PRS is a ray refracted at R, and after refraction appearing to come from Q. Then $\mu = \sin ORQ/\sin ORP$. If P is so chosen that $OP/OR = OR/OQ$,

the triangles ORP and OQR will be similar, and therefore the angle ORP will equal the angle OQR, and the angle ORQ will equal the angle OPR.

Thus $$\mu = \sin ORQ/\sin ORP$$
$$= \sin OPR/\sin ORP = OR/OP,$$
also $$= \sin ORQ/\sin OQR = OQ/OR,$$

and the positions of P and Q will be fixed and independent of the position of R. In other words, every ray that issues from P will appear after refraction to come from Q, or P and Q are conjugate aplanatic foci. If a sphere CDE be described, with P as centre, every

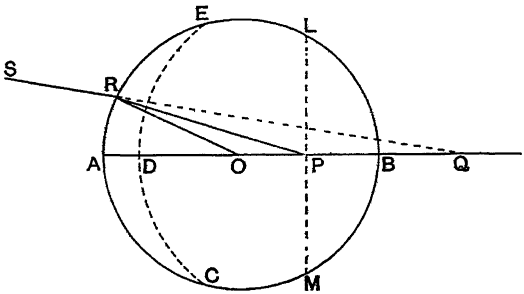


FIG. 207.—Aplanatic Points.

ray from P will strike the surface normally; the glass on the right of the surface could therefore be removed, and P (now in air) and Q will still be aplanatic. Or the glass to the right of a plane PLM drawn through P normal to the axis may be removed, and P and Q will still be aplanatic. Thus a lens of this shape will produce an image of the point P entirely free from spherical aberration. It will of course not be achromatic. By slightly altering the position of P the image may be given negative spherical aberration, and then by adding a lens which has ordinary positive spherical aberration and which is over-corrected chromatically, a compound lens free from spherical aberration which is also achromatic can be constructed. This is the principle upon which micro-objectives of large aperture have been constructed.

Aberrations of Eye-pieces.

174. **Two-lens Eye-piece.**—Except in the use of the erecting eye-piece of a terrestrial telescope, which will be dealt with later, an eye-piece usually consists of two lenses at a distance apart which

depends upon and is comparable with their focal lengths. They are called the "field lens" and the "eye lens," the former being the one nearer the objective of the microscope or telescope, as the case may be, and the latter the one nearer the eye. The field lens is so called because (in the absence of a stop) its size will practically determine the extent of the image that can be viewed by the eye at the same time, and in any case its size will mainly depend upon the area of the stop that is to be used to limit the field.

It is sometimes supposed that an eye-piece consisting of two lenses will embrace a larger field than a single lens of the same equivalent power could. This, however, is not so if the eye is placed in the proper position in each case. Suppose, for instance, that a single lens of one-inch focus is to be used as the eye-piece of a microscope. It will have to be placed at a little less than an inch beyond the image formed by the objective, so as to form an image at such a distance that it can be seen by the eye, viz. at something between 10 inches and infinity for a normal adult eye. The lens will form an image of the stop of the objective at a little more than an inch beyond the lens (at $1\frac{1}{6}$ ins. from the lens if the objective is 7 ins. away); this image of the stop, or "exit-pupil," will be a circle about a seventh of the diameter of the objective stop, and all rays issuing from the latter must pass through it. Thus it will probably be smaller than the pupil of the eye, and if the eye is placed there, the field (so far as the eye-piece is concerned) may extend out to the edge of the eye-piece lens. This image of the stop of the objective is called the exit-pupil, or Ramsden circle, or eye-ring. Of course, if the eye is placed either nearer the lens or further from it than this circle, the field will become limited by the pupil of the eye.

The two-lens eye-piece has not been adopted therefore to increase the field of view, but because it has been found possible with it to correct for spherical and chromatic aberrations. It is proved in books on geometrical optics that two convex lenses of the same kind of glass, placed at a distance apart equal numerically to half the sum of their focal lengths, will be achromatic for parallel light. In practice, the light pencils diverge from the objective, and the lenses have to be put

further apart than the mean of their focal lengths. The actual distance is always found experimentally. It is more nearly equal to the mean of the focal length of the eye-lens and of the distance from the field-lens at which that lens forms an image of the objective. Or the distance apart is $\frac{1}{2}(f_1+v)$, where v is given by $1/v+f/d=1/f_2$; f_1, f_2, and d being the focal lengths of the eye-lens, the field-lens, and the distance of the field-lens from the objective respectively.

But as stated above, the distance is always found experimentally, being varied to correct the residual chromatic error of the objective and to give as flat a field as possible.

By varying the shape of the lenses (though plano-convex lenses are nearly always employed), and by using different glasses for the lenses, further variation in the nature of the images produced can be obtained.

In practice, series of observations have to be made, and the results of each series noted. Thus, the effect of each kind of alteration is known, and in any given case the kind of change required to improve the image will also be approximately known.

175. *Apparatus.*—A sheet of perforated zinc about 1 ft. square supported in front of a sheet of ground glass and well illuminated from behind, say with five 8 c.p. lamps arranged in the form of a cross with horizontal and vertical arms. About 10 feet away set up a telescope-objective of about a foot focal length to form an image of the perforated zinc. Or use a good $\frac{1}{4}$ or $\frac{1}{2}$ plate camera objective. It is well if possible to mount this lens on the same stand as the eye-piece lenses to be tested, in order to be sure that their axes shall be collinear. Probably the most convenient mounting is the one of angle-brass described in § 39. In this case each of the eye-piece lenses will be mounted in a short tube, as also will be the objective and the stop. They can then be very easily and rapidly adjusted.

Suppose the eye-piece consists of a plano-convex eye-lens of 1″ focal length and a similar field-lens of 2″ focal length. Begin by placing them about $\frac{1}{2}$″ apart and with their plane sides towards the eye; focus and note (in tabular form) (*a*) the chromatic error; (*b*) the amount of spherical aberration; (*c*) the nature of the astigmatism towards the margin of the field; and (*d*) the curvature. By using coloured screens measurements may

be made of the amount the objective has to be moved to focus the red and the blue respectively near the centre of the field, and to focus in yellow light the centre and the margin successively, and (*a*) and (*d*) can be stated numerically. Repeat with the lenses, 1″, $1\frac{1}{2}$″, 2″, $2\frac{1}{2}$″ apart. Try the effect of reversing each lens. Repeat with a 1″ and a 3″ lens. Repeat with a 1″ lens of flint, and the 2″ of crown, and *vice versa*. Of course in all cases the eye must be placed at such a distance from the lens that the whole field of the image is seen.

176. **The Erecting Eye-piece.**—This is the eye-piece used in a terrestrial telescope in order to obtain an erect final image. It may be looked upon as a low-power microscope consisting of two pairs of lenses, of which the first pair is the micro-objective forming a slightly magnified image of the image which has been formed by the telescope objective, and the second pair is the eye-piece of that microscope. The distances are again always found by trial.

In a telescope which I measured, calling the objective A, the erecting lenses B and C, and the eye-piece lenses D and E, I

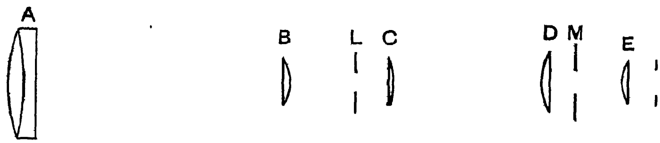


FIG. 208.—Erecting Eye-piece.

found the focal lengths as follows, A = 48 cms., B = 3·8 cms., C = 4·75 cms., D = 4·6 cms., E = 3·0 cms. The distances apart were approximately AB = 48 cms., BC = 5·8 cms., CD = 10·2 cms., DE = 5 cms. There was a stop L between BC such that BL = 4·1 cms., and a stop M between DE such that ME = 3 cms.

It will be found that the image of the lens A formed by the lens B coincides with the stop L. The image of L formed by successive refractions through the lenses C and D is 7·18 cms. to the far side of D. The distance apart of D, E, very nearly equals the mean of this number 7·18, and of 3 the focal length of E.

The final image of L is 1·26 cms. beyond E, which is the exit-pupil or Ramsden circle.

The stop M is at the principal focus of E. Following this stop back-

wards, its image formed by successive refractions through D and C is 7·26 cms. from C towards the objective. The distance apart of B, C approximately fulfils the relation $BC = \frac{1}{2}(BL + \text{this } 7{\cdot}26) = 5{\cdot}6$ (instead of 5·8). I have found similar results in other cases, but I do not believe the lenses are adjusted by any such rule but by trial and error; nor do I know if the rule has any foundation in theory.

It will be at once seen on trying the experiment that the relative positions of the lenses, and especially the position of the stop L (*i.e.* the position of the eye), are very important, and that with a given set of four lenses a good image can be obtained only in one way.

Experiment.—Set up four lenses of which the focal lengths are about in the ratio 4 : 6 : 7 : 5, in that order; place a stop about $\frac{1}{8}''$ diameter between the last two; place the eye in front of the first lens, and conjugate to that stop; and examine some fine print on a white card placed beyond the last lens. Try and adjust them to obtain an achromatic image free from distortion.

Instead of the white card, this combination may be used with the objective and perforated zinc of the last experiment. In any case it will be found desirable to mount the lenses in short tubes, and adjust them in a **V** as there described. The enormous effect of the position of the stop is very striking. It is of course due to the fact that there is a large amount of spherical aberration present in the case of each of the lenses, and the stop determines what zone of the lens shall be used in each case; thus, by suitably arranging the stop and the several lenses, the zone of each lens which is made use of to form a given part of the image is determined, and the aberrations can be made to neutralise one another to a very large extent.

CHAPTER X

THE DIFFRACTION GRATING

Determination of the Wave-Length with a Wire-Gauze Screen.

177. On looking at a distant flame through a piece of wire gauze or any similar structure, *e.g.* a stretched handkerchief, two series of flames will be observed at right angles to one another, having the actual flame at the centre. The bordering flames are coloured, and get fainter as they become more distant. The finer the gauze the more widely will these flames be separated. If the angular separation and the number of wires to the centimetre on the gauze be determined, the wave-length of the light can be found.

FIG. 209.

Let O be the light and AB the gauze, and suppose O sufficiently distant from the light for the light to be considered to be approximately parallel. Then, after passing the gauze, the light, in addition to proceeding in a straight line as OMP, will also appear to come from O′. For draw MR_1 perpendicular to $O'M_1$, then the light will appear to come from O′ if the vibration at M_1, the next aperture in the gauge, is in the same phase as that of R_1. This will be the case

if M_1R_1 is a wave-length. It is obvious if O is sufficiently distant for the light at the gauze to be considered parallel, that if a perpendicular M_2R_2 be drawn from the next aperture $O'M_1$ the distance M_2R_2 will be two wave-lengths, and so on. Thus the light through each aperture would be in the same phase as if it had come from O′. There will be a similar point O_2 on the other side of O. Beyond these on each side will be seen still further images. As O′ is bright if M_1R_1 is a wave-length, if the angle PM_1P' be θ, we have

$$\sin \theta = \frac{M_1R_1}{MM_1} = \frac{\lambda}{c},$$

where c is the distance in centimetres from one wire to the next, or is the reciprocal of the number of wires to the centimetre.

But $$\sin \theta \text{ also } = \frac{OO'}{OM'};$$

therefore $$\frac{\lambda}{c} = \frac{OO'}{O'M}.$$

178. *Apparatus.*—A piece of fine wire gauze which should have from 150 to 200 wires to the inch.[1] It can be soldered on a sheet of brass or zinc about four inches square, covering a hole two inches in diameter in the middle of the sheet. It is important that the wires should be straight; thus a new piece of gauze should be secured, and care taken that it is not damaged or strained in any way.

To determine the angular separation: paste some tinfoil on a piece of glass about 3″ square. Before the paste is quite dry, with a sharp knife and straight edge cut a letter A on this foil. The width of the A at the cross-bar should be about 1 cm. A telescope and sodium flame and a reading microscope will also be required.

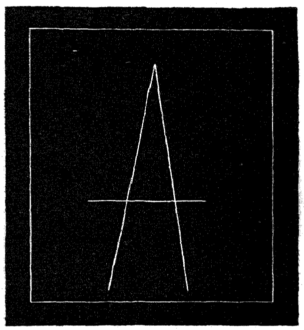


FIG. 210.

Experiment.—Set up the A with the cross-bar horizontal, close to the sodium flame. At a distance of eight or ten feet place the telescope, and focus it upon the A. Now introduce the wire

[1] Such gauze can be obtained from Messrs. Bryan Corcoran of Lime Street.

gauze held in a Bunsen clip between the telescope and the A. On looking through the telescope the A will be seen bordered with diffraction lines. The gauze must be adjusted so that one set of its wires is vertical. The cross-bar of the A will now be most distinct, being bordered by a bright line top and bottom due to the horizontal wires of the gauze only. If the one set of wires be not quite vertical, the cross-bar of the A will have diffraction fringes, formed by both sets of the wires.

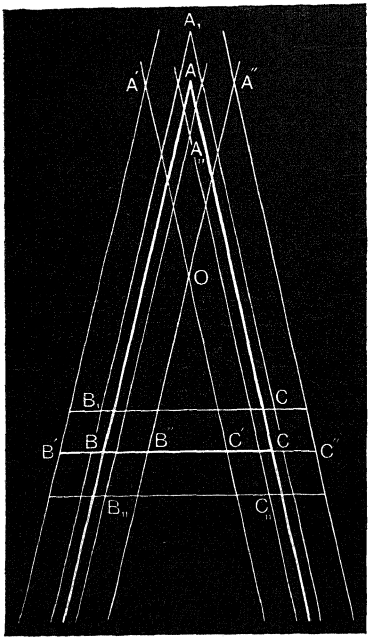


FIG. 211.—Appearance of the A through Wire Gauze.

The sides of the A will have diffraction fringes formed by both the horizontal and vertical wires. If the side lines are nearly vertical, the diffraction lines formed by the vertical wires will be much further separated, and no confusion need arise. If the gauze be now approached close to the A, these fringes will close in upon their respective lines; if it be withdrawn and brought close to the telescope they will open out. At a certain distance from the A, the fringes of the sides of the A formed by the vertical wires will cross one another on the cross-bar of the A. Fix the gauze at this distance. In the figure 211 the wire gauze is not far enough from the A, as the crossing point O is above the cross-bar BC.

Then OO′ of the formula is the distance from the centre of the cross-bar to either end, *i.e.* half the length of the cross-wire; and the distance from A to the gauze is the distance O′M. Measure this distance *d*. With a reading microscope find the distance apart of the wires. To do this, set the cross-

wire on a vertical wire of the gauze, and moving it in one direction only read 4 wires, then miss 6, and read 4 more. Enter them thus:

First Four Readings.	Second Four Readings.	Difference.
1.		
2.		
3.		
4.		

The last column should be constant and be the width of ten wires; c is therefore its mean value divided by 10. With the reading microscope also measure the width of the cross-bar of the A $= w$ (say) $= 2 \, . \, OO'$,

then $$\frac{\lambda}{c} = \frac{OO'}{O'M},$$

or $$\lambda = c \, . \, \frac{w}{2d}.$$

Considering the simple nature of the apparatus, this experiment gives remarkably good results.

The Diffraction Grating.

When truly parallel light, a perfectly ruled series of lines, and an accurate method of determining the deviation is substituted for the above rough apparatus, we obtain an exceedingly accurate method for determining the wave-length of light.

179. *Theory.*—Let L be a lens and F its principal focus, then if PQ is a plane perpendicular to the axis of the lens the time taken by every point on a plane wave-front at PQ to reach F will be the same. If a diffraction grating DD′ be placed parallel to PQ and a parallel beam of light fall perpendicularly upon DD′, the wave-front of the light is parallel to DD′. Let AB be such a wave-front, then the light at every point of AB is at any moment at the same phase of its vibration. It will reach every aperture in DD′ at the same moment, and if PQ be parallel to DD′, it will reach PQ everywhere in the same phase, and therefore finally arrive at F together, and F will be a bright point.

If now the lens be turned through an angle θ as in figure 213 and the principal focus be now at F′, the time from every point in the plane

P′Q to F′ will be constant. The times from the apertures in the grating to F′ will no longer be the same. But if θ be such an angle

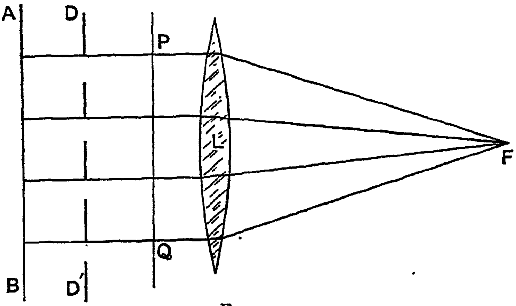


FIG. 212.

that the distance from M_1 to P′Q′ is one wave-length greater than the distance from M to P′Q′, the light, which was in the same phase at M as at M_1, will be in the same phase when it reaches P′Q′, therefore also when it finally arrives at F′. It is obvious from similar triangles that if M_1N_1 is one wave-length, M_2N_2, M_3N_3, M_4N_4, etc., will be 2, 3, and 4 wave-lengths respectively, and the light from all these apertures will arrive at F′ in the same phase; thus F′ will be a bright point.

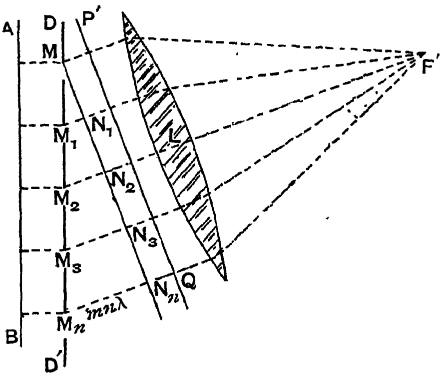


FIG. 213.

From the figure, $\sin\theta = \dfrac{M_1N_1}{MN_1} = \dfrac{\lambda}{c}$, where c is the width of the ruling.

In the same way F′ will be bright when M_1N_1 is 2, 3, 4, ... n wave-lengths; thus the position of the n^{th} bright point F_n is given by

$$\sin\theta_n = \frac{n\lambda}{c}$$

or

$$\lambda = c \, . \, \frac{\sin\theta_n}{n}.$$

Definition.—When the difference of path from two successive apertures is $n\lambda$, the spectrum produced is said to be of the n^{th} order.

180. **Minimum Deviation.**—If the normal to the grating makes an angle ϕ with the axis of the collimator, and θ_n with that of the telescope (see Fig. 215), it is obvious that λ is given by

$$\lambda=\frac{c(\sin\theta_n+\sin\phi)}{n}.$$

If the deviation is a minimum, a slight rotation of the grating will not alter it, *i.e.* $\theta_n+\phi$ is a constant for an infinitely small rotation.

∴ we have, differentiating the above equation and using $\delta\theta_n+\delta\phi=0$,

$$\cos\theta_n\,\delta\theta_n+\cos\phi\,\delta\phi=0,$$

$$\cos\theta_n=\cos\phi \text{ or } \theta_n=\phi.$$

Or at minimum deviation, the grating is equally inclined to the collimator and the telescope, and if δ be the deviation

$$\theta_n=\phi=\tfrac{1}{2}\delta.$$

181. *Apparatus.*—The diffraction grating may be either a reflecting grating ruled on speculum metal, or a celluloid cast of such a grating. These latter made by Thorpe of Manchester are hardly inferior to the original, and are very much less costly; they are transparent. The film must be treated with great care and must on no account be touched. If necessary it may be dusted with a camel hair pencil. It must be protected from fumes of alcohol or other solvent of celluloid. The spectrometer and a sodium flame will also be required.

The diffraction grating must be mounted on an independent table to allow of the proper adjustment of its surface. It should be held up by a clamp as shown in the figure. The table must be on three levelling screws. The position of these screws is important. Let A, B, C be the three screws; the line joining two of them, AB for instance, must be perpendicular to the surface of the grating. The clamp also should be perpendicular to this line as shown in the figure.

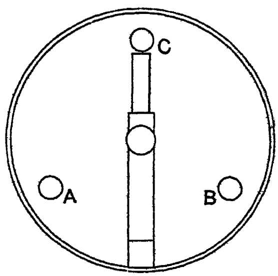


FIG. 214.

If the grating is mounted in this way, it can be seen that the screw C will rotate the grating in its own plane only. It is used for setting the ruling of the grating vertical, and as this is the last adjustment, it is obvious that we must be able to effect it without interfering with the previous adjustment of the plane of the grating. Only two of the screws need have milled heads,—B,

for instance, may be merely a point. This table should fit on to the table of the spectrometer with the "hole, slot and plane." In this case B might be placed in the hole and A in the slot. The point of the screw A should be concentric with the screw itself.

Adjustment.—The spectrometer must be very carefully adjusted for parallel light (preferably by the method of Shuster already described in dealing with the refractive index of a prism, page 122).

The adjustments of the grating are four.

(i.) Its plane must be vertical.

(ii.) It must either be perpendicular to the axis of the collimator or be set for minimum deviation.

(iii.) Its rulings must be vertical.

(iv.) Also the slit of the collimator must be vertical. (By vertical is here meant, perpendicular to the plane of the divided circle of the instrument.)

182. **First Method.**—Grating normal to the axis of the collimator.

i. and ii. The first and second adjustments are effected together. Point the telescope at the collimator, lift the grating away, and focus the slit on the cross-wire, read the position of the telescope circle. Limit the vertical length of the slit, and notice the height of the image of the slit in the field of the eye-piece,—for instance, notice where the slit appears to be cut by the cross-wire. Now turn the telescope through an angle of 90° by the divided circle, clamp it, and replace the grating on the table of the instrument; turn the grating until the reflected image of the slit is formed on the cross-wire. If the plane of the grating is not vertical or very nearly so, this image will be tilted up or down by the reflection, and may not even be included within the field of the eye-piece of the telescope. The reflection must be formed by the surface upon which the grating is ruled. [The back surface of the glass will also form a second equally bright image which will not coincide with the first unless the front and back surface of the glass are parallel. These two images may be distinguished by cutting a vertical slit about $\frac{1}{8}''$ wide, in a black card and placing this against the front surface of the grating. The card will cut off the light reflected from the back surface entirely, whilst it will

still allow enough light from the front surface to form a very faint image.] This image must be brought exactly upon the cross-wire. The screw A must be adjusted until the image is exactly the same height in the field as it was when the telescope was directly pointed at the collimator. When this is the case, the plane of the grating will be vertical and will make an angle of 45° with the collimator. Now turn the grating through an angle of 45°, using the divided circle attached to the table of the instrument, and it will be perpendicular to the collimator.

iii. Turn the telescope once more into line with the collimator, then by moving it either to the right or left, find the first diffracted spectrum on that side. This, with yellow light, will be a pair of lines very close together. With one of Thorpe's gratings of about 14,000 lines to the inch, they appear with the usual magnification about as far apart as two lines ruled $\frac{1}{16}''$ apart would be when held at a distance of about a foot. The screw C must be adjusted until the spectrum is at the same height in the field of the eye-piece as the slit appeared. The rulings will then be vertical.

Readings.—If the deviation produced by the grating with a grating of only 3000 lines to the inch, such as those ruled by Nobert, is required, there is no difficulty in using the first four or five diffraction lines on each side of the centre, the deviation being comparatively small. Take three or four independent readings of the central image, and use their mean value. Enter the readings in columns as follows:

Order of Spectrum $=n$.	Reading. Left. Right.	Deviation. Left. Right.	Mean Deviation.	$\sin\delta$.	$\frac{\sin\delta}{n}$.
0.					
1.					
2.					
3.					
4.					

The last column should be constant. Take its mean value,

then
$$\lambda = c \cdot \frac{\sin\delta}{n},$$
where c is the width of the ruling.

The width of the ruling of these gratings which have 3001 lines to the Paris inch, which is 2·707 cms. is ·0009023 cms.

With the Thorpe grating the deviation is so much greater that it is only possible to obtain the first two spectra on each side. The first one can usually be seen fairly easily. The second is sometimes so faint that it can be found only with difficulty. If this is the case, its position should be roughly calculated, using the deviation obtained for the first one, and the telescope placed in the right position by means of the divided circle. The spectrum should now be in the field, and the cross-wire can be set upon the lines. As the number of lines to the cm. with a Thorpe grating is not necessarily the same as that of the original, owing to a possible contraction or expansion of the film, it is best to assume the value of the wave-length of each of the two sodium lines, and to determine the number of lines to the centimetre, or the width of the ruling. The accuracy of the experiment will be shown from the agreement or otherwise of the results. Take at least two readings of each line. Enter the results thus:

	Central Line.	First Order.				Second Order.			
		D_1		D_2		D_1		D_2	
		Left.	Right.	L.	R.	L.	R.	L.	R.
1. Readings									
2. Mean Deviations									
3. Nat. Sines of Deviations									
4. $\frac{\sin\theta_n}{n}$									

In the first line is to be entered the actual readings. The deviation is found by subtracting the mean reading of the centre from each of the others. The deviation of the corresponding lines to right and left of the centre should be the same, their mean is to be entered on the next line. The natural sine of this is to be found from the table-book and entered on the third line. In the last line this sine is divided by the order of the spectrum, so that the numbers under the columns D_1 should now be the same as should those under the columns D_2. Divide the mean

of the $\frac{\sin\theta_n}{n}$ for D_1 by the wave-length of that line, that is ·000058962, and the mean of the numbers under D_2 by the wave-length of that line, ·000058902. The results should be the same, and will be the number of lines to the centimetre.

183. **Second Method. Minimum Deviation Method.**—This method differs only in that the grating, instead of being adjusted with its plane normal to the axis of the collimator, is rotated until the deviation is a minimum. It has to be set again for minimum deviation for each order of the spectrum, and for the readings on the opposite side of the central image, also for each colour; but this setting is easily and rapidly accomplished, and the readings can be taken with great accuracy. As already shown (p. 249) λ is given by

$$\lambda = \frac{c2}{n} \sin\frac{\delta_n}{2},$$

where δ_n is the deviation and n the order of the spectrum. All the other adjustments and the method of taking and working out the readings are similar to those of the first method, except that in making the preliminary adjustments it is not necessary to turn the telescope exactly through 90° or the grating through 45°, and therefore it is not necessary to take any scale readings in making these adjustments.

The wave-lengths of the hydrogen lines with the diffraction grating.

184. *Apparatus.*—In addition to the spectrometer and grating, a Rhumkorff induction coil and a hydrogen tube will be required.

Experiment.—Having adjusted the grating with the sodium flame as already described, and found the number of lines to the centimetre, the sodium flame must be replaced by the hydrogen tube. This must be set with the narrow part of the tube opposite and close to the slit. Point the telescope directly towards the collimator in order to make sure that this is the case. Then, on turning the telescope round, three bright lines will be visible in the first spectrum on either side,—a red, a blue and a violet one. The readings can be entered as above directed. It will be sufficient to take one reading of each line. Use the width of the grating already determined.

The Reflecting Grating.

185. *Apparatus.*—The original gratings ruled by Professor Rowland were mostly ruled upon speculum metal with a diamond point. Such a grating can be mounted upon a stand similar to the one described on page 249 for the transmission grating, with its plane perpendicular to the line joining AB for the same reason as before.

Let DD′ be the grating and AB the wave-front of the incident light, P′Q′ the wave-front of the diffracted light. Draw AL'_2 on the other side of DD′, making the same angle with DD′ as AL_2 does. Then it is obvious that the distance from any point L on AB to P′Q′ will be the same as from a corresponding point L′. If LM and L_1M_1 be reflected from two successive lines of the grating, there

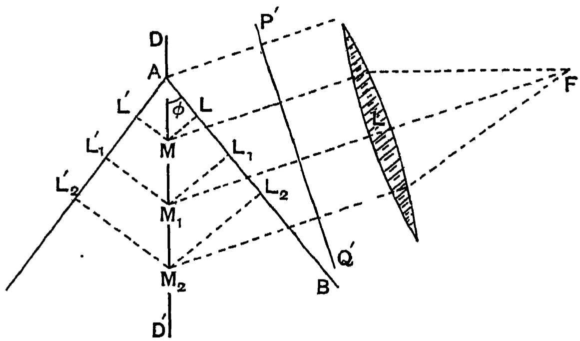


FIG. 215.

will be brightness at F′ if the distance from L'_1 is greater than the distance from L′ by a whole wave-length. It is obvious from the figure that this distance is

$$c(\sin\theta \sim \sin\phi).$$

It will therefore be necessary to know the angles the grating makes with both the telescope and the collimator. It will be most convenient to make the latter 45°, which may be done in the way already described for the transparent grating. The results may also be entered in the same way.

The Concave Grating.

186. By ruling a grating on the surface on a concave speculum mirror, it is possible to obtain a diffraction spectrum without the aid of any lenses; thus the light will proceed from the slit to the spectrum without having to pass through any absorbing medium. This is of great importance in researches upon the infra-red or ultra-violet portions of the spectrum, to both of which glass is strongly absorbent.

It is shown in books on Physical Optics (Edser, p. 460), that if a circle be drawn on the radius of curvature of the mirror as diameter, and the slit is adjusted on this circle, the spectrum will also be in focus upon the same circle. To realise this in practice the grating is usually mounted at one end of a beam, with its axis parallel to the beam. The eye-piece (or photographic plate) is mounted at the opposite end of the same beam, exactly at the centre of curvature of the mirror. The radius of curvature of the mirror (and therefore the length of the beam) is generally twenty to twenty-five feet. The ends of this beam are mounted to slide on two other fixed beams at right angles to one another, so that it always forms the hypotenuse of a right-angled triangle. Above the junction of the two fixed beams is mounted the slit, which is therefore at the right angle of the triangle, and so always lies on the circumference of a circle of which the movable beam is a diameter. These gratings require a large room to themselves, and are to be found in only a few laboratories. For directions for mounting and using these gratings see *Phil. Mag.* (5), vol. 27, 369 (1889), also *Proc. Roy. Soc. Dublin* (1), vol. 8; 711 (1898).

Thorp has made gratings which have a radius of curvature of only two or three feet, which can be used in an ordinary dark room, and which are comparatively cheap. The gratings are celluloid casts of a flat grating mounted on the convex surface of a concavo-convex lens. This glass of course absorbs the light as an ordinary lens would, and they will not be much if any better than an ordinary grating for ultra-violet light.

187. *Apparatus.*—The Thorp grating of about two feet radius of curvature. Measure the radius of curvature R of the grating. Fix two brass plates, about 6 inches square and $\frac{3}{16}''$ thick, at this distance R apart, by half-inch brass tubes through the four corners. The grating is to be mounted on one of these,

ABCD, and the photographic plate or eye-piece on the other, EFGH (Fig. 217). Each plate is to be mounted on pivots P, Q; R, S, in the centre of its top and bottom edges, so as to be able to rotate about a vertical axis through the middle of its face in the frames described below. Fix two lengths of $1\frac{1}{4}''$

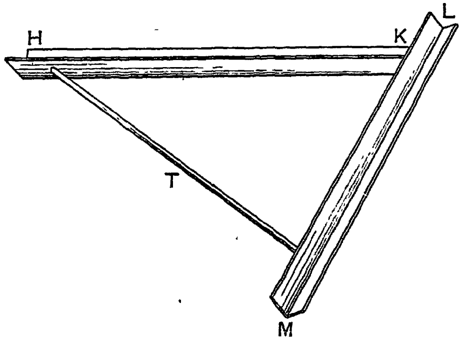


FIG. 216.—V's for Concave Grating.

angle brass, at right angles to one another. They should each be about 9″ longer than R. They can be brazed together at the corner and strengthened by a third side of brass tube. Let them have their hollow side uppermost forming a **V**. In these **V**'s are

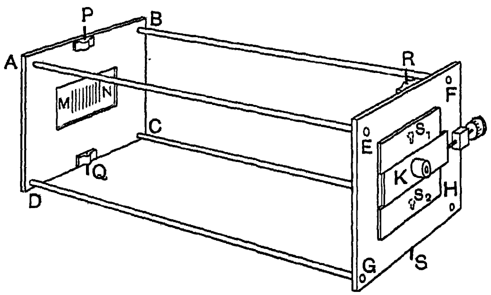


FIG. 217.—Mount for Concave Grating.

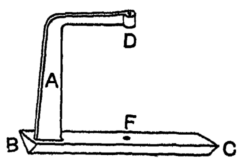


FIG. 218.—Arm.

to slide the frames in which the plates are pivoted. Each of these consists simply of a casting BC in the form of a solid bar about 9 inches long, of which the section is a right-angled triangle, with an arm A rising from one end to a height of a little more than six inches and then projecting some four inches and ending

over the centre of the bar (Fig. 218). The pivots of the plates carrying the grating and eye-pieces work in holes in the end of this arm, D, and in the bar underneath, F, the two holes being in the same vertical line.

The slit can either be mounted on another similar triangular bar, and be merely stood in the **V**, or can be permanently attached

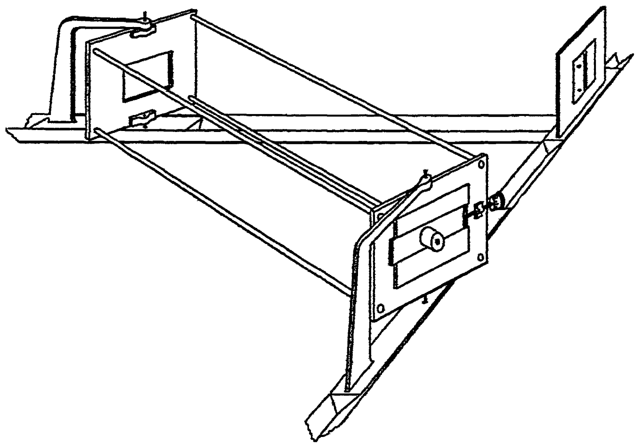


FIG. 219.—The Concave Grating.

to the **V** at the right angle. The slit should in either case be mounted on a larger sheet of brass, so as to screen the grating from all light except that coming through the slit; and there should be a support at the top for a rod or tube, so that a black cloth may be placed over it to exclude all extraneous light. A great deal of the space between the grating and the eye-piece can be permanently closed in with sheet zinc or cloth.

Note that the white image of the slit is formed in the air to the left of the apparatus as this is drawn in Fig. 219. The spectrum observed is between the slit and its image. The violet end is to the *left*; yet to pass from the red to the violet the frame carrying the eye-piece has to be moved to the right. The spectrum travels twice as fast as the eye-piece. This movement may be produced by a milled head, as in Fig. 217, or better, by a screw which moves the arm (Fig. 218) and the frame with it.

CHAPTER XI

INTERFERENCE AND DIFFRACTION

Interference.

188. The simplest method of producing interference was described by Lord Rayleigh in 1893. Silver two plates of glass; rule a fine line on one, using a knife and straight edge, and two parallel lines on the other as close together as possible. Hold the double slit so formed close to the eye, and the other a short distance away and parallel to it. The fringes will be formed on the retina. The two glasses may of course be mounted in a tube.

Instead of silvering a mirror, the red varnish on the back of an ordinary silvered mirror may be removed with alcohol or benzene, leaving the silver bare; the lines may then be made as above, or they may be cut in tin-foil pasted on glass; the cut is best made before the paste is quite dry.

189. It is also not very difficult to arrange a simple experiment to show the interference of light with a bi-prism, the only piece of apparatus required that cannot be easily constructed being the bi-prism, and this can be obtained for a small sum.

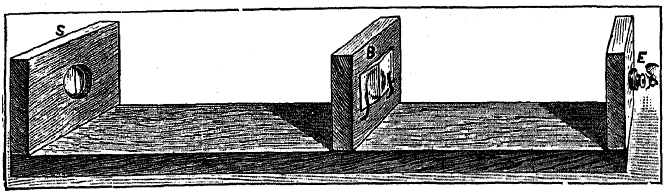


FIG. 220.—Simple Apparatus to observe Interference. A black cloth is to be tacked over it.

Apparatus.—The bi-prism: slit: an eye-piece: supports for same. Cut a board about 1 foot long and 4 inches broad.

Fasten to this board three upright blocks, each about 4 inches square with a hole about 1 inch in diameter through the centre of each; the hole should just fit the eye-piece. Fix one of the blocks at each end of the board and one in the centre, with a single screw each, so that they may be rotated if desired, and also so that they may easily be placed at other distances from one another. Glue a piece of velvet or smooth-faced cloth to the face of the central block. For the slit two pieces of printers' "rule" will do excellently, each about 2 inches long. Fasten one against the outside of the end block so that its edge is along the vertical diameter of the hole. Clip the other against the block under a spring of stout brass wire, it may then be adjusted to any desired distance from the fixed one, and the width of the slit varied. Clip the bi-prism in the same way against the central block under wire springs. A piece of black cloth should be cut to just cover in the space between the blocks, and tacked to the upper surface of each block, so that the hand may be inserted at either side to adjust the bi-prism.

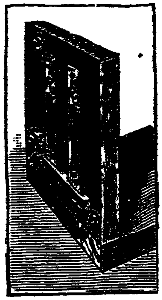


FIG. 221.—Slit.

Set the apparatus up opposite a good sodium flame, look through the centre of the hole in which the eye-piece is to be inserted and move the bi-prism sideways, until its ridge is in a line with the slit, when two images of the slit will be seen: one in each half of the bi-prism. On moving the head slightly to the right or left, one of the images will reach the ridge of the bi-prism and will be cut off. If the ridge of the bi-prism is exactly parallel to the slit (if, for instance, they are both exactly vertical), this image will be cut off along its whole length at the same instant; but, if they are not parallel, it will be cut off gradually, beginning, say, at the top. The bi-prism must be turned round until it is exactly parallel to the slit. Now insert the eye-piece, and the fringes should be seen. Very carefully rotate the bi-prism to and fro, and see if they can be improved; also see if by adjusting the width of the slit they can be improved. It will be found that if the slit is too wide they disappear, whilst, of course, if it is too narrow there will not be sufficient light to see them. The experiment should, if possible, be performed in a dark room. The

sodium flame must be very bright to give good effects. Instead of the sodium flame, it may be pointed at an incandescent gas light, when the fringes due to white light will be seen. As the incandescent light is much brighter than the sodium flame, these are much more easily seen than the ones due to the sodium. They are, however, less numerous, only four or five being visible with white light, whereas forty or fifty can be seen with the sodium flame.

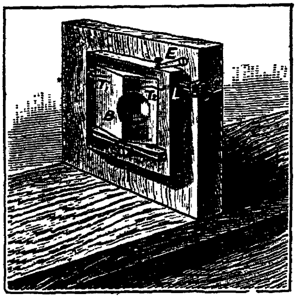


FIG. 222.—Mount for Bi-prism.

The most important thing in this experiment is to adjust the bi-prism parallel to the slit. Unless this is done nothing but diffraction fringes will be visible.

To make the final adjustment of the bi-prism easily, some more sensitive means of rotating it is wanted. If a piece of wood, somewhat smaller than the block, be fastened to its face by a single screw, S, as in Fig. 222, and the bi-prism attached to this, it can be rotated about this screw; and by attaching an ordinary levelling screw, L, to the side of the block, a very slow motion can be obtained, the wood being kept against the end of the levelling screw by an elastic band, E. The bi-prism can stand on a small ledge attached to this block, being held in position by wire springs, T_1, T_2. If the hole in the block, that is to carry the slit, be made to fit the inner tube of the collimator, that slit can of course be used, though owing to the small height of the slit it will be at a sacrifice of light.

Diffraction Experiments.

190. *Apparatus.*—A similar piece of apparatus to that already described for the interference experiments can be used for the diffraction experiments, but with the distance, from the block carrying the slit to the central block, increased to 18 inches. No adjusting screw will be needed on the central block. The bi-prism is to be replaced by pieces of thin wood 1½ inch square and ⅛ inch thick, each having a hole about ½ inch in diameter

through its centre. They should be blackened and fitted as follows:

A. A piece of *printers' rule* fastened vertically to the face with shellac varnish.

B. Break off $\frac{3}{4}$ inch of the points of four sewing needles of different sizes, and attach these, at intervals of $\frac{1}{16}$ inch, vertically to the face so that the four points are on a horizontal diameter.

C, D, E. Three pieces of thin sheet zinc about 1 inch square with circular holes about $\frac{1}{20}$ inch, $\frac{1}{50}$ inch, $\frac{1}{100}$ inch in diameter. The holes must be drilled perfectly clean, preferably by twist drills, all burrs being carefully removed.

F. A flat spectacle glass is to be fastened against the face, and on the wood side are to be attached 4 or 5 clean and new bicycle

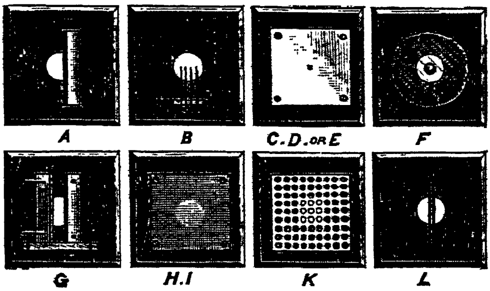


FIG. 223.—Objects for Diffraction.

balls of different diameters, from $\frac{1}{8}$ inch downwards. The selection of these should be as spherical as possible; they can be attached with a single drop of very thick shellac varnish, or by red wax; of course, the varnish must be entirely hidden by the balls.

G. A variable slit made as before with one fixed piece of brass rule, and another clipped by a wire spring.

H, I. Pieces of fine wire gauze of about 70 and 150 strands to the inch respectively.

K. A piece of perforated zinc.

L. Two pieces of thin knitting needle fixed parallel to one another.

For the end block, in addition to the slit (which will be used with numbers A, G, L only), a circular aperture will be required. This can be drilled in a piece of zinc about 1 inch square, and should be about $\frac{1}{70}$ inch in diameter. It can be inserted under

the wire spring, the slit being opened sufficiently to allow the light to pass.

Pointing the apparatus at a strong light, sketch the appearance seen in the eye-piece with each piece of apparatus, A—K. If it can be pointed at the sun, the fringes will be far more brilliant.

When using the slit with numbers A, G and L, care must be taken that the slit and the edges are accurately parallel to obtain the best effects.

The Bi-prism with the Spectrometer, used instead of an Optical Bench.

191. *Apparatus.*—An ordinary spectrometer with the table that usually carries the prism mounted on levelling screws; a bi-prism having a more obtuse angle than usual;[1] sodium flame.

Remove the lenses from both collimator and telescope. Set up the bi-prism on the small levelling table, with its refracting edge coinciding with the axis of the instrument (*i.e.* over the centre of the table), and with its plane perpendicular to the line

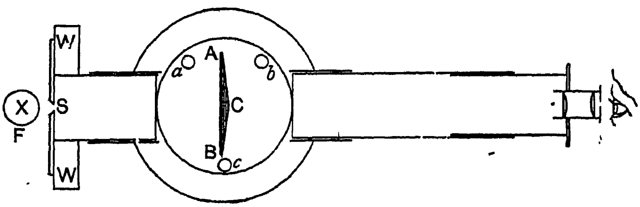


FIG. 224.—Spectrometer arranged to give interference Fringes with a Bi-prism. The collimator has been removed, and replaced by a short tube carrying a wooden ring, W, W, on which a slit, S, of printers' rule is mounted. The rule is mounted like a parallel ruler, so that its width may be adjusted.

joining two of the screws. These two screws should be in the hole and slot respectively of the spectrometer table (p. 117). The third screw will then enable the bi-prism to be rotated slowly in its own plane, and thus allow the refracting edge to be set accurately parallel to the slit of the instrument.

[1] The lines given by an ordinary bi-prism, so far from the slit as this has to be, are too fine, and generally cannot be seen at all; the ordinary bi-prism may be used, if the slit is brought near it, as in Fig. 224.

Remove the eye-piece from the telescope, point it directly opposite the collimator, and look through it, when two images of the slit should be visible, one in each half of the bi-prism. By moving the eye, one of these can be caused to reach the refracting edge, and if the slit is parallel to this edge, it will disappear from top to bottom at the same instant. If far out, rotate the collimator tube; but, if nearly correct, adjust with the levelling screw of the prism table. Replace the eye-piece, cover up the centre of the instrument with a piece of black velvet, and the fringes should be visible. A slight adjustment of the levelling screw, and of the width of the slit, will make them perfectly distinct. The distance apart of the bands will be equal to the distance, b, from the axis of rotation to the cross-wire, multiplied by the circular measure of the angle, θ, through which the telescope has to be moved to pass from one band to the next, and this can be easily determined. The distance apart of the image and the slit can be found by multiplying the distance, a, from the bi-prism to the slit, by the circular measure of the sum of the deviations produced by the halves of the bi-prism.

Thus, we require to measure:

(i) The distance, a, from the bi-prism to the slit.

(ii) The distance, b, from the axis of the instrument (which must coincide with the edge of the bi-prism), to the cross-wire of the telescope.

(iii) The angular distance, θ, from line to line.

(iv) The angular distance, A, subtended at the edge of the bi-prism by the images formed in the halves of the bi-prism.

The first two of these measurements are straightforward, though it is necessary to caution the student to be exceedingly careful in measuring the distance to the cross-wire of the telescope not to destroy it, as it is formed of two very delicate fibres.

To find the angular distance, θ, a series of 10 or 12 lines should be selected which are as clear as possible, and free from diffraction fringes which, it will be remembered, are superposed upon the interference lines. Set the cross-wire on the extreme line of the set selected, and take the reading of the vernier. Move the telescope, using the tangent screw so that the cross-wire is carried across the field, counting the lines as they pass. Again

take the reading of the extreme line on the other side. The difference divided by the number of intervals will give the angular width of each.

To obtain the angular distance, A, subtended by the images formed in the halves of the bi-prism, the lenses must be replaced in both collimator and telescope, and the instrument focussed for parallel light, by one of the methods in § 93. Then on pointing the telescope at the collimator with the bi-prism in position, two images of the distant slit will be observed. Set the cross-wire on each of these in succession and take the two readings. The difference will be the angle A required.

It is proved in books on Physical Optics that if c be the distance between the two images of the slit, x be the linear distance apart of the interference fringes, and d the distance from the slit to the plane in which the fringes are produced, the wave-length λ is given by

$$\lambda = \frac{cx}{d}.$$

In our case the distance apart of the images of the slit c becomes $a . \mathrm{A} . \frac{\pi}{180}$ (since A will be given in degrees by the spectrometer scale), x will be $b . \theta . \frac{\pi}{180}$, whilst d equals $a+b$.

Substituting we obtain :

$$\frac{ab\mathrm{A}\theta\pi^2}{(a+b)(180)^2}.$$

As the interference fringes are very faint, owing to the small vertical height of the collimator slit, their distance apart is very difficult to obtain; for, if the instrument is well covered in so as to see the fringes with the greatest distinctness, the cross-wire is generally invisible; whilst, if sufficient scattered light be allowed to enter the telescope to illuminate the cross-wire, the fringes become invisible. By lifting and dropping the covering over the spectrometer table, the slit and the fringes can be seen alternately, and the cross-wire approximately adjusted upon a line. Of course, if the ordinary slit of the collimator be replaced by a temporary one of a vertical height as great as the diameter of the tube will allow, this difficulty will not be so great.

It is very important that the sodium flame should be as bright as possible. A good flame is produced by bending an iron wire

of about number 22 B.W.G. into a circle about ½ an inch in diameter, and coating the wire with a little asbestos, which is then dipped into a strong solution of salt in water. If this is placed over the top of a Bunsen burner so that the flame passes through the centre of the ring, it will everywhere be in the hottest part of the flame, and will give a powerful light which will last for a long time.

Lloyd's Mirror with the Spectrometer.

Apparatus.—Spectrometer; prism; sodium flame; white light (*e.g.* incandescent gas).

192. Remove the lenses of collimator and telescope, and place a prism on the levelling table, with one face across a diameter and

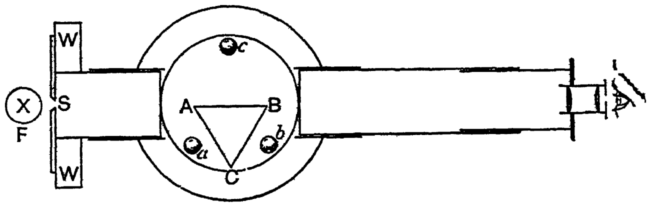


FIG. 225.—Spectrometer arranged to produce Lloyd's Fringes. A slit made of printers' rule has been substituted for the collimator as in Fig. 224.

parallel to the line joining two of the screws. Turn the table until this face is nearly parallel to the axis of the collimator. Remove the eye-piece, and, looking through the tube, turn the table until an image of the slit illuminated by the sodium flame is seen in this face of the prism, almost coincident with the slit itself. With the third levelling screw, adjust the face of the prism until this image is parallel to the slit.

Replace the eye-piece, and the interference fringes should be visible. They can be finally adjusted with the levelling screw of the table. Observe the effect of a slight rotation of the prism. As this will alter the distance from the slit to its image as seen in the face of the prism, it will also alter the width of the interference fringes. The reading of the width of the bands may, if desired, be taken and treated as before. The distance of the slit from its image is more difficult to determine, as it depends

upon the relative position of the face of the prism and the axis of the collimator. Thus, the insertion of the lenses will have to be made without disturbing either of these. This being done, the distance can be found and treated as before. As, however, these readings are very trying to the eyes, it is better to be content with the adjustment and observation of the fringes.

193. Replace the sodium flame by white light, and insert a piece of mica or micro-cover glass perpendicular to the reflecting face of the prism, so that the *direct* light from the slit has to pass through the mica on its way to the eye-piece. This will shift the centre of the system of fringes and enable it to be seen. Note that in white light the central band is *black*.

Diffraction with the Spectrometer.

194. *Apparatus.*—As in the last Experiment; the objects of Experiment, § 190, and an extra ring, W, W (Fig. 225), fitted with a pin-hole.

In the same way, the straight edge and narrow wire may be observed with the spectrometer, and their fringes sketched. Either of these diffraction experiments can be arranged as easily or even more conveniently than with the optical bench, as the instrument can be so easily covered in, a point of great importance.

Replace the slit by a pin-hole, either by opening its jaws and fixing a piece of tin foil over them with a hole in it; or, very much better, by withdrawing the tube carrying the slit, and replacing it by a tube with a pin-hole properly drilled in thin brass. Examine the circular aperture, opaque disc, and others of the set A to I on p. 261. All these diffraction experiments must be performed with the lenses of the telescope and of the collimator removed.

Herschel's Diffraction Experiments with the Telescope.

195. *Apparatus.*—Spectrometer with a pin-hole in place of the slit of the collimator; circular stops of varying size to pull over the objective of the telescope; triangular apertures; perforated zinc.

A pin-hole mounted at the principal focus of the lens of the collimator is equivalent to a distant star and gives a beam of

light which is parallel, not only in a vertical plane, as is the case with the slit, but is parallel in all azimuths. When a star is observed by a telescope, if the objective is a good one and well corrected for spherical aberration, the image will consist of a central bright disc surrounded by diffraction rings. The size of this disc, and the relative intensity of the disc and rings, depend upon the ratio of the diameter of the glass to its focal length. When the aperture is small the disc will be large, and the brightness of the rings will also be comparable with that of the disc. As the aperture of the lens is increased, the disc and rings shrink and a greater proportion of the light is concentrated into the disc itself.

These effects can be easily observed by reducing the aperture of the telescope of the spectrometer with diaphragms containing circular holes of varying size. The collimator must be adjusted so that the pin-hole shall be at its principal focus and the telescope focussed upon it. If the lenses of both collimator and telescope are good and the telescope is sharply focussed, even without any reduction of its aperture, a diffraction ring will be faintly visible, surrounding the image of the pin-hole. Now reduce the aperture by a diaphragm with a $\frac{1}{2}''$ hole. The diffraction ring will be much more clearly visible and will be larger; it should be quite circular. Use next an aperture $\frac{1}{4}''$ in diameter, then $\frac{1}{8}$, $\frac{1}{16}''$ and so on. The effect upon the disc and diffraction rings can thus be traced and the above statements verified. If other than circular apertures are used, the diffraction will vary accordingly; for instance, place a diaphragm with an aperture the shape of an equilateral triangle about $\frac{1}{4}''$ side over the lens of the telescope and a series of circles will appear in the form of a star with six arms.

A regular series of apertures such as those in a piece of perforated zinc will produce a strong diffraction effect. A series of irregular apertures punched at random will give circular diffraction patterns. These diaphragms can be made of metal or card, but the easiest material to make them from is *stout* tinfoil (*i.e.* sheet lead about No. 28 B.W.G.).

ADDITIONAL EXERCISES ON CHAPTER XI

1. Adjust the bi-prism as in Experiment, § 189, using an incandescent gas light (instead of adjusting them with a sodium flame, and afterwards substituting the white light).

2. Alter the position of the block carrying the bi-prism (Fig. 220), placing it about 4 inches from the one carrying the slit. Obtain the fringes, and see that they are much larger than before.

3. Remove the screw from the central block (Fig. 222), and insert the wire gauze object (Fig. 223). While looking through the eye-piece, gradually slide the block along the bench towards the slit. See that the fringes slowly close in.

4. In a dark room, look at a distant bright point of light (an arc, if possible, or an incandescent gas light behind a small circular aperture), and hold first a hair, and then a straight edge, in front of the eye. Observe and draw the diffraction fringes.

5. Obtain a piece of old plate-glass, which has been scratched by repeated cleaning, and hold it near the eye while looking at a distant bright point. Diffraction fringes will be formed along lines which seem radial to the distant bright light. Explain these.

6. On withdrawing the eye, and fixing the attention upon the glass plate, bright lines will be seen which seem concentric with the distant point. Explain these.

7. Hold a small bunch of glass wool, or ordinary curly wool, near the eye, and observe it in the light of a distant arc, or the sun. A number of tiny circular coloured rings will be visible. Withdraw it a greater distance, and see that the wool forms numerous point images of the source of light, and that each of these point images when the wool is brought nearer, gives rise to a tiny circular system of rings. (These rings can frequently be seen on a sunny day in the loose nap of a woolly coat, or in the eyebrows.)

CHAPTER XII

FURTHER EXPERIMENTS ON INTERFERENCE

The Optical Bench and Accessories.

196. **The Bench.**—To obtain any measurement of the fringes which are produced by interference, a firm and easily adjustable optical bench is a necessity owing to the delicacy of the fringes. The bench is usually a kind of lathe bed furnished with sliding stands. These stands should be perfectly rigid, and for some experiments should be capable of being clamped. Four stands will be required. The first will carry the slit, the next the bi-prism or other similar apparatus, another the micrometer eye-piece. The fourth stand will be required temporarily to carry a lens which is used to form an image of the two slits. One or more of these stands usually has a rack motion to raise or lower the head, and a clamp. If this is the case, care must be taken that the clamp is free before the rack is used, or the teeth will be torn off.

One at least of the heads of the first two stands must have a means of rotation in its own plane by a tangent screw, or similar device, as it is impossible to perfect the adjustments without this screw; indeed it generally forms the most important adjustment. One stand must have a transverse motion to allow the head to be moved out of line with the other stands for some two inches, and should either move in a perfectly fitted dove-tail slide, or on a geometrical slide, and be able to be clamped in any position. If the movement is produced by a screw, it is an advantage in one or two experiments to have this of definite pitch, with a divided head. The eye-piece must be a positive one, with a fine single spider-line in its focal plane. To avoid back lash the readings must always be taken in one direction only.

The slit must have true knife edges, either of steel or quartz, which can be adjusted to any width. If they are of steel a greasy

rag should be kept to rub them over after use, especially when working with a sodium flame.

197. **The light** is of very great importance in interference work. A good bunsen is essential, with a large supply of air. If the salt is supplied on wire gauze it should be on very fine gauze, preferably of platinum, and it must only be allowed to touch the edge of the flame. The light depends upon the vaporisation of the salt, and therefore upon the temperature of the flame. If the gauze is allowed to penetrate far into the flame it will lower the temperature and prevent a proper vaporisation of the salt, and the light will be very poor. The gauze should touch the flame edge-wise as seen from the bench, that is the edge of the gauze should be parallel to the length of the bench; it should touch the flame about $\frac{1}{4}''$ above the top of the burner. A good light can be obtained by making a ring of wire, about half an inch in diameter, coated with asbestos fibre. This is then dipped in powdered rock salt, or in a strong salt solution, and supported so as to encircle the flame from a bunsen burner (or the burner of an incandescent gas light) about a quarter of an inch above the burner. The flame must be protected from draught by a metal chimney, cut away where necessary to allow the flame to be seen and the gauze to be inserted.

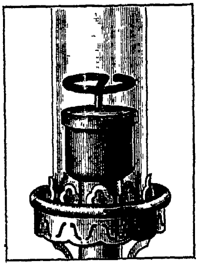


FIG. 226.—Sodium Light.

Powdered *rock salt* is much better than common salt, as it does not splutter on to the bench so much. A sheet of glass between the flame and the slit, whilst in no way spoiling the light, is a great protection to the slit and brass work, which the salt will very readily attack.

When the adjustments are completed and before taking the readings, it is always advisable to replenish the salt, and to see that the light is as good as possible.

198. **The Lens** above referred to for producing an image of the slits in the focal plane of the eye-piece should be a good one. A single lens is frequently supplied with the bench, this is quite useless for the purpose. A good photographic lens should be obtained. It should have a focal length of about 5 inches (a "quarter-plate" lens). It ought to give a good definition with full aperture. It should be so mounted that it projects somewhat beyond the base of the stand and can be rotated easily to face either way.

199. **The Bi-prism.** This is a piece of worked glass, flat on one face, and rising to a slight ridge along the centre of the other face. The angle at the ridge is about one degree.

200. **Lloyd's Mirror.** A piece of plane unmounted black glass is sufficient for this. It can be attached to the head of the stand with the transverse motion by three little pellets of wax or plasticene. It is, however, better if mounted on a separate head of its own, which need have no adjustment. To avoid distortion the mirror should be held loosely by three clips (Fig. 227), against which it is pressed by two springs (shown by the dotted lines) in the form of arcs of circles. One of these springs must press at its *middle* against the back of the mirror, its ends resting against the supporting plates. The other spring must press the mirror with its ends just behind the clips, and its middle must press against the brass plate. The clamp by which this is to be attached to the stands of the bench may most conveniently be in front as shown in the figure, so that the plane of the mirrors is in line with the axis of the stand.

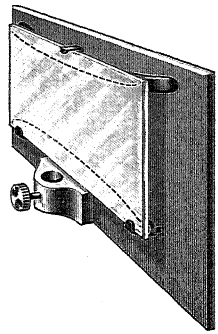


FIG. 227.—Lloyd's Mirror.

201. **Fresnel's Mirrors.** These are two pieces of black glass about two inches square, they are usually mounted on a separate head. The mirrors are carried in small brass frames which can be adjusted by screws from the back of each mirror. One of these frames is mounted to rotate on the points of two screws, the axis of rotation being in the plane of the front surface of the mirror, and coincident with the edge of the mirror. The other frame is mounted to press against three screws against which it is pressed by a spring (at the centre of the triangle formed by these screws); the screws enable the plane of

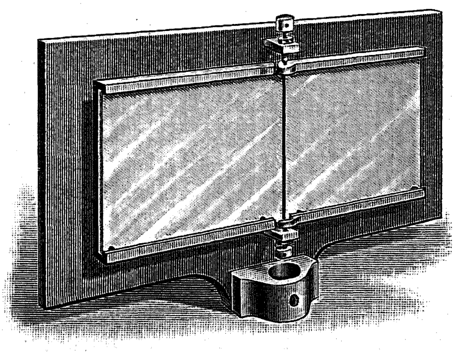


FIG. 228.—Fresnel's Mirrors.

this mirror to be adjusted to coincide exactly with the plane of the first mirror. Then by rotation of the first mirror about the axis above described, the mirrors can be set at any inclination to each other, and the axis of rotation will be the line of intersection of their planes.

FIG. 229.—Bi-plate.

202. **The Bi-plate** is made from a piece of parallel glass about two inches by one, which is cut in halves, and after the edges have been bevelled the two halves are cemented together again so as to form a slight angle with one another. It is placed in the ordinary head.

Determination of the Wave-length of Sodium Light with the Bi-prism.

Apparatus.—The optical bench; the bi-prism; sodium flame; a white light; a good photographic lens of about 5 inches focal length; millimetre scale.

203. **Preliminary Adjustment.** Set up the sodium flame with the slit, the bi-prism, and the eye-piece in a line; the stand carrying the slit being at one end of the bench, the bi-prism about 6″ or 8″ from it, and the eye-piece about another 6″ or 8″ away. Withdraw the eye-piece from its tube, look through the tube and see that the flame is properly lighting the slit. With the transverse motion of the stand carrying the bi-prism, adjust the bi-prism until its ridge is in line with the slit, so that by a slight movement of the eye the slit may be seen in either half of the bi-prism. Set the slit as nearly vertical and the plane of the bi-prism as nearly perpendicular to the axis of the instrument as possible. Then on looking through the tube of the eye-piece, if the head is slowly moved sideways, one of the images of the slit will reach the ridge of the bi-prism. If this ridge is parallel to the slit, the whole length of the slit will reach it at the same instant, and moving the eye a little further it will disappear at the same instant. If the ridge is not quite parallel to the slit, this disappearance will occur first, either at the top or bottom of the slit; adjust the tangent screw until this no longer happens. This can be done most accurately, finally, by placing the eye so that the slit has almost disappeared from one half of the bi-prism, and by then noticing

whether the light is *equally intense* along its length. On moving the eye the other way, a similar disappearance of the slit seen in the other half of the bi-prism should occur. With the eye in the centre of the eye-piece tube, two slits will now be seen, equidistant from and parallel to the ridge of the bi-prism. The slit should be almost closed. Now replace the eye-piece in its tube and look through. If the preliminary adjustments have been carefully performed, both the interference and diffraction fringes will be visible.

To see these fringes properly, either the experiment should be conducted in a dark room, or the apparatus must be covered in so that no light may reach the eye-piece but that coming from the slit. If the experiments are not performed in a dark room, much time will be saved if a sheet of black card about a foot square, with a hole two inches in diameter in the centre, is attached to the head of the stand carrying the slit, and a second similar card to the eye-piece, by thin wire. Two thin wooden rods[1] can be laid across from one of these to the other at their upper corners, and a piece of black velveteen spread over them, reaching down to the table, or the velveteen may be held in place and prevented from sagging by securing it to the cards or stands with ordinary gentleman's tie-clips. This will allow the hand to be inserted to adjust the tangent screws without admitting any appreciable amount of light. It is obvious that a very much better result can be obtained in a dark room, as the pupil of the eye will then be larger, and therefore the faint light forming the interference fringes will be more readily perceived.

204. **Final Adjustment.**—If the fringes are not seen at all upon inserting the eye-piece, the preliminary adjustment must be repeated; when once found the fringes can be made distinct with the eye-piece in. Having covered the apparatus in, look through the eye-piece and very slowly turn the tangent-screw one way or the other, watching the effect upon the fringes. They will usually be found to be greatly improved by a *slight* motion of the screw.

[1] In place of wooden rods, a pair of parallel brass tubes about 1 cm. diameter and say 50 cms. long, fixed at one end about 10 cms. apart into a brass cross-bar, and telescoping into a pair of similar tubes of slightly greater diameter can be used. In this way the length can be varied from 50 to about 90 cms., and over a long range it can always be made exactly to reach from the eye-piece to the slit.

The slit will probably have to be still further narrowed. It is easy to determine the best width for the slit; when it is too wide there is a blaze of light, but the fringes are very hazy; when too narrow, the fringes will be sharp, but it will not be possible to see the cross-wire and adjust it upon them. The eye-piece should be pushed in or out until the cross-wire is in focus. The cross-wire must be rotated until it is parallel to the fringes. Usually some dozen or twenty fringes in the centre of the field can be sharply distinguished. These will be continued on either side by other fringes superposed upon the diffraction fringes, which makes it difficult to observe them. If the fringes are too fine they may be made larger, either by bringing the bi-prism nearer to the slit, or by withdrawing the eye-piece to a greater distance from the bi-prism. Both of these involve certain disadvantages. Withdrawing the eye-piece will diminish the amount of light, but if the light is powerful enough this is the better way. Placing the bi-prism nearer the slit increases the size of the fringes, by making the apparent distance between the two images of the slit smaller. As this distance will have to be measured, any decrease in it will mean an increased percentage error in its determination. The larger it can be kept the better.

205. **Résumé of above Adjustments.**—i. Remove the eye-piece, look through its tube and see that the bi-prism, slit, and flame are in line, so that two images of the slit can be seen, one in each half of the bi-prism, both well illuminated.

ii. Set the bi-prism and slit vertical, and adjust them parallel to one another, by seeing that the image of the slit seen in a half of the bi-prism vanishes at the same instant throughout its length as the eye is moved transversely.

iii. Insert the eye-piece, cover up the apparatus if not working in a dark room, and see if the fringes are visible. If not repeat ii. If visible, adjust the tangent-screw and the width of the slit to obtain the best possible definition.

iv. If the fringes are too narrow, withdraw the eye-piece stand as far as the illumination will allow.

These adjustments should be entirely thrown out, and repeated several times, until starting with the apparatus entirely out of adjustment, the fringes can be found in one or two minutes.

206. **Fringes with White Light.**—The sodium flame may be replaced by an ordinary flame, the best being an incandescent gas light, and the coloured fringes due to white light will be visible. A sketch approximately to scale, showing the distribution of the interference and diffraction fringes, and the arrangement of the colours should be made. The fringes may be found with the white light at once by the same steps as above. They are of course best seen with an arc lamp or sunlight as source of light.

Determination of Wave-Length.

207. The following measurements have to be made:

i. The distance between the two images of the slit formed by the bi-prism.

ii. The width of the fringes.

iii. The distance d from the slit to the cross-wire of the eye-piece.

Having seen that it is possible to set the cross-wire upon the fringes—that is, that the cross-wire is visible and the fringes large enough to measure—**it is better to find the distance c between the images of the slit before proceeding to determine the width of the fringes.** If this is not done the time is frequently wasted, for it may be found impossible to determine c without rearranging the apparatus. Begin then with the determination of the distance apart of the images of the slit.

208. **To find c.**—For this a good photographic objective will be needed as already mentioned. Withdraw the stand carrying the eye-piece to the extreme end of the bench, but be careful not to disturb in the slightest degree either the bi-prism or the slit. Place the lens between the bi-prism and the eye-piece, and adjust it to form a reduced image of the slits in the focal plane of the eye-piece. If any difficulty is found in this, hold a sheet of white paper against the eye-piece to act as a screen, and focus the slits upon it. A very slight movement either of the lens or eye-piece will now bring them into focus. Without moving the eye-piece, move the lens until an enlarged image of the slits is formed.

It may be found that this would require the lens to be brought nearer the slit than the stand of the bi-prism will admit. If

this is so, put the lens as near as possible to the slit, and bring the eye-piece up until it is in focus. Make sure that you have the *enlarged* image by sliding the lens once more towards the eye-piece (without moving the latter) until a diminished image is again formed. If an enlarged image cannot be obtained, either a longer focus lens must be used, or the bi-prism stand must be placed nearer the slit. Let c be the distance between the images we wish to determine, and c_1 the distance between the reduced images formed by the lens at the eye-piece, and u and v the distances from the principal points of the lens to the slit and eye-piece respectively, then $\frac{c}{c_1}=\frac{u}{v}$.

If the lens be moved until the enlarged image is formed, and c_2 be the width of this image, then as u and v are now interchanged $\frac{c}{c_2}=\frac{v}{u}$. Therefore $\frac{c}{c_1}=\frac{c_2}{c}$; and $c=\sqrt{c_1c_2}$. Thus u and v will not have to be determined. The accuracy of this method depends upon the ability to focus the image perfectly. If a very slight movement of the lens does not at once affect their definition so that the position of the lens can be determined accurately, the lens is not good enough for the purpose. To find c_1 and c_2, use the micrometer eye-piece, being careful to take the readings in one direction only in order to avoid the back lash of the screw. At least four independent determinations of c should be made, the eye-piece being moved between each set of readings, each time about a centimetre. If the error is large, a still greater number of readings will be necessary. The mean of these readings is to be used. The best readings will be obtained when c_1 and c_2, and therefore u and v_1, are nearly equal.

209. **Alternative Method for determining c.**—If the focal length of the lens is too small, it may be impossible to place it sufficiently near the slit to obtain an enlarged image at all. In this case c can still be determined from the equation $\frac{c}{c_1}=\frac{u}{v}$, if u and v can be found. Therefore, having placed the lens so as to obtain the diminished image and found its width c_1, measure the distance from the slit to the nearest part of the mount of the lens, and also from the cross-wire to the end of the mount nearest it. The

distances of the principal points from the ends of the lens mount can be determined as described under heading of "Compound Lenses," § 132, and added to these two distances; this will give u and v.

Fairly accurate values of u and v are obtained by measuring to the diaphragm of the lens from the slit and the cross-wire respectively.

Great care is required in measuring to the cross-wire not to push anything up the tube to break it. It will be quite near enough if u and v are measured to the nearest millimetre. Again, at least four determinations should be made.

It is usually in the measurement of the distance c that the greatest error appears.

210. **To find x.**

See that the light is in the very best condition. Remove the lens used in the last determination, and withdraw the eye-piece stand as far as the illumination will allow; again being careful not to disturb the other two stands. If the fringes are not distinct, the tangent screw may be readjusted, as may also the width of the slit.

Let us suppose that there are some fifteen fringes sharply defined. Commence at one end, take the reading for each fringe across the field. It is very easy in making these readings to miss one fringe, and the series of readings should be examined to see if there is any sudden jump, and be numbered accordingly. Care will have to be exercised that the eye-piece head is not displaced by the hand in rotating the screw. It should of course be firmly clamped.

Arrange the readings thus:

No. of fringe.	Reading on scale.	Reading on head.	Reading in cms.	Difference of readings.	Width of six bands.

If fifteen readings are taken the first reading should be subtracted from the seventh, the second from the eighth, and so on,

and the mean taken. This will give the average value for the width of six fringes. These numbers are to be entered in the last column, and the mean at the foot of the column.

Or read each of the first five fringes, then without taking readings move the cross-wire on (counting carefully) to the eleventh fringe, and read the eleventh to the fifteenth fringes. Enter them as above and subtract the first from the eleventh, the second from the twelfth and so on. The difference will be the width of ten fringes, and $\frac{1}{10}$th of the mean will be the probable width of one fringe.

211. **The distance from the slit to the cross-wire of the eye-piece.**—This may be taken with sufficient accuracy with an ordinary millimetre scale. The distance from the slit to the brass work of the nearest point of the eye-piece should be measured, and the distance from this point to the cross-wire added. The cross-wire is usually mounted at the end of a short tube. This tube can be withdrawn and its length found without touching the cross-wire. *On no account must any pencil or rod be inserted in the tube*, as it would be almost certain to destroy the cross-wire. If these distances are found to the nearest millimetre it will be sufficient.

The wave-length can now be found from the formula

$$\lambda = \frac{cx}{d}.$$

Lloyd's Mirror.

212. **Preliminary Experiments.**

Apparatus.—The optical bench, with slit and eye-piece (page 196); Lloyd's mirror (page 271); sodium flame and white light; millimetre scale; black velveteen.

Adjustment.—The mirror must be mounted on the stand which carried the bi-prism in the last experiment, with its plane vertical and parallel to the optical bench. The stand must be set by its transverse motion so that the plane of the mirror shall contain nearly, when produced, the slit and the cross-wire of the eye-piece. Remove the eye-piece and look through its tube. The slit should be seen directly and its image be visible in the mirror. If this is not the case and if the image and the slit do not appear very

close together, either the mirror may be moved forward or backward with the tangent screw, or the head carrying the mirror may be slightly rotated round a vertical axis, and then again clamped.

The slit and its image have to be perfectly parallel to one another. This adjustment can be made by inclining the mirror, if it is mounted with adjusting screws, or if not by using the tangent screw on the stand carrying the slit. The images can usually be set sufficiently parallel by eye (especially if the slit and its image are brought very close to one another by the transverse motion), for the interference bands to be at once seen on inserting the eye-piece. Now look through the eye-piece, and even if the fringes are invisible, we know that the slit and mirror are practically parallel, and therefore that a very slight motion of the tangent screw must make them so. It is therefore easy to find them, and to obtain well-defined lines.

The effect of an alteration in the distance apart of the slit and its image can be very readily observed with this apparatus. Looking through the eye-piece, turn the screw attached to the transverse stand, and as the plane mirror is brought nearer to the slit so that the distance c diminishes, the fringes will be seen to rapidly broaden. On withdrawing the mirror and increasing the distance, the fringes will become narrower and narrower; the field over which they extend increasing in width at the same time. The reason of this increase can be easily seen if the eye-piece be removed, remembering that fringes will be formed only over the part of the field from which both the slit and its image are visible.

Observe, and sketch the fringes, both diffraction and interference, formed with yellow and also with white light, and notice the effect of the transverse motion on these fringes.

213. **Determination of the Wave-length of Sodium Light with Lloyd's Mirror.**—The mirror must be close to the slit, and the eye-piece should be withdrawn as far as the illumination will allow. The width of the fringes should be adjusted, until it is about as great as it was in the case of the bi-prism, for it must be remembered that an increase in this width though making this measurement easier to determine, at the same time means a decrease in the distance between the slit and its image, and therefore an increased

error in the determination of that distance. The determination of the values c, d and x is precisely similar to that already described in the case of the bi-prism.

As these readings are much more trying (owing to the feeble illumination) than those with the bi-prism, it is not advisable as a rule actually to make them.

214. **Effect of widening the Slit with Lloyd's Mirror.**—The corresponding points are the slit and its image, the right side of the image corresponds with the left side of the slit, and *vice versa*. Hence an increased width in the slit adds corresponding points symmetrically situated with respect to the plane of the mirror. The central fringe will therefore be in the same position for the right and the left hand sides of the slit, but the fringe formed by the one side of the slit and its image will be wider than those formed by the other side and its image. The central bands, therefore, will remain sharp with a wide slit, but those at a distance from the centre will become more and more hazy and finally disappear as the width is increased. This can be easily tested when the apparatus has been adjusted.

215. **Range of field over which the figures are formed.**—Draw a figure as on page 283, it would appear that there should be fringes only over a short range. Remove the eye-piece but not the cross-wire, and looking through the eye-piece tube set the cross-wire, the image of the slit and one vertical edge of the mirror in a line. This setting should be done in an ordinarily lighted room to avoid diffraction, when it can be made sufficiently well for the present purpose. Replace the eye-piece, and see if the cross-wire is or is not at one end of the interference system. Repeat with the other edge of the mirror.

Fresnel's Mirrors.

216. A very **simple form of Fresnel's mirrors** has been described by Quincke. Lay a sheet of plate-glass about 6″ × 2″ on a flat table. Spread ¼″ of plasticene over it. Place two pieces of plate-glass, each 3″ × 2″, on the plasticene. Lay another sheet of glass on the top, and press it firmly down. Apply rather more pressure in the middle than at the ends. This will cause the two small pieces to make a slight angle with one another, which can be

varied by the way the pressure is applied; the angle is, of course, due to a small flexure of the top plate. Thus, when the top plate is lifted off, the two small plates are in a condition to produce interference.

The mirrors so made can be laid on their back on one of the stands of the optical bench described in § 37, adjusting the mirrors in a horizontal plane, and at right angles to the length of the bench. The plane can be varied by the screw P. The slit, which must be a good one, must be put up horizontally at one end of the bench, and the eye-piece at the opposite end; they may each be a foot from the mirror. A bridge to cut off the direct light from the slit must be set up about a $\frac{1}{4}''$ above the dividing line of the mirrors. The eye-piece must be so placed that some light from each mirror enters it (*i.e.* seen from the eye-piece tube, the image of the slit should seem double). Then, on adjusting the screw P so as to make the slit and the line of intersection of the mirrors parallel, the fringes should appear.

217. *Apparatus.*—The optical bench, with slit and micrometer eye-piece (page 269); sodium flame and white light; Fresnel's mirrors (page 271); black velvet.

Adjustment.—The experiments with these mirrors are far more difficult than with either the bi-prism or Lloyd's mirror, and should not be undertaken until the student is familiar with the other two methods of producing interference. There are two mirrors, each forming an independent image of the original slit. To obtain the fringes, the following conditions are necessary:

i. The images must be parallel to one another. Unless this is the case the fringes formed by the top and bottom of the slit will be of unequal width, and therefore only a few of the fringes in the centre of the field will be sharp, and overlapping will occur to a greater and greater extent towards the edge.

ii. The original slit must be parallel to each of the images. This is necessary in order that the corresponding points in the images may be symmetrically situated.

We have thus two independent adjustments, and of these the latter is the more important and the more difficult; but unless

this is secured, no fringes are visible in any part of the field. It is best first to adjust the mirrors parallel to one another by eye. Hold the frame, carrying the mirrors in the hand, and look at a distant window bar reflected in the mirrors. Adjust the screws B, C, and D until the image of the window bar is straight in every position of the mirror, *i.e.* until the mirrors are in one plane. A distant lamp-shade flame will do even better, for if the mirrors are tilted until the shade is seen first in one mirror and then in the other, it is easily perceived if there is any sudden displacement as the image passes from the one surface to the other. See also that the plane of neither mirror is above that of the other. This may be noted by looking at grazing incidence along the surface of each mirror in turn; the junction of the two mirrors should appear as a very fine line only. Now place this head on the stand of the optical bench that previously carried the slit. Place the latter on the transverse stand, this stand being, of course, changed to the end of the bench, and set the head about an inch out of the central line of the bench, so that the light from the slit may fall obliquely upon the mirrors. Remove the eye-piece, and looking through its tube rotate the head carrying the mirrors until the image of the slit is formed at about the junction of the two mirrors. Place the flame behind the slit in such a position that the image (or images if two are seen) is brightly illuminated. The slit itself, as seen from the eye-piece, will probably appear dark; if not, a screen must be set up to cut off the direct light from it. On moving the eye transversely the image will pass from one mirror to the other; adjust the tangent-screw on the stand carrying the slit until the image of the slit reaches the dividing line of the mirrors at the same instant throughout its length. Now turn the screw A, which alters

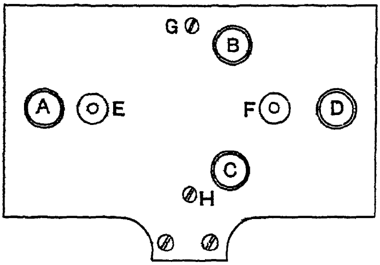


FIG. 230.—Back view of Fresnel's mirrors. E and F are springs to hold the frames carrying the mirrors against the points of the screws A, B, C, D and E.

the angle between the mirrors, until two images of the slit are visible at the same time, one in each mirror. One of these images ought to reach the dividing line of the mirrors at the same instant throughout its length, when the eye is moved in either direction. If all these preliminary adjustments have been carefully executed, on replacing the eye-piece and looking through it the fringes ought to be visible in the middle of the field, and can be rendered more distinct by carefully adjusting the width and inclination of the slit, and finally and exceedingly carefully, the inclination of the planes of the mirrors to the vertical.

218. **The Limits of the Interference Fringes.**—As with Lloyd's mirror, so for Fresnel's mirrors, both the width of the fringes and their

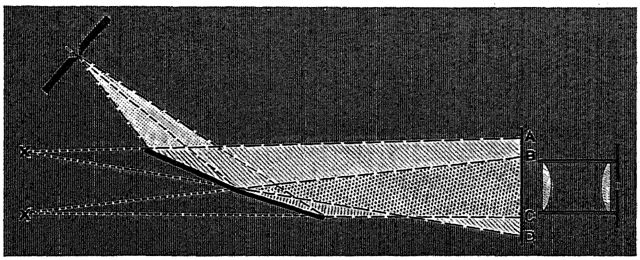


FIG. 231.—Limits of Fringes with Fresnel's Mirrors.

number can be altered. It is an instructive exercise, to compare the extent of the fringes with the amount they should extend on the ordinary geometrical theory of the formation of these fringes. The fringes should be visible over all the field which is illuminated by both the images of the slit, that is the piece of the field where the fan of light from the one image of the slit which is bounded by the edges of the mirror in which it is formed, overlaps the similar fan from the other image.

To find the limits of the illuminated fields, remove the eye-piece, and with the micrometer screw, set the cross-wire in line with an edge of the mirror, and with the image of the slit formed by that mirror. Take the readings. Move the cross-wire until it reaches the line joining the image to the other edge of the same mirror. Repeat this for the image of the slit formed by the other mirror. The adjustments must be made in a light room.

Or the amount of the overlapping can be very easily calculated, if the size of the mirrors is measured, and the distance of the slit from the plane of the mirrors is found (by measuring with an ordinary scale the distance from the slit to its image in the mirror). Then knowing the distances from the mirrors to the slit and also to the cross-wire, the overlapping can be found by similar triangles.

It will be seen from the figure that the overlapping portion is from B to C. The length of this portion is of course given by the readings. Replace the eye-piece, get the fringes as clearly defined as possible, and see if they extend over an equal distance. The edges of the diffraction fringes will of course correspond with the same readings. Repeat this with the broad fringes, formed by making the two images of the slit nearly coincident.

219. **The effect of an increase in the width of the slit.**—As the two "corresponding points" are each of them images of the slit, the right hand side of the one image corresponds to the right hand side of the other, and the left to the left. Thus an increased width in the slit will result in a lateral displacement of the fringes which will be the same for the central ones as for the marginal ones. An increase in the width will therefore produce an equal haziness in the fringes all over the field.

If no fringes are visible rotate the slit by its tangent screw a very little, watching carefully the effect. There ought to be no difficulty in obtaining some trace of fringes. **Do not touch the screws of the mirror until some faint fringes are visible.** The slit must of course be fairly narrow, so that the light is quite faint. The previous experience with the bi-prism will be a guide in this respect, although the fringes with the mirrors must not be expected to be so bright or so easily seen as they were with the bi-prism. When the fringes have been made as distinct as possible with the tangent screw and by the adjustment of the width of the slit, there should be several clearly visible in the centre of the field. The number will depend upon the accuracy with which the planes of the mirrors were preliminarily adjusted parallel to one another. A very slight movement may now be given to one of the screws of the mirrors, watching the effect all the time. This should increase the number of the fringes. The tangent screw of the

slit must be again adjusted to render them distinct. By proceeding in this way, first with one and then the other, the number of visible distinct fringes can be greatly increased. In every case the amount the screws are moved must be very small.

If any light reaches the eye-piece from the slit other than that which was reflected by the mirrors, a black screen must be interposed.

Owing to the very small luminosity of these fringes and the double adjustment required to obtain them, it is very difficult to set the apparatus up in a room not properly darkened; although when once found they may be seen (if not measured) in an ordinary room if the apparatus is properly covered in as directed in the case of the bi-prism. With the eye-piece removed, observe whether any light reaches its tube other than that passing through the slit, and place extra screens to cut off all such light.

220. **Effect of variation of angle of mirrors.**—When the fringes have once been found their width can be altered by rotating the screw which controls the inclination of the mirrors to one another, as this has the effect of changing the distance between the two images of the slit formed by the mirrors. If the fringes are sufficiently broad and distinct, the measurements of the wave-lengths may be made as with the bi-prism, but the readings are exceedingly trying to the eyesight, and it is better to be content with obtaining, observing and sketching them only.

221. **White Light.**—Having found them, substitute white light for the sodium flame, and again sketch the fringes. Only a few coloured interference bands will now be visible in the centre of the field, with some diffraction bands on the two edges of the field. The alteration in these bands should be observed and sketched as the angle between the mirrors is varied. These observations of the behaviour of the fringes are much more instructive than the mere mechanical "taking of readings" so frequently attempted, and are much less trying to the eyes.

222. **Corresponding Points.**—Notice that rotating the slit by the tangent screw does not alter the parallelism of the images formed by the two mirrors, and if the planes of the mirrors meet in a vertical line, these images will remain parallel to one another for all inclinations of the slit, and the corresponding points will

be on the same horizontal line. By "corresponding points" is to be understood the two images formed of the same point of the slit. It is these corresponding points which, being in the same phase, will produce interference bands. Thus the interference bands of any pair of corresponding points will be vertical. If the slit is not also vertical, the fringes formed by the different pairs of corresponding points along the length of the slit will overlap one another and produce uniform illumination.

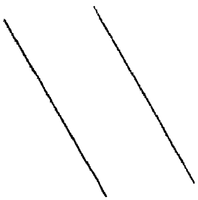


FIG. 232.

If the slit is vertical, but the planes of the mirrors are each equally inclined to the vertical in opposite directions, the images of the slit will also be equally inclined to the vertical and no longer be parallel to one another. The corresponding points will again be on the same horizontal line, and the fringes formed by each pair of corresponding points will again be vertical; but this time the central one of the fringes formed by every point of the slit will be coincident and therefore distinct. As the distance between the corresponding points is a varying one, the width of the fringes will vary, and they will be confused at a short distance from the centre. Thus, even if the planes of the mirrors are not both vertical, *if the slit be adjusted to bisect the angle between these planes* (or rather between the intersection of these planes and the plane of the slit), a few lines will be obtained in the centre of the field. Hence, if the mirrors are adjusted approximately correctly, these central fringes can be obtained by adjusting the slit only, and the final adjustment of the mirrors may be made afterwards. (The student should think out for himself the reason why the bi-prism and slit, and the Lloyd's mirror and slit, have to be so carefully adjusted parallel to one another, before any interference fringes are visible.)

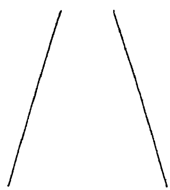


FIG. 233.

The Bi-Plate.

223. *Apparatus.*—The optical bench with slit and micrometer eye-piece (page 269); the bi-plate (page 272); sodium flame; a good lens (page 270).

If the course of the rays is traced from the slit through the bi-plate, it will be seen that there is no region of overlapping, and therefore no fringes can be seen in an eye-piece placed beyond it. To obtain fringes a lens is necessary, which must be so placed as to produce a real image of the slits.

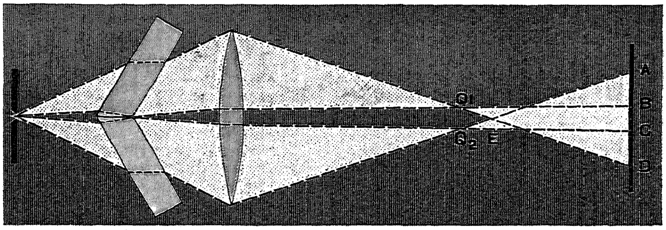


FIG. 234.—Locus of Fringes with the Bi-plate.

From the above figure (Fig. 234), it will be seen that for a short distance from B to C beyond E there is a region of overlapping of the two beams of light transmitted by the plate. In this region

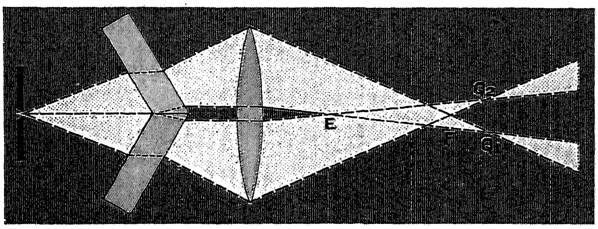


FIG. 235.—Locus of Fringes with the Bi-plate.

interference fringes can be formed. If the bi-plate is reversed, as in Fig. 235, this will be between the lens and the image from E to F only. The lens should be so placed that the images Q_1 and Q_2 are formed within the length of the bench, so that the

cross-wire of the eye-piece may be adjusted upon them. The distance Q_1Q_2 can then be measured directly by the micrometer screw. This is the distance c of the formula (page 278).

Let BC be the position of the focal plane of the eye-piece (*i.e.* the cross-wire) when the fringes are observed. The distance from Q_1Q_2 to BC is the d of the formula. The fringes formed by the bi-plate can be found as directed for the bi-prism, taking care only that BC is within the region in which overlapping occurs. The fringes are very bright, quite easily found, and therefore can be readily measured.

EXERCISE ON CHAPTER XII

Loss of Phase by Reflection in Lloyd's Mirror.—*Apparatus.*—The optical bench with the ordinary slit and eye-piece and Lloyd's mirror; an arc lantern; an additional slit; an achromatic lens about 6″ or 8″ focus and 2″ diameter; a good prism and stands; squared paper and black velvet.

When light is reflected from a glass surface there is a sudden change of phase caused by the reflection. If we assume this loss of phase to be independent of the colour, it can be determined approximately in the following manner:

Project the spectrum formed by an arc light upon the slit as in Fig. 236. By moving the bench slightly, any part of the spectrum, and

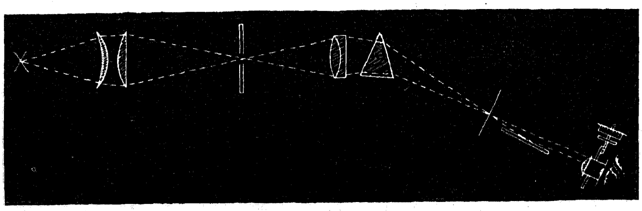


FIG. 236.

therefore any colour desired, can be used with which to form the fringes. It will not be necessary to know what the colour is. Set the mirror so that the fringes are fairly broad. Placing the slit in the red end of the spectrum, observe the positions of the dark bands. Repeat with the slit in the yellow, green, blue and violet (if there is sufficient light). If the readings are plotted on squared paper

those of any one colour will be found to lie upon a straight line. These straight lines will meet in a point. If the loss of phase is half a wave-length, this point would be exactly upon a line; if there is no loss of phase, this point would be half way between; by the position of this point of intersection, the loss of the phase can be estimated approximately.

CHAPTER XIII

COLOURS OF THIN AND THICK PLATES

Newton's Rings—Theory of the Method.

224. As the inclination of the surfaces used to produce the rings is very slight, where the rings are formed at any one point we may consider the two surfaces to be parallel.

Let AC and DB be the two surfaces separated by a layer of air of thickness e. A ray of light incident along PA will be refracted at A, reflected at B, and refracted at C, and emerge along CR. Another ray QC of the same parallel bundle as PA will also partly emerge along CR. Draw CN at right angles to AB produced.

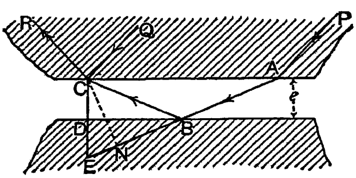


FIG. 237.

Draw CD at right angles to BD and produce to E making DE equal to CD. Then, as AB, BE are in the same straight line,

$$EN = EC \cos CEN = 2e \cos i.$$

Then the light will reach N from P, and C from Q together. Thus the difference of path at C is the difference between BC and BN ; that is, EN, since BC = BE.

There is a difference of phase of half a period introduced, as one reflection (at C) is in glass from air, and the other (at B) is in air from glass.

Thus, the whole difference of path is $2e \cos i + \frac{1}{2}\lambda$.

If this difference is any whole number of wave-lengths, the rings will be bright at that point. They are therefore

$$\textit{bright} \text{ if } 2e\cos i + \tfrac{1}{2}\lambda = n\lambda,$$
$$\textit{dark} \text{ if } 2e\cos i = n\lambda. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

225. To get e in terms of the radii of the surfaces.

If both surfaces are convex the thickness e at any point is the sum of e_1 and e_2, the amounts the surfaces are separated from the tangent plane at the point of contact.

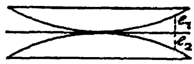


FIG. 238.

Let x be the radius of the ring, *i.e.* $=$OM ; R_1, R_2, the radii of curvature of the two surfaces at O (Fig. 239).

Then, as OM is a tangent,

$$OM^2 = MP \,.\, MQ,$$
$$x^2 = e_1 \,.\, 2R_1 \text{ (nearly)},$$

since MQ is nearly equal to the diameter of the sphere.

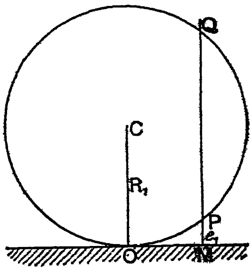


FIG. 239.

Or, $$e_1 = \frac{x^2}{2R_1}.$$

So $$e_2 = \frac{x^2}{2R_2};$$

$$\therefore\ e \equiv e_1 + e_2 = \frac{x^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \quad \ldots\ldots(2)$$

226. To calculate $\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ from the spherometer readings.

When the spherometer (Fig. 242) stands on a sphere, the three legs lie on a small circle PS of radius ρ, and the central point touches the sphere at the pole of this circle, O (Fig. 240).

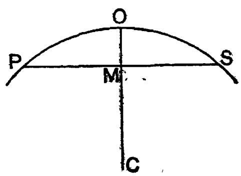


FIG. 240.

Let M be the centre of this circle and let OM $= h$ (h is the *reading* of the spherometer). Then as PS is a chord, we have

$$PM \,.\, MS = OM(2R - OM),$$

or $$PM^2 = \rho^2 = h \,.\, 2R - h^2$$
$$= h \,.\, 2R \text{ (nearly)}$$

(as in our case the curvature is very small); therefore

$$\frac{1}{R} = \frac{2h}{\rho^2}.$$

To get ρ we may either measure the distance, a, between two of the legs, PQ, of the spherometer, or we may measure it directly by screwing the central point down to the plane PQR, and then measuring OP (Fig. 241).

If we use the former,

$$\rho=\frac{2}{3}a \cdot \cos 30^\circ$$

$$=\frac{2}{3}a \cdot \frac{\sqrt{3}}{2},$$

$$\rho=\frac{a}{\sqrt{3}}$$

and

$$\frac{1}{R}=\frac{6h}{a^2}.$$

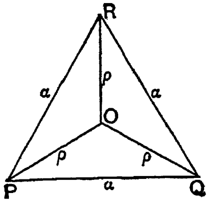


FIG. 241.

Thus
$$\frac{1}{R_1}+\frac{1}{R_2}=\frac{2}{\rho^2}(h_1+h_2)\ ; \qquad (3)$$

or
$$=\frac{6}{a^2}(h_1+h_2).$$

227. Then, if we take the dark rings,

$$\lambda=\frac{2e}{n} \cdot \cos i \qquad \text{by (1)}$$

$$=\frac{2 \cdot \cos i}{n} \cdot \frac{x^2}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \qquad \text{by (2)}$$

$$=\frac{2 \cdot \cos i}{n} \cdot \frac{x^2}{2} \cdot \frac{2}{\rho^2}(h_1+h_2) \qquad \text{by (3)}$$

$$=\frac{2 \cdot \cos i}{\rho^2} \cdot \frac{x^2}{n}(h_1+h_2) \qquad (4)$$

Newton's Rings by Transmission.

228. *Apparatus.*—A plano-convex lens of about 10 inch focus; a flat glass surface (one face of a prism or cube, or a piece of plate glass, *not* a mirror); a reading microscope and sodium flame; red wax; silvered mirror; blocks; scale; spherometer.

Make two small pellets of red wax about the size of small shot. Clean the surfaces of the lens and plate. Holding them parallel to one another with the curved surface of the lens facing the plate, blow smartly between them. Wait a moment for any condensed moisture to evaporate, put them together and place them on the table. Push the two pellets in between the lens

and plate until they just touch both, forming a small equilateral triangle with the point of contact of the lens and plate. Press the lens down gently upon the pellets. This will cause them to adhere and the lens will now be firmly held, by its weight and the two pellets, upon the point of contact. The rings may be viewed either by reflection or transmission. If by transmission, the system must be mounted on blocks and a mirror placed so as to reflect the light from the lamp vertically upwards through the lens. The reading microscope must then be pointed vertically down and adjusted so that it is focussed upon the system of rings. As the curvature of the lens is considerable, the system of rings will be very minute, and therefore rather difficult to find with the microscope. It can easily be seen with the naked eye as a little black speck if the microscope is moved aside, and a small triangle of paper is laid on the lens with one corner pointing at the rings. If the microscope is focussed upon this corner of the triangle, and then slightly lowered, the rings will come into view.

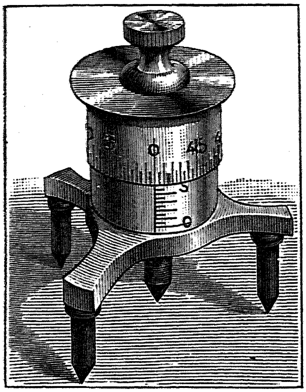


FIG. 242.—Spherometer.

When the rings are found they appear slightly hazy, and will now and then be seen suddenly to alter their position, and it is impossible to take good readings. This is due to the fact that the size and position of the rings depend upon the angle of incidence of the light. As the sodium flame has considerable area, if the rings are allowed to be lighted by the whole of this area, it is obvious that the angle of incidence is not definite, and the accidental variations in the brightness in different parts of the flame will determine the apparent position of the rings. It is therefore necessary to limit the extent of the source of light. This can be easily done, if a hole about $\frac{1}{4}$ inch in diameter is cut in a piece of zinc, and after the rings have been found with the microscope, this is inserted near the flame between it and the

mirror. By moving the diaphragm about, its best position can be determined. The rings will now appear perfectly sharp and steady. It is an advantage if the glass plate and reading microscope are mounted on a single slate slab isolated from the floor to avoid vibration, but this is not absolutely necessary. The readings must be taken from left to right only or from right to left only, to avoid the back lash of the screw.

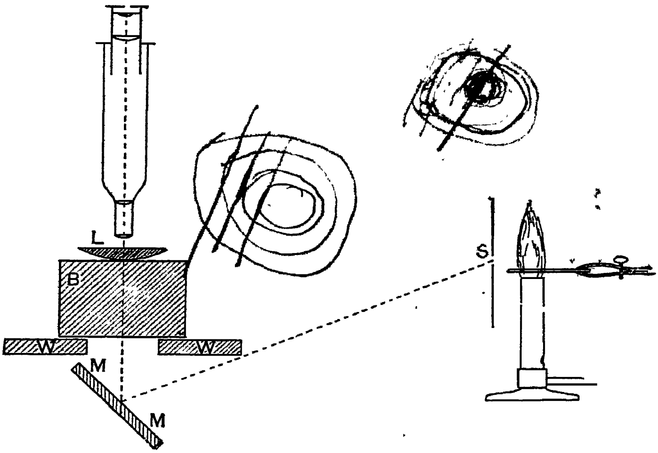


FIG. 243.—Newton's Rings by Transmitted Light.
L, lens; B, glass block; W, W, support; M, M, silvered mirror; S, diaphragm.

The positions of the dark part of the rings can be entered in the following table, beginning with the tenth ring on the left, then reading the 9th, 8th ... in succession across the centre to the ... 8th, 9th, 10th rings on the right. These entries will thus be made in the second column of the following table from bottom to top, and in the third column from top to bottom.

No. of ring = n.	Left.	Right.	Diameter.	(Diam.)^2.	$\frac{(\text{Diam.})^2}{n}$.
0.					
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10.					

The last column should be constant. One quarter of its mean value is to be substituted for the $\frac{x^2}{n}$ in the formula above.

Newton's Rings by Reflection.

229. *Apparatus.*—As in Exercise 228. To adjust the rings for reflected light, it is necessary to be able to tilt the reading microscope.

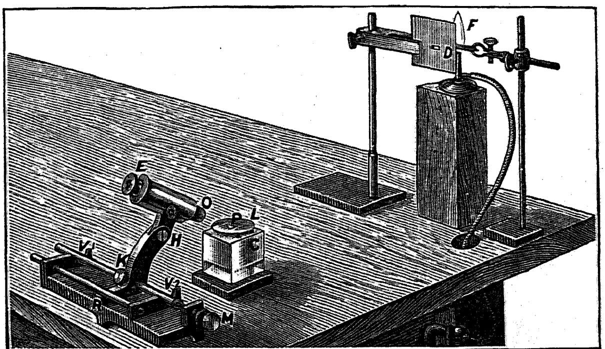


FIG. 244.—Newton's Rings by Reflection.

L, lens; G, glass block; E, eye-piece, and O, objective of reading microscope; M, divided head of microscope; V^1, V^2, slots in which the rods carrying the microscope rests; K, clamping screw; the microscope is hinged at H to its support, and the support rests at R against the under-side of a fixed rod; F, flame; D, diaphragm.

Having attached the lens to the glass as already described, mount it horizontally on a block in such a position that the surface can be focussed with the reading microscope (this can be tested by placing a small piece of paper on the glass). Arrange a pointer opposite the eye-piece of the microscope. Remove the microscope without disturbing this pointer. Adjust the sodium flame, distant about 18 inches, so that its reflected image is seen by an eye placed at the tip of the pointer. In doing this, it will be found that at least three images of the flame are visible, one from the upper plane surface of the lens, one from the lower convex surface, and one from the plate glass beneath. (It is an advantage if the lens is tilted sufficiently to throw the first of

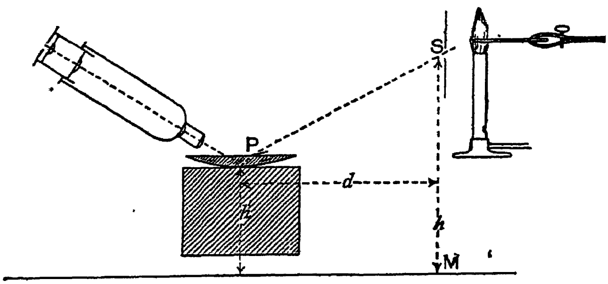


FIG. 245.—Newton's Rings by Reflected Light.

these, which is much larger than either of the others, to one side, as we shall then only be dealing with the light reflected from the interfering surfaces, and shall be able to get perfectly black fringes.) The image from the convex surface of the lens will not coincide with that from the glass plate, unless the reflection takes place at the point of contact of the two surfaces, for only then is the tangent plane to the surface of the lens parallel to the plate. The flame must, therefore, be adjusted until these two images appear to coincide as seen from the pointer. When this is the case the small black dot, which represents the Newton's rings system will be just visible. Place one corner of a small paper triangle, P, to appear to coincide with this spot as seen from the pointer. Replace the microscope, and focus it upon the corner of the paper triangle, when with a slight alteration of focus, the rings will be visible and a very small movement of the glass

plate will bring them into the centre of the field. The flame should be now slowly moved until the illumination is as powerful as possible, and finally the stop in the zinc plate must be inserted in front of the flame, and adjusted to produce the best possible effect, as described above for the transmitted system. The readings will be taken and tabulated as before. This time the angle of incidence will be wanted. It is best found from its *tangent*. Measure the height, h, of the slit above the table, and also h', that of the system of rings. Measure also the horizontal distance, d, from the system to the slit. Then

$$\tan i = \frac{d}{h-h'}.$$

Mica Films or Selenite.

230. *Apparatus.*—A thin film of mica or selenite, which should be as flat as possible, mounted on a table capable of rotation, for instance the table of a spectrometer; the film may be semi-silvered on each side; a semi-silvered mirror, which must be supported at an angle of 45° in front of the film.

The light from a Bunsen burner is to be reflected from the mirror normally on the film, from which it will be reflected back and received by the eye placed behind a small hole, or by the low-power telescope above described focussed at infinity. If the film is normal, the interference fringes form a series of concentric circles whose centre will be in the centre of the field. If the film be now slowly rotated, the fringes will cross the field. By counting the number which pass the needle point while the film is rotated through a given angle i, the optical thickness of the film may be found.

For let e be the thickness of the film. The difference of path between the light reflected from the front surface of the film and that which passes through the film, making an angle r with the normal and reflected at the back surface, is given by a formula differing from that in § 224 by the introduction of the refractive index, viz.:

$$\begin{aligned}\delta &= 2\mu e \cdot \cos r + \frac{\lambda}{2} \\ &= (p+\tfrac{1}{2})\lambda \quad \text{(say)}.\end{aligned}$$

If the incidence be normal this becomes

$$\delta_0 = 2\mu e + \frac{\lambda}{2}$$

$$= (p_0 + \tfrac{1}{2}) \cdot \lambda.$$

Thus $$\delta_0 - \delta = (p_0 - p) \cdot \lambda = 2\mu e(1 - \cos r),$$

where $p_0 - p$ is the number of fringes that has passed the cross-wire during the rotation from the normal, and r is given by

$$\mu \sin r = \sin i.$$

Substituting for r, the optical thickness, μe, can be calculated.

N.B.—Both mica and selenite are doubly refracting, but the difference of refractive index in a direction normal to the natural cleavage does not greatly matter for purposes of ratio.

Determination of Wave-Length by the Interference of Thin Films.

The same experiment may be formed in a slightly different way, by which the wave-length of the light of any colour can be found.

231. *Apparatus.*—Spectrometer; prism; thin film of selenite, or better, the double plates of § 243; white light; semi-silvered mirror (page 323).

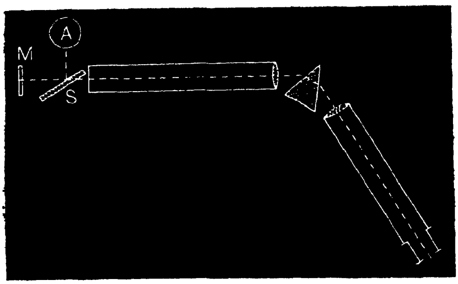


FIG. 246.—Interference of Thin Film.

The spectrometer is set up with a prism and the spectrum obtained as usual. A white light A is then placed to one side of the collimator, and reflected by a semi-silvered mirror S on to a double plate or selenite M, placed normal to the collimator,

from which it is reflected back into the collimator. If now the double plate be slowly rotated through a known angle, fringes will be seen to travel along the spectrum. (The mirror S must be rotated sufficiently to keep the light in the field.) By setting the cross-wire on any bright line in the spectrum, and counting the fringes that pass that point, counting from the time that the mica was normal, the wave-length for that colour can be found.

If it is only desired to observe the interference fringes and not to measure the wave-length, they may be seen by reflecting the light directly from a mica film into the collimator and observing the spectrum, or by placing a film in front of the eye-piece of the telescope at an angle of 45° and observing the spectrum by reflection in that.

The fringes may also be obtained by reflecting white light by a soap film at an angle of about 45° down the collimator and observing the spectrum. The fringes in this case are very broad, and may each extend nearly half way along the visible spectrum. As the film gets thinner they travel rapidly along the spectrum towards the violet, and look like shadows passing over the spectrum. As the film is thicker at the bottom, the shadows lie obliquely across the spectrum.

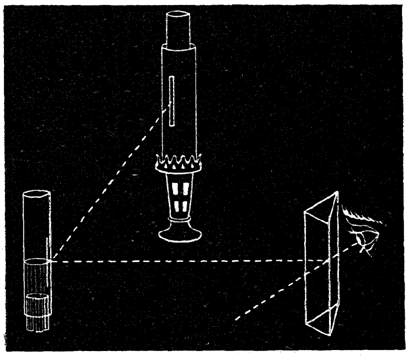


FIG. 247.

232. A still simpler way of seeing the interference fringes caused by the reflection at the front and back of a mica film is to roll the film round a cylinder, and observe with a prism the image of a bright light, reflected by the mica cylinder so formed. (It will appear to the naked eye as a bright line.) In a prism of which the edge is parallel to the generating lines of the cylinder, a spectrum will be seen crossed with a large number of dark bands. These are due to the fact that the image of the light is really a double line, the two lines being separated from

one another by a very small interval, and since they are images of the same source of light, they consist everywhere of corresponding points. The difference of path is, as before,

$$\frac{2\mu e}{\cos r}+\frac{\lambda}{2}.$$

Tie this mica cylinder round a cork, and make it water tight by shellac-varnish, or anything similar. Half fill it with liquid of greater refractive index than the mica and observe the spectrum. It will be found that the dark and bright parts of the spectra, formed by the upper and lower halves of the tube, are interchanged. If the prism is held in such a position that the two spectra are in the field at the same time, the dark parts of the one will be found to correspond throughout with the bright parts of the other. This is because, if the refractive index of the liquid is higher than that of the mica, both reflections are from a rarer to a denser medium, and thus the difference of path is only $\frac{2\mu e}{\cos r}$, without the $\frac{\lambda}{2}$.

To Scale the Spectrum, using the Colours of Thin Plates.

233. *Apparatus.*—Spectroscope, as before; two plates of optical glass, each semi-silvered; they need only be about an inch in diameter; some red wax, or plasticene. (The plates of § 243 will of course do.)

The colours of thin plates are produced by the removal of certain colours in the spectrum, which depend upon the distance apart of the plates. As the distance is increased, more and more bands of colour in the spectrum will be removed, with the result that using white light all trace of colour will disappear, for the colours removed being distributed throughout the spectrum, and the colours left also distributed throughout the spectrum, the light transmitted appears to the naked eye identical with white light. Examined with a spectroscope, however, the removal of the colours is still perfectly evident. When the plates are very close, one or more large bands of colour are removed from some region or regions of the spectrum. But as the distance is

increased, the width of the bands diminishes, and their number increases; with a separation such as is produced by the thickness of a thread of cotton, a very large number of narrow bands are seen extending from one end of the spectrum to the other. The effect is very much more marked, if the two surfaces of the glass that are placed together are semi-silvered, as then the bands are almost black. Messrs. Edser & Butler have suggested the use of this for "scaling" the spectrum produced by a prism.

Owing to anomalous dispersion, the prism spectra formed by different varieties of glass do not agree with one another, even if the total dispersion is the same. The positions of the dark bands produced by the colours of thin plates depend, however, upon the wave-length only. So that if two Fraunhofer lines in the spectrum are known, and the space between them is divided up by these interference lines, the scale so obtained is independent of the glass of the prism.

Let two ordinary pieces of plate glass be semi-silvered, and let them be placed together with the semi-silvered surfaces inwards and separated only by three or four minute pellets of red wax or plasticene at the edges. Look at the reflection of a distant incandescent filament, and by pressing the surfaces, adjust them until the images seem to coincide. Then view from a great distance, or with a telescope, the fringes formed by reflected sodium light. These fringes are formed at a distance in front of the plates, when the adjustment is nearly complete, and therefore can only be seen at a distance. They must be made as broad as possible, and their direction noted.

The double plate so formed is placed between the right angle "comparison" prism of the spectrometer and a source of white light, with the direction of the broad fringes that still remained vertical. A great number of dark interference lines will now be found in the spectrum. The number of the interference lines can be increased or diminished to anything that is desired, by altering the distance apart of the silvered surfaces.

At the same time observe a hydrogen tube by direct light, and count the number of fringes between the red and blue lines. This will completely determine the whole scale of the instrument.

234. If d is the distance between the reflecting surfaces, λ_0 the wave-length of a dark band produced by these plates, λ_1 that of the next band, λ_2 that of the third band, and so on, we have

$$2d=n\lambda_0=(n+1)\lambda_1=(n+2)\lambda_2=\dots$$
$$=(n+r)\lambda_r=\dots=(n+s)\lambda_s.$$

If λ_0 and λ are known, *e.g.* the wave-lengths of the red and blue hydrogen lines, n can be found from the equation

$$n=\frac{s\lambda_s}{\lambda_0-\lambda},$$

λ_s being the sth band towards the violet from λ_0; then if λ_r is some unknown wave-length, it is given by $\lambda_r=\frac{n\lambda_0}{n+r}$.

This method of sealing the spectrum is particularly useful in photographing absorption and other spectra, as by using a comparison prism the absorption spectrum may be adjusted alongside the fringes formed with this double plate, and the wave-length thus automatically registered on the plate in a single operation, it being only necessary to mark with scratches the situations of any two known wave-lengths.

The Refractive Index of a Liquid by Newton's Rings.

235. In the system of Newton's rings formed by reflection, a dark ring is produced at such a distance from the centre that the length of path between the two surfaces between which the system is produced, is 1, 2, 3, ... wave-lengths. If the space between the plates is filled with a medium other than air, in which therefore the wave-length will be shorter, it is obvious that the rings will be smaller; and as the refractive index is the ratio of the wave-length in air to that in the liquid, by determining the wave-length in the liquid we can find its refractive index. As the intensity of the light reflected from the surfaces is very much less when the space between is filled with a liquid whose refractive index is near that of glass, the rings are not so easily seen as they are when formed with air between the surfaces, and it is almost impossible to see them at all in transmitted light. If reflected light is used, the rings formed by the reflections from these two surfaces will be alternately bright and black, but will be very

faint. If, therefore, there is no scattered light, and no light from any other surface reflected up the tube of the microscope at the same time, the rings will be visible though faint. But if the light reflected from the upper surface of the lens is also allowed to reach the eye, they will become invisible. As already pointed out, this light can be eliminated by sufficiently tilting the lens, so that the light reflected from the upper surface shall be oblique. The best way to perform the experiment is to find the rings with air first, taking care to tilt the lens, and then, without disturbing any of the apparatus, carefully insert a small quantity of the liquid to be examined. This will, by capillary attraction, be drawn into the centre and the reduced system of rings will be still visible.

In order to find the refractive index, it is merely necessary to know the ratio of the diameters of the rings in the two systems. The mean value of $\frac{D^2}{n}$ should be found, as directed in § 228, both for the system in air and for the system in the liquid; the refractive index will be the ratio of these two means.

The Colours of Thick Plates, Brewster's Bands.

236. *Apparatus.*—Cut a piece of worked parallel glass into two, or cut from a piece of plate glass, selected by the method described on page 38, two circles about $1\frac{1}{4}''$ in diameter. If the piece of plate glass is not quite parallel, if for instance it is thicker at the top than at the bottom, the two discs should be cut out side by side on the same horizontal line, so that their thickness may be the same. Then, although they may be both wedge-shaped, it will still be possible to obtain the fringes. In this case an arrow should be scratched on each disc indicating the direction of the wedge.

Experiment.—Clean these two circles and place them together. On looking at the surfaces by reflected light, the bright Newton's bands should be visible; in addition one or two systems of narrower straight bands going obliquely across the field are usually visible.[1] On rotating the pieces of glass over one another,

[1] An ophthalmoscopic mirror (*i.e.* a concave mirror with a small hole in the middle) is very useful to reflect the light from a lamp down on to the plates, which are then observed through the hole in the mirror.

these two oblique systems will generally rotate, but not at the same rate, and they will also alter in width. They are affected very much less by an alteration in the distance apart of the plates than the Newton's rings. In fact, if the plates be slowly separated at one edge the Newton's rings will disappear long before the others. These fringes may also be seen by transmitted light, using a distant flame.

They are more brilliant if the plates are placed in a tube and the light admitted through a small hole at one end, the eye being placed at the other. If the glasses can be kept parallel to one another the fringes are equally well seen when the plates are some distance apart. If mounted in a tube the one glass should be as nearly as possible perpendicular to the axis of the tube, and the other glass mounted perpendicular to the axis of an inner tube which can be rotated in the outer one. By tilting the glass in the outer tube by a micrometer screw, the effect on the fringes can be better observed than by rotation only. The heads of the screws in this case should be outside the tube, to enable them to be adjusted while the fringes are actually being observed. If sodium light is used the fringes will extend right across the plate; with white light only a few are visible. It will be seen that the fringes are formed along lines where the difference of path is a constant, that is, if the glasses are mounted in contact with or parallel to one another, along lines parallel to the bisector of the angle between the two scratches on the wedges.

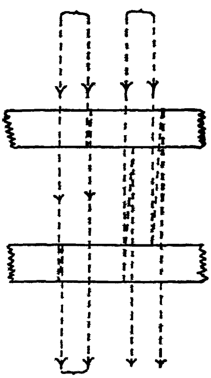


FIG. 248.

If the surfaces are a long way apart only these two systems are seen.

The fringes are formed as indicated in Fig. 248. It will be seen that they may be formed in at least two independent ways and that the differences of path by either method will be small. If the two pieces of glass are not in contact and not quite parallel these two methods will give rise to two independent sets of fringes. When the glasses are in contact, Newton's rings will be formed, and also one other system of

fringes is possible, as shown in Fig. 249. Thus, in this case, there are four systems of fringes. Semi-silvered glass would not improve the fringes, unless all four surfaces were very lightly silvered, for, as an inspection of the figures shows, reflections take place at each of the four surfaces.

If the fringes are viewed by somewhat oblique reflected light, it is possible with a sodium flame screened by a card in which a slit about $\frac{1}{8}''$ wide has been cut, to obtain the systems independent of one another and free from other light. In this case their appearance is very interesting.

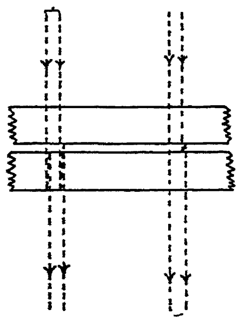


FIG. 249.

Examination of a Sheet of nearly Parallel Glass by the Fringes of Thick Plates.

237. Milner (*Phil. Mag.*, Sept. 1908) showed that these fringes can be easily used to examine a sheet of nearly parallel glass. Let AB and CD be two pieces of glass of nearly equal thickness (*e.g.* two pieces cut from one sheet of plate glass), let sodium light fall on them as shown at an angle of about 45°, and put a screen S to shade the further portion of the glass from the directly reflected light. Several systems of fringes are formed. Any fringe represents the loci of points at which the thickness of one plate is a constant amount greater than that of the other, hence the fringes do not move across the plates as they are pressed together or separated slightly.

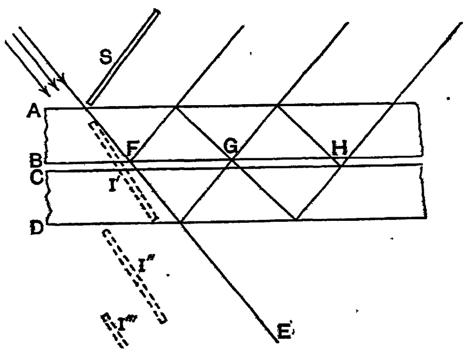


FIG. 250.

Consider the interference of any two rays ABCDE and AGDF, each of which passes through the point D and enters an eye at EF.

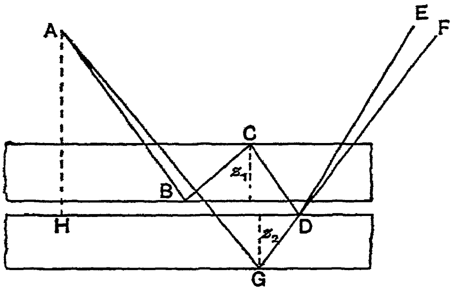


FIG. 251.

Let r and $r+\delta r$ be the angles which they make in the glass with the normals to the plates (the bending by the glass is omitted in the figures for the sake of simplicity, and HAB, HAG are in reality the angles of refraction in the glass). Let Z_1 and Z_2 be the thicknesses of the glasses at C and G respectively. Then the relative retardation at D will be

$$\Delta=\frac{2Z_1+AH}{\cos r}-\frac{2Z_2+AH}{\cos(r+\delta r)}$$

$$=\frac{2(Z_1-Z_2)}{\cos r}-\frac{2Z_2+AH}{\cos^2 r}\sin r\,.\,\delta r.$$

But from the figure,

$$HD=(2Z_1+AH)\tan r=(2Z_2+AH)\tan(r+\delta r),$$

giving $$\frac{2Z_2+AH}{\cos^2 r}\,.\,\sin r\,\delta r=\frac{2(Z_1-Z_2)}{\cos r}\,.\,\sin^2 r\,\delta r.$$

Consequently, $$\Delta=2(Z_1-Z_2)\cos r. \qquad \text{(i)}$$

As Δ is independent of the position of A, the fringes will be seen at the air film whatever be the size of the light. But if the eye is moved towards the normal so as to make the angle r less, the equation shows that the same retardation Δ will be produced by a smaller difference in the thickness of the plates (*i.e.* of Z_1-Z_2); thus the fringes move towards the central one of the system. Thus the **position of the central fringe** (for which $Z_1-Z_2=0$) can be easily located.

Over a small area each plate may be looked upon as a wedge of very small angle. Let Fig. 252 represent the plan of the two wedges superposed; and let the outline be supposed so drawn that the thin ends AB and A′B′, and the thick ends CD and C′D′ each have equal thickness. Produce these ends to meet at A and

C respectively. Then the line AC is a line of equal thickness in the two plates, it represents the central fringe of the system, and the other fringes are parallel to it. If AB and A′B′ are parallel, and the wedges slope the same way, the fringes will be broadest and parallel to AB. If one wedge is rotated over the other, the fringes also rotate, and the fringes become parallel to AB once more when the plate has been rotated through 180°, but they are now very fine.

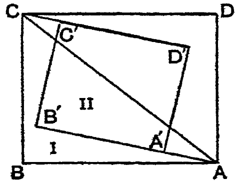


FIG. 252.

238. If either plate is displaced parallel to its contour line the fringes will, of course, not move. In this way the **contour lines may be easily mapped out.** If, for instance, a small plate, cut from a large sheet, is placed on the latter, and lighted by a sodium flame at an angle of about 45°, and shaded by a card, the fringes can usually be seen. They may be too fine; if so, they will probably appear on rotating the little plate. Then fix attention on a particular fringe, and move the small plate by trial in such a direction that the fringe remains apparently fixed to it; a contour line of the large sheet will be traced out.

To find the **angle of the wedge** of the large sheet at any point, move the small piece at right angles to the contour line and count the number of fringes that pass a fixed point on the small plate for a given displacement. The alteration in thickness is given by $\frac{\lambda}{2 \cos r}$ for each fringe.

To obtain the **fringes in white light** the central fringe must be found as above directed, using sodium light, and then the white light be substituted. If the plates are now in contact, and then one is slightly tilted perpendicular to the plane of incidence, the one system will break up into several, which are produced by to and fro reflections at the air film. If the plates are now slightly tilted in the plane of incidence, each of these will split into two. It is possible to have a large number of sets of fringes visible at one time.

Newton's Diffusion Rings.

239. *Apparatus.*—Either an arc lamp, or a good white light, or a sodium flame enclosed in a light tight box in a dark room; a white card a foot square, and a sheet of zinc of the same size; a large concave mirror of 10″ to 20″ focus and 6″ diameter of glass, silvered at the back (a "shaving mirror").

The rings are best observed with a powerful source of light such as that from an arc lamp or a lime light. But it is possible to see them with an ordinary white light or sodium flame, if the experiment is conducted in a dark room and the sodium flame itself is enclosed. A small hole about $\frac{1}{8}''$ in diameter is made in the centre of a sheet of white card about a foot square. (If an arc light is used the hole should be protected by a piece of zinc on the back of the card with a similar hole in it.) The large concave mirror is set up at such a distance that this hole shall be at its centre of curvature, and the light placed behind the card in the case of the sodium flame, or concentrated upon the hole with the ordinary lantern condenser if the arc is used. An image of the hole will now be formed on the card. By tilting the mirror slightly this image can be produced at about an inch from the hole. If now the mirror be lightly breathed upon, the rings will be seen. With an arc light they are very brilliant and can be seen from a considerable distance. As the moisture evaporates the rings fade away, and if the surface of the mirror is perfectly clean, and free from scratches or dust, disappear almost entirely. When the rings are formed with white light, there will be one white ring, of which the diameter is the line joining the centre of hole in the card and that of its image. This will be bordered both inside and outside by a number of coloured rings. The rings are produced by the interference of the rays scattered at the surface.

One ray starting from the hole in the card is scattered at a point P, say, on the mirror, and after reflection at the silvered surface at the back of the mirror at R and regularly refracted at S reaches the screen at Q. Another ray starting from the centre is first refracted regularly, then reflected at the back at N, and after being scattered at the same point P, will also arrive

at Q. As these two rays are scattered from the same point they will be approximately equally powerful, and their difference of path will be small. They will be in a condition to interfere. Q is in this case not the image of the centre O, it will only be the image of O when the light is regularly refracted at P instead of scattered there.

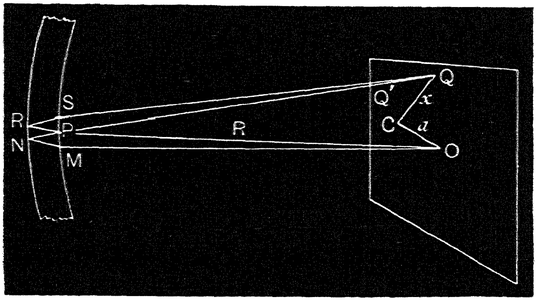


FIG. 253.

240. It is proved in Preston's *Light*, p. 206, that if R be the radius of curvature of the mirror, and $OC=a$, $CQ=x$, where C is the point half-way between O and its image O′ (and therefore the centre of curvature of the mirror).

The difference of the two paths is

$$\frac{d}{\mu R^2}(x^2 \sim a^2).$$

If this is a whole number of wave-lengths, the point Q will be bright, that is, if

$$\frac{d}{\mu R^2}(x^2 \sim a^2) = n\lambda,$$

$\frac{d}{\mu}$ is the apparent thickness t of the mirror, which can be determined by the microscope, as in determining the refractive index of a glass plate in § 25. If the silvered surface is so perfect that no spot can be seen upon it, the image of a particle of dust on the surface may be used, the distance from the actual particle to its image being taken to be twice the apparent thickness of the glass. Then R can be measured directly and a series of readings for x taken with a pair of compasses, using a sodium flame as the source of light.

Enter the readings thus:

Number of Ring.	Diameter.	The Square of the Diameter $=(2x)^2$.	$(2x)^2 \backsim (2a)^2$.	$\dfrac{(2x)^2 \backsim (2a)^2}{n}$.
Inner 0				
1				
2				
3				
4				
Outer 1				
2				
3				
4				

The readings of both the internal and external rings should be taken, numbering them from the ring which passes through the object and image, the one next to this both inside and outside being called *one*. The mean value of the last column must be multiplied by $\dfrac{t}{4R^2}$ to give λ. If the mirror is tilted so that the object and image approach one another, the internal system of rings grows smaller; and they vanish into their centre successively. When the object and image coincide, only the outer system remains.

Optical Thickness of a Plate of Glass.

241. Let AB be a glass plate supposed to be of uniform optical thickness, *i.e.* the time taken by a wave of light to go normally through it should be constant at all parts of the surface.

Place the plate at the bottom of a wooden box. Let the light from some monochromatic source S (the green mercury line) enter the box by a small hole; and be reflected by a semi-silvered mirror CD down on to the glass AB. Then it will be returned from both the upper and the lower surfaces, and after passing through the semi-transparent mirror CD, will reach the eye E. Thus interference fringes should be seen. The eye may be assisted by a low-power telescope focussed on the plate, and furnished with a cross-wire in the eye-piece. If the plate is optically uniform and

is normal to the line of sight, a series of circular fringes should be seen. If the plate is well made it will probably give nearly circular rings even if it is not perfectly uniform. To see if it varies slide the plate about in its own plane, the rings will probably move also. The number of rings that pass over the cross-wire during any movement of the plate, gives the number of half-wave lengths by which the optical thickness has changed. By so moving the plate that no fringes pass the cross-wire, contour lines can be drawn on the plate showing the loci of equal thickness. Then, if necessary, the plate may be polished again, rubbing down the thicker parts, and so an even thickness obtained. Lord Rayleigh found that it is possible to reduce the thickness by painting the surface with weak hydrofluoric acid and that the surface remained optically good.

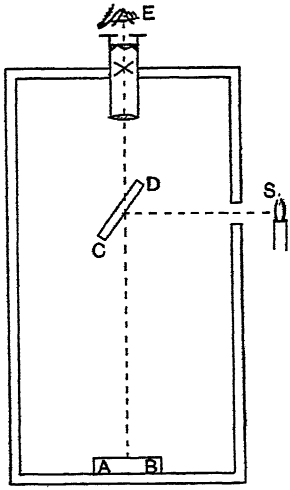


FIG. 254.

ADDITIONAL EXERCISES ON CHAPTER XIII

1. Obtain the reflected system of rings, under the microscope (Fig. 244). Allow a drop of water to flow into the space between the lens and the glass block. It will be drawn in by capillary attraction. The rings will nearly disappear, but can still just be seen—especially if the light reflected from the upper surface of the lens has been eliminated by tilting the lens so as to reflect it to one side. The system will be so faint, that if once lost, it is practically impossible to recover it.

Note the reduced size of the rings, showing that the wave-length of light is less in water than in air. This experiment cannot be explained on the Emission Theory.

2. Place an ordinary plano-convex lens on a face of a highly refractive glass prism or plate. Focus the rings by reflection, under the microscope. The system should have a black centre. Without

disturbing the apparatus, allow a single drop of oil of cedar to flow between the surfaces. The rings should now have a white centre, as the refractive index of cedar oil is intermediate between that of ordinary crown glass, and the highly dispersive glass prism.

3. The adjacent surfaces of the concave and convex components of an ordinary achromatic lens have a nearly equal curvature. Should the curvature of the convex surface be slightly greater than that of the concave surface, Newton's rings will be formed when they are placed in contact; and the system will have a black centre when seen by reflected light. Allow a drop of cedar oil to flow between the lenses; the centre will become white.

4. **Newton's Diffusion Circles.**—These circles can also be produced with a plane mirror silvered at the back, and a lens.

Place the lens at its focal length from the hole in the screen, and the mirror behind it; the light after reflection at the mirror will be returned and refocussed on the aperture. Turn the mirror slightly, the image will be formed to one side of the aperture. Then by breathing upon the front surface of the mirror so as to produce a scattering at that surface, the rings will—as in the case of the concave mirror—be seen. The lens should be achromatic if white light is to be used.

CHAPTER XIV

INTERFEROMETERS

An Interferometer may be described as an instrument in which interference fringes are produced by two beams of light which have travelled along paths sufficiently distinct for the one path to be capable of variation while the other is unaffected.

Rayleigh's Interferometer.

242. The simplest method of producing interference under these conditions is Rayleigh's, in which light from a slit is parallel-

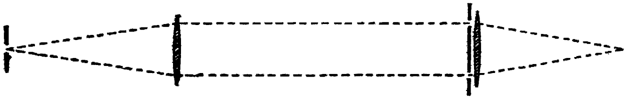


Fig. 255.—Simple Interferometer.

ised by a collimator and viewed by a telescope of 3 cm. aperture. In front of the objective is a screen with two apertures. In order to get good magnification with the least loss of brightness he used a glass rod, 4 mm. diameter, as an eye-piece. He used it to compare the refraction indices of gases. The gas to be experimented upon was enclosed in a tube 20 cm. long, 6 mm. bore, closed by worked glass ends, which were large enough to cut both beams. The pressure of the gas in the tube was varied until the systems (one through the tube and one above it) were coincident.

Then $\frac{p_1}{p_2}=\frac{\mu_2-1}{\mu_1-1}$ when $p_1 p_2$ and $\mu_1 \mu_2$ were the pressures and refractive indices of the gases.

Fabry and Perot's Fringes. Interference of Parallel Plates.

243. *Apparatus.*—The following is a simple method of mounting the mirrors by which these fringes may be formed: Obtain two pieces of glass optically flat on one face, and about two or two and a half inches square (two pieces of ordinary plate glass can be selected, giving *straight* and parallel "Newton rings"). The front and back surfaces of the plates should not be quite parallel, but should make an angle of $1'$ or $2'$ with one another. On adjacent corners of the best side of one, cement with Canada balsam three pieces of cover-glass (cut off the same piece so as to be as nearly as possible of equal thickness), each about three-sixteenths of an inch square. When dry, semi-silver the best surface of each glass, and place the second glass on the first so that the best and silvered surfaces are adjacent, but separated by the pieces of cover-glass.

Now examine by oblique incidence the images of a distant lighted object (*i.e.* the opal shade over a good light). If the silvered surfaces are not perfectly parallel, there will be a series of images as on page 36. Glue or pin a sheet of No. 1 or No. 0 emery cloth on a flat board, wet it with turpentine, and carefully grind down whichever of the bits of cover-glass is too thick. This is easily done by placing the glass on the emery cloth so that it rests on the three pieces of glass, and then pressing gently on the one to be ground down while moving the plate about in circles of two or three inches diameter, going round these say once a second. The grinding is quite rapid, and the plate must be repeatedly rinsed, dried, and the images examined as above to see when parallelism is obtained. If the silvering has been damaged in the grinding, it should be cleaned off (with a few drops of nitric acid) and renewed, but if care has been taken this should not be necessary. Now place one drop of Canada balsam dissolved in xylol or benzol on each of the three corners and cement the plates together, applying just enough pressure to bring the surfaces parallel, as tested by the images coinciding. To get perfect coincidence towards the end of the grinding and in the final mounting, a telescope focussed on the image may be employed with advantage. Or the fringes formed with sodium light by transmission may be observed. The fringes should be observed from a distance as they are formed at a distance in front of or behind the plates (see § 254). It will be found that they are very fine and narrow; if the plates are not parallel, they

become broader as parallelism is approached, and finally circular fringes are obtained. The circles should be central when the light is coming through normally.[1]

The pair of plates constructed as above are now to be mounted on a stand similar to that used for the diffraction grating (Fig. 256),

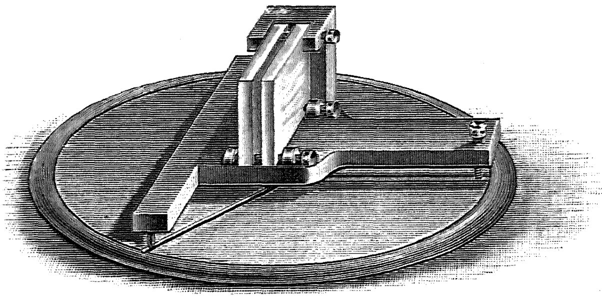


FIG. 256.—Mounting for Fabry and Perot's Mirrors.

and stood on the table of the spectrometer. The collimator and telescope will not be used, and they must be turned aside or removed. A sodium flame; three convex spectacle lenses about 10, 8, and 6 diopters, mounted on supports that will allow them to be set up in a line with the middle of the glass plates; and a needle-point to serve as a "cross-wire" will also be required.

244. The plates P are to be set up on the table of the spectrometer in a vertical plane. The 8 and 10 diopter lenses, A and B, are to be set up in a line normal to the plates, to form a very low-power telescope and focussed at infinity. The 6 diopter lens C is to be set up in the same line on the other side of the plates, and the sodium flame put at its principal focus, so that approximately parallel light shall fall on the plates, and after transmission through them shall be received by the telescope. If all is in good adjustment, a system of concentric circular fringes should be seen in the telescope. If the centre of the system is not in the centre of the field, the plates must be turned round or inclined forwards or backwards

[1] Good results can also be obtained with plates separated at the *four* corners with pellets of plasticene. By pressing on the edges while watching the fringes as above, it is quite possible to set the mirrors parallel to one another.

to get it there. Lastly, the needle-point Q is to be adjusted between the two lenses forming the telescope to coincide with the central fringe.

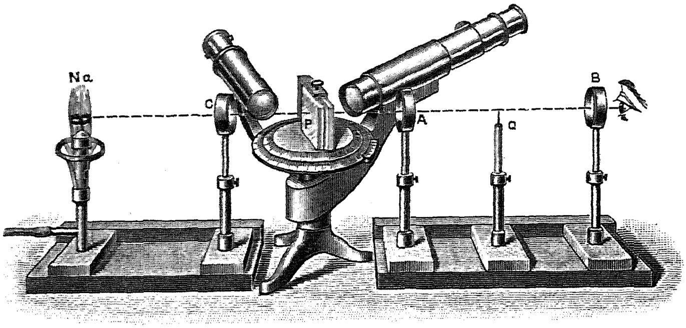


FIG. 257. – Fabry and Perot's Fringes.

By counting the number of fringes n which pass the needle-point, while the table with the plates on it is rotated through an angle i, we can find the optical distance between the silvered surfaces. For let e be the distance between the surfaces. The difference of path between the light reflected from the two surfaces is given by

$$\delta = 2e \cos i = p\lambda \quad \text{(say)}.$$

When the incidence is normal, this becomes

$$\delta_0 = 2e = p_0\lambda \quad \text{(say)}.$$

Thus $$\delta_0 - \delta = (p_0 - p)\lambda = 2e(1 - \cos i),$$

where $p_0 - p$ is the number n of fringes that have passed the cross-wire during the rotation i from the normal. As it is unlikely that p_0 would be a whole number, it is better to get the first ring (at an angle i_1) on the cross-wire, and count on from there to the angle i. The formula then is

$$\delta - \delta_1 = 2e(\cos i - \cos i_1) = n_1\lambda.$$

Thus if this number of fringes is known, and λi_1 and i are known, e can be found, and p calculated.

245. **Analysis of nearly Monochromatic Light.**—With sodium light, the appearance above described is modified by the fact that the light consists of two wave-lengths; this results in the fringes becoming sharply defined for a certain distance and then comparatively indistinct, to become sharp again further on. It is obvious that the indistinctness occurs when the difference of path contains a whole number of wave-lengths of one of the constituents of the light, and contains an odd half wave-length of the other constituent. The region of greatest distinctness will be at a place where the difference of path is approximately a whole number of wave-lengths of each constituent.

Thus, if p be the number of wave-lengths of the one constituent λ_1, and q be the number of the other λ_2, we have

$$\delta = 2e\cos i = p\lambda_1,$$

and this is approximately equal to

$$q\lambda_2.$$

At the next region of greatest distinctness,

$$\delta' = 2e\cos i' = p'\lambda_1,$$

and this is approximately equal to

$$q'\lambda_2.$$

Subtracting $\qquad 2e(\cos i' - \cos i) = (p' - p)\lambda$

and this is nearly equal to $\qquad (q' - q)\lambda_2,$

where $\qquad (p' - p) \sim (q' - q) = 1.$

Thus, nearly, $\qquad \dfrac{\lambda_1}{q' - q} = \dfrac{\lambda_2}{p' - p} = \dfrac{\lambda_1 - \lambda_2}{1}.$

Or $\qquad \lambda_1 - \lambda_2 = \dfrac{\lambda_2}{p' - p} = \dfrac{\lambda_1\lambda_2}{2e(\cos i' - \cos i)}.$

The light may also be analysed by using Fabry and Perot fringes at normal incidence, if the mirrors are so mounted that their distance apart can be varied while the mirrors remain strictly parallel to one another.

This can be managed by attaching the one mirror to a support

similar to Fig. 265, and the other to a support by which its orientation can be varied until it is parallel to the first mirror. Such a support is shown in Fig. 267. The mirror is attached to this with red wax or plasticene, with its semi-silvered face in the front. The mirrors having been adjusted parallel to one another, the second one is moved by the long screw and the *clearness* of the fringes estimated. A curve is then plotted showing the distinctness of the fringes at each position of the mirror. This curve will be periodic, and is due to the interference of the various components of the light used. In this way a line apparently monochromatic (*e.g.* the green mercury line) may be found to be multiple, and the components may be ascertained.

The two chief components of sodium light can be easily followed (*each* line is probably really compound).

Thus having found e as already described, the difference of wave-length of a nearly monochromatic light, which consists of two constituents of nearly equal wave-lengths, can be found.

246. **Ratio of the Wave-lengths of two Monochromatic Lights,** *e.g.* the red and green cadmium lines. Find the value of p, as in § 244, for each wave-length for a pair of Fabry and Perot plates at a fixed distance apart. Then adjust the plates normal to the telescope and observe the rings. Suppose the p, as above found, refers to the first ring from the centre in light of wave-length λ, and is therefore a whole number. So let p' be the order of the first ring in light of wave-length λ'. Then

$$\lambda' = \lambda \frac{p}{p'}\left(1 - \frac{x^2}{8} + \frac{x'^2}{8}\right),$$

where x and x' are the angular diameters of the ring in each case.

For the centre of the system, p_0 will not in general be a whole number, and will differ from p by a fraction ϵ, less than one. Thus we may write:

$$p = p_0 - \epsilon \text{ for the wave-length } \lambda$$

and $$p' = p'_0 - \epsilon' \quad ,, \quad ,, \quad \lambda'.$$

If x is the angular *diameter* of the ring (which can be measured by the telescope), then $i=\frac{x}{2}$.

Then, as in § 244, we have

$$\left.\begin{aligned} 2e\cos i &= p\lambda, \\ 2e &= p_0\lambda. \end{aligned}\right\}$$

$$\epsilon = p_0 - p = p\left(\frac{1}{\cos i} - 1\right)$$

$$= p \cdot \frac{x^2}{8} \text{ nearly.}$$

So $$\epsilon' = p'\frac{x'^2}{8}.$$

Thus, as $$\lambda(p+\epsilon) = \lambda'(p'+\epsilon'),$$

we get $$\lambda' = \lambda\frac{p}{p'}\left(1+\frac{x^2}{8}-\frac{x'^2}{8}\right).$$

The ratio of $\lambda' : \lambda$ can be found to a high order of accuracy.

The Zeeman Effect.

247. This can be exhibited by means of a Fabry and Perot etalon, *i.e.* a pair of parallel semi-silvered reflecting surfaces at a fixed distance apart (see § 243). The etalon is most easily made by semi-silvering the two surfaces of a piece of parallel glass about $\frac{1}{4}''$ or so thick. It must be used with monochromatic light—say from a helium or argon tube, and a spectroscope.

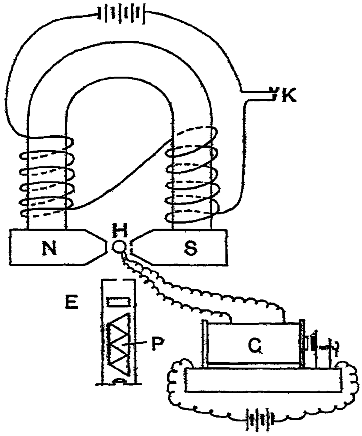


FIG. 258.—The Zeeman Effect. Diagram of Connections.

Apparatus. — An electro-magnet with conical pole-pieces to give a strong concentrated field. The pole-pieces must be separated just sufficiently to allow the narrow part of

the helium tube to be inserted between them. The magnet need not be a very large one, if it is only required to show the broadening of the lens. The etalon may be mounted in a direct vision spectroscope on either side of the prisms, or it may follow the train of prisms of an ordinary spectroscope. The slit should be opened fairly wide: a helium tube and stand, and a coil to work it: a battery and key for the electro-magnet.

The spectroscope is first to be adjusted, so that the helium lines are in focus. Then the etalon is to be inserted, and adjusted so that the centre of the ring system is in the middle of the field of each bright line. The fringes will, of course, only be seen in the bright patch formed by each bright line, the size of which may be increased by widening the slit. A nearly similar system will appear in each line, and if the etalon is made of a parallel plate, and is normal or nearly normal to the light, the fringes will form a number of arcs of circles as in the figure.

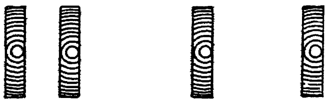


FIG. 259.

Now turn on the current in the electro-magnet. Each of the fine bright arcs in the systems of fringes should become broader. The broadening can be seen when the current is alternately switched on and off while the observer is watching the fringes.

By observing the lines with a nicol or a double-image prism, it may be possible to separate the components of a broadened fringe, and to see that they are polarised.

Jamin's Interferometer.

248. *Apparatus.*—If it is required merely to obtain the fringes and to observe them, it will be sufficient to have two pieces of plate glass mirror cut from a single piece of parallel glass, so that they may be of exactly equal thickness. The glass may be selected by observing the image of a distant flame at grazing incidence. If the glass is parallel, only one image will be visible and not a series of images (see page 36). Very often in a large sheet some part can be found where this occurs; the mirrors should be cut from such a part,—or, of course, specially worked glass can be

used. In this latter case also, the two mirrors must be cut out of the same piece of glass. They must be silvered on the back and can be about 1 cm. thick and 5 cms. square. Two stands upon which to mount these mirrors will be required. These can be very easily made as indicated in Fig. 260. A piece of thin brass about $2\frac{1}{2}''$ square is soldered upright on a base about $2\frac{1}{2}''$ by $1\frac{1}{2}''$. Fine screws, with milled heads, are put near three of the corners. A plate about $2\frac{1}{4}''$ square, with a hole, slot, and plane, is held against the points of these screws by a spring at the centre of the triangle formed by the first three screws. The mirror is attached to the plate with red wax or plasticene, or Chatterton's compound. As the angle of the triangle formed by the first three screws is a right angle, a movement of the screw on the right will rotate the mirror about a vertical axis, whilst a movement of the bottom screw will rotate it about a horizontal axis, and the plane of the mirror can therefore be adjusted. Two pieces of zinc will be required, about $2''$ by $3''$, bent at right angles parallel to one of the shorter sides. One of these (Fig 261) has a hole $\frac{3}{8}''$ in diameter in the centre, furnished with a pair of cross-wires, and the other a hole about $\frac{1}{16}''$ in diameter. A slate slab about $18''$ square and an inch thick will do to mount them upon. They can be attached to the slab by the cement about in the positions indicated in the figure, where A is the piece of zinc with the cross-wire, B and C are the mirrors parallel to one another, and D is the piece of zinc with the small hole.

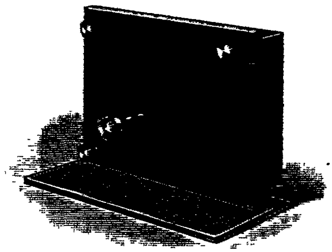


FIG. 260.—Mount for Mirror.

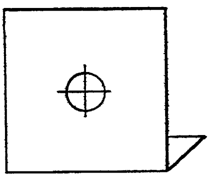


FIG. 261.

249. A sodium flame F is put behind A and the eye applied to the hole in D. Two bright images of the cross-wire will probably be visible. The screws attached to the mirrors are to be adjusted until these two images exactly coincide. When this occurs the fringes will appear. These fringes will form parts of circles, and further adjustment of the screws will bring the centre of the system of

circles into the field. When this is the case, if the mirrors are of exactly equal thickness, white light may be substituted and a few coloured fringes will be visible, but if the mirrors are not of exactly equal width these fringes cannot be seen.

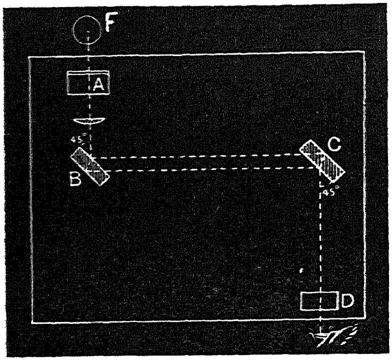


FIG. 262.

The adjustments should be disturbed and the fringes found several times until they can be easily obtained in a few minutes.

The fringes are greatly improved by the introduction of a short focus lens between F and the mirrors, the zinc plate A can then be removed and put at the principal focus of the lens, the flame being immediately behind. Also the plate D being removed, a low-power telescope magnifying only about two diameters, focussed for infinity, should be used to observe the fringes as described on page 316.

For actual work, the glass plates require to be optically worked. They should each be about 6 cms. by 3 cms., and about 4 cms. thick, and must be silvered at the back. For illumination a Nernst filament can be used, placed close behind a small circular hole about 2 mm. diameter; the filament is to be in the focal plane of a collimating lens. The re-combined white light which comes from the second plate can be focussed on the slit of a sprectroscope. Fringes will then be formed in the spectrum, and any optical variation introduced into the path of either beam will cause the fringes to move along the spectrum. The position of the fringes in the spectrum can be determined by using a comparison prism and auxiliary spectrum in the usual way.

Lodge's Interferometer.

250. *Apparatus.*—Two stands similar to those already described will be required, with two plane mirrors silvered on the *front*

about 2″ square,[1] and also a semi-silvered mirror. The amount of silver on the latter must be very small, so that the light reflected is about equal to that transmitted at an angle of about 45°. The stand for this mirror consists only of a piece of brass in the form of a right angle, one side being about $2\frac{1}{2}''$ and the other about 3″ long, supported on a fine milled head at the extremity of the longer arm, and upon two round headed screws, one at each end of the shorter arm.

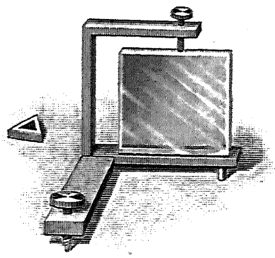


FIG. 263.—Stand for Mirror.

The mirror is cemented upright to the shorter arm with red wax; or if preferred, an upright angle piece may be attached to the stand, with a little side screw to clamp the mirror as indicated in the figure. The collimating lens and telescope of § 244 should be used.

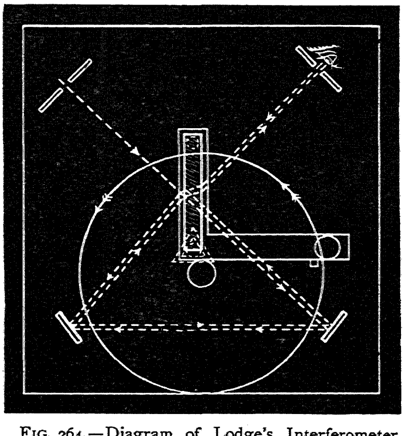


FIG. 264.—Diagram of Lodge's Interferometer. The semi-silvered mirror is shown much enlarged, so that the path of the light may be more easily traced.

A small triangular piece of brass, with a triangular hole, is fastened to the slate slab for one of the round-headed screws to rest in; a small brass block is also cemented to the slab for the side of the long arm to be pressed against. The two silvered mirrors are seen just outside the circle which indicates the rotating discs; and the two pieces of zinc previously described are mounted as shown in the figure, or the collimating lens and telescope may with advantage be substituted as in Fig. 257.

On looking through the pinhole in the one piece of zinc (or the telescope) similar fringes are seen to those with Jamin's

[1] Two selected pieces of silvered plate glass, from the back of which the red varnish has been removed with alcohol, the cleaned faces being used as the reflecting surfaces.

apparatus. The light forming one image of the cross-wire has gone round the triangle clockwise, whilst that forming the other has gone round the triangle counter-clockwise.

Lodge used an interferometer on this principle to try and find whether matter carried ether with it or not. He rotated a pair of circular saws in the triangle, as indicated by the large circle, the saws being on the same axis and a short distance apart, and the light passing through the space between the two. The result was to produce a circular motion of the air between the saws. If this and the saws had any effect upon the ether it should accelerate the light passing round in one direction and retard that going in the other direction, and a displacement in the fringes should result. When he had eliminated all the spurious displacements caused by the stress in his apparatus, he could find no trace of any movement.

Michelson's Interferometer.

251. *Apparatus.*—Four right angle pieces of brass will be required, three of them are to be similar to those already described, but the milled head of the fourth should be rather large, and must have a very fine threaded screw. Two large milled heads with fine screws, each working in an angle piece of stout brass, will be wanted; and two pieces of brass about 4″ long and ½″ wide, bevelled on one edge, which placed with their bevelled edges together will form a groove. The angle piece MPT_1 (Figs. 265 and 266) which has the large milled head M must also have a projection ME on one side at the end of the arm, to which the milled head is attached, bent up as shown in the figure, for the screw of the milled head K_2 to press against. Two mirrors silvered on the front, as in Lodge's interferometer, will be required, also a semi-transparent mirror and a compensator. These two latter are to be two pieces cut out of a single piece of worked glass, so that they may be of exactly the same thickness, one being semi-silvered and the other clear. Either

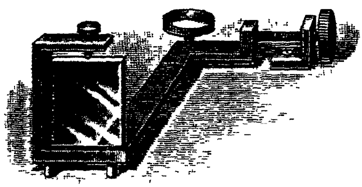


FIG. 265.

two pieces of zinc A and D, as in the figure, or a sodium flame and collimating lens and a low-power telescope, are required. The apparatus is set up on a slate slab as shown in the figure.

All the apparatus can be attached to the slab with cementium. Begin with the grooves G; then lay the angle piece in them and attach the block of the milled head K_1; then that of K_2 and the triangle T_1, so that the mirror P may be at right angles to the slab, and exactly the same distance from B that Q is. Attach the triangle T_3, so that the mirror B may be at the junction of the lines from the mirrors P and Q, parallel to the sides of the slab, and at 45° to these lines. Fix A and D on the continuation of these lines, and T_2 so that C shall be on the line BP.

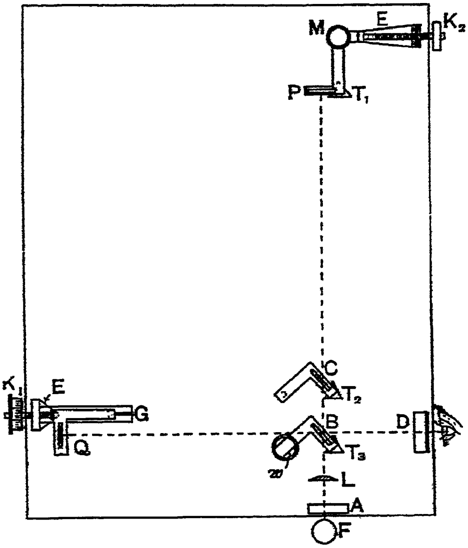


FIG. 266.—F, flame; A, piece of zinc with cross-wire; D, zinc with small hole; B, semi-silvered mirror; C, compensator; P and Q, mirrors silvered on the face; K_1, K_2, the milled heads in thick angle blocks; G, grooves; E and E, elastic bands; T_1, T_2, T_3, brass triangles; *w*, brass block to prevent the angle piece rotating; M, large milled head in angle block.

On looking through D two bright images of the cross-wire will be seen, and other faint ones. Adjust B until the bright ones approximately coincide, and cement on the block *w*. The final coincidence must be obtained by adjusting the milled heads M and K_2. When the coincidence is perfect, fringes will be seen

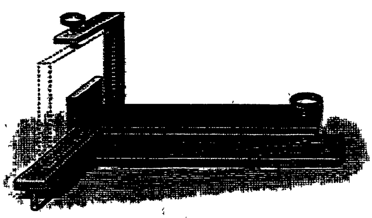


FIG. 267.—Enlarged view of the grooves G, and the stand which slides on them.

with the sodium flame. Further adjustment of M and K_2 will enable the centre of the system of rings to be found. If the distance BP is exactly equal to BQ, on substituting white light at F for the sodium flame, coloured fringes will be visible. This will, however, probably not be the case, but the distance can be adjusted either by the screw K_1 or, if nearly right, by rotating the compensator C.

As the two interfering pencils BP and BQ are widely apart, this arrangement has been used to measure the optical thickness of a thin plate or of a tube containing gas whose refractive index is to be determined.

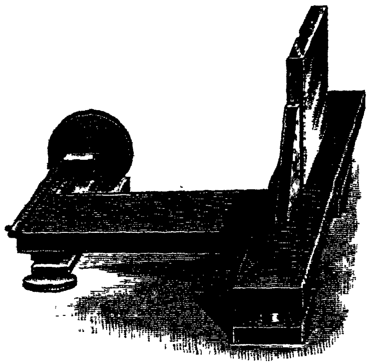


FIG. 268.

252. An alternative method of mounting the mirror P is shown in Fig. 268; the milled head M is here on the block that carries the screw K_2, shown separately in Fig. 269. A pivot which has a triangular pyramid for apex (Fig. 269), fitting into a circular hole, may be substituted for the triangle T_1. An elastic band keeps the stand against the screw K_2.

The stand in Fig. 269 also shows a method by which very slight movements may be produced by a coarse-threaded screw. The screw is put through a thin piece of the stand which is partly separated by a saw cut from the other and more solid portion on which the arm of the mirror stand rests. On tightening the screw both of these parts spring to extents which depend upon their thicknesses; and thus the movement of the thick part may be exceedingly slow—if its thickness is much greater than that of the other.

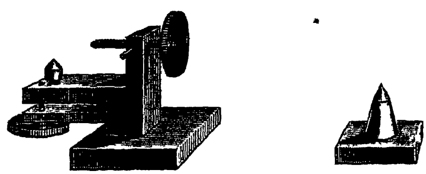


FIG. 269.

Fabry and Perot used a very simple and effective device for the *fine* adjustment for parallelism (Fig. 270). One of the mirrors was mounted on the end of an arm of only moderate rigidity, and of rectangular cross-section. Along, and parallel to, two adjacent sides of this arm, were two projections from the stand, and between each of the projections and the arm was a small hollow rubber ball (the teat sold for a baby's comforter would no doubt

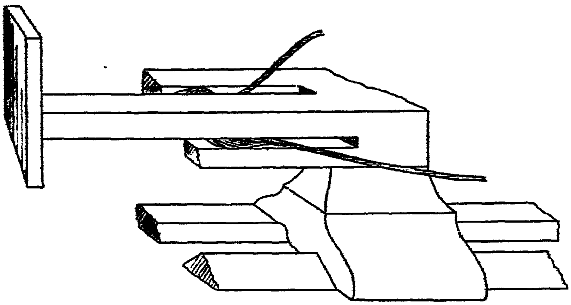


FIG. 270.—Adjustment by Flexure of the Bar carrying the Mirror.

do). Each ball was connected by a long thin rubber tube with a small reservoir (*e.g.* a thistle funnel), and the systems were filled with water. Each rubber ball of course exerted a pressure, forcing the arm away from the projection it was pressing against, and this pressure could be varied by adjusting the height of the reservoir. Thus the arm carrying the mirror was given a flexure which would be varied at will, and the final setting of this mirror into parallelism with the other mirror of the pair was effected with ease, and without risk of disturbing the stand which the manipulation of a screw would produce.

Exercises with the Interferometer.

1. **To compare the Refractive Indices of two Gases.**—Prepare two brass tubes of the same length with worked glass ends. Fill one with each gas. Let the glass ends be large enough to project beyond the tubes. Insert one tube in each beam and get two interference patterns, one for the light which passes through the tubes and one for the light which comes through the glass ends but not through the tubes (*i.e.* through the parts that project beyond the tubes). Vary the

pressure until the systems are coincident, that is until with white light the fringes are coincident. Then

$$\frac{p_1}{p_2}=\frac{\mu_2-1}{\mu_1-1},$$

where μ_1, p_1 and μ_2, p_2 are the refractive indices and the pressures of the respective gases.

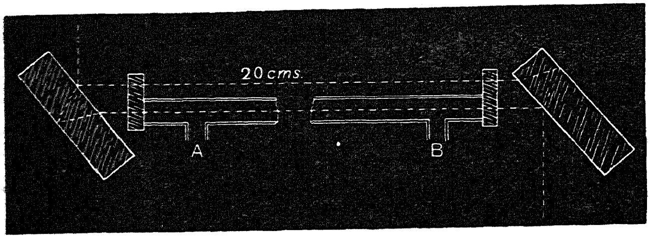


FIG. 271.

The absolute value of the refractive index (μ_0) at 0° C. and 76 cms. pressure for light of wave-length λ is given by

$$\mu_0-1=(1+at)\frac{76}{P}\cdot\frac{f\lambda}{L},$$

where t is the temperature, a is the coefficient of expansion, f is the number of bands that pass the wire, while the pressure changes by P cms., L is the length of the gas path in the tube. The pressure can be varied by first pumping it up, and then allowing the gas to escape. a may be taken to be 0·00367. If a Nernst lamp and a spectroscope are used, as described in § 249, f will be the number of fringes that passes some standard wave-length.

If μ_1 and μ_2 for two known wave-lengths λ_1 and λ_2 have been determined as above (Ex. 1) the constants in Cauchy's equation can be found, for we have

$$\left.\begin{aligned}\mu_1-1&=a+\frac{b}{\lambda_1^2}\\ \mu_2-1&=a+\frac{b}{\lambda_2^2}\end{aligned}\right\}$$

in which a and b are the only unknowns.

2. The Determination of the Thickness of the "Black Spot."—This apparatus was used by Reinold and Rücker to find the thickness of the black spot in a soap film. Two pieces of flat brass, each about 8″ or 10″ long, had a series of shallow grooves filed across one face, at distances

of $\frac{1}{4}''$. A glass rod was drawn out to fine threads, and one of these, about $1\frac{1}{2}''$ long, was placed in each of the grooves of one of these pieces of brass. The other piece was then laid upon it, and they were wired together. By passing a small flame along close to the brass while the glass rods are in a horizontal plane, these latter are bent down by their own weight at right angles, making a series of small hoops. Some solder was then melted in a shallow trough, and the points of the glass rods dipped into it whilst hot, and allowed to remain there till cold, the pieces of brass were then removed. In this way a large number of hoops were left standing up vertically in the solder.

Some soap solution was poured in a glass trough. The trough with the hoops was immersed in it, and then raised, so that the hoops were lifted above the liquid. Each hoop now supported a vertical soap film. This was then placed in the path of the beam PC so that the light was transmitted through all the films. An air-tight glass cover with worked ends was placed over the whole to stop evaporation. The fringes were found as usual. As the films drained, some broke, but many of them formed black spots. When nothing was left but black spots the position of the fringes was taken and the number of black spots counted. (The counting was done by watching the reflection of a candle in the films.) Then assuming the refractive index of the soap solution known, since the displacement of the fringes gives the total difference of path, the thickness of each film can be calculated. He was led by the results of his experiments to the conclusion that the spot was either of thickness $12\mu\mu$ or $6\mu\mu$, the thickness changing abruptly from the one value to the other.

3. Determination of the Optical Thickness of a Thin Film, e.g. a Sheet of Mica or a Micro-cover Glass.—Adjust the apparatus to obtain vertical fringes. Insert in one of the beams of the interfering light a piece of mica. The fringes, will, of course, be displaced, and they will be displaced by a number which cannot be directly determined, as it is not possible to identify the bands. Suppose, first, the film to be placed normally to the light, then the difference of path will be $2(\mu-1)e$, e being the thickness of the film and μ its refractive index. If this be supposed to have displaced N fringes, we have

$$2(\mu-1)e=N\lambda.$$

Now rotate the film through a definite angle, i (say $10°$), and count the number of fringes that cross the field, n. The difference of path will now be $2e\left(\frac{\mu-\cos(i-r)}{\cos r}\right)$, and thus

$$(N+n)\lambda=\frac{2e}{\cos r}(\mu-\cos(i-r)).$$

Subtracting, $$n\lambda = 2(\mu - 1)e - \frac{2e}{\cos r}(\mu - \cos(i-r)),$$

where r is given by $\frac{\sin i}{\sin r} = \mu$.

This will enable e to be found if μ is known, or μ if e is known.

Dividing, we obtain $\frac{N+n}{N} = \frac{1}{\cos r}\,\frac{\mu - \cos(i-r)}{\mu - 1}$.

Thus, if μ is known approximately, we can substitute for $\cos r$, and can find N, the retardation produced by the film.

NOTE.—If λ be the wave-length in air, the wave-length in the film is $\frac{\lambda}{\mu}$. The distance travelled in the film is $KL = \frac{e}{\cos r}$, thus the number of waves in the film is $\frac{e}{\cos r} \div \frac{\lambda}{\mu}$. The number of waves in the air before the insertion of the film was

$$\frac{KM}{\lambda} = \frac{e\cos(i-r)}{\lambda \cos r}.$$

Thus the difference on passing through the film is $\frac{e\cos(i-r)}{\lambda\cos r} - \frac{e}{\lambda}\frac{\mu}{\cos r}$. In the case above it passes through twice, and therefore

$$N + n = 2\frac{e}{\lambda}\left(\frac{\mu}{\cos r} - \frac{\cos(i-r)}{\cos r}\right).$$

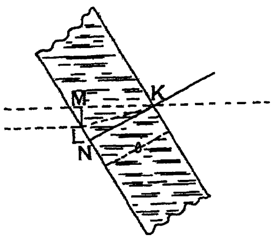


FIG. 272.

Other Methods of Producing Interference with a Small Difference of Path.

253. *Apparatus.*—A mirror with parallel faces silvered at the back; a semi-silvered mirror; a pair of plane mirrors silvered on the front. These mirrors may all be about $2\frac{1}{2}''$ long and $1\frac{1}{2}''$ high; they should be mounted as in Fig. 263.

Set up the mirror A, silvered at the back, and the semi-silvered mirror B, parallel to one another, so that the light from a source S, after reflection at B, shall fall at an angle of 45° on A. Adjust a pair of plane mirrors silvered on their face at O, at right angles to one another, to receive this light. It will be seen that a ray of light starting along KLM, after transmission through the semi-silvered surface, will emerge from K′; whilst a ray along K′L′M′, which will traverse the same path in the opposite direction, will emerge at K. If the line of intersection, O, of the mirrors is exactly parallel to the plane of A, and this path be exactly equal,

there will be no fringes, and the whole surface will appear equally illuminated. But if the edge be slightly inclined, or if the paths are not quite equal, fringes will be formed.

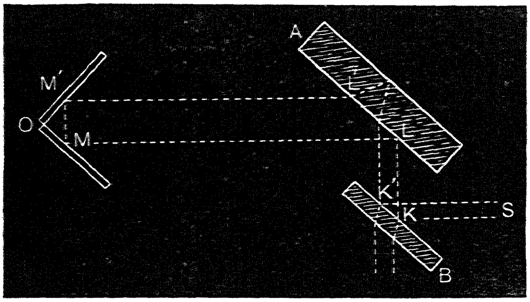


FIG. 273.

It is not possible to apply this to any practical purpose, as it is obvious that anything placed in the path of one ray is at the same time placed in front of the ray going the other way round.

A simpler arrangement is shown in the next diagram, in which the light after reflection in the mirror A passes through a lens

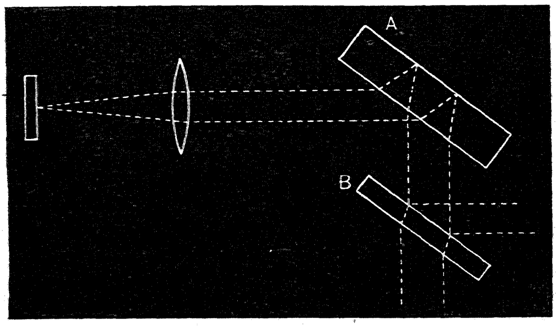


FIG. 274.

and falls on a plane mirror situated at its principal focus. If this is exactly adjusted at the focus of the lens, no fringes are formed. But if it be placed a little within or without the focus, fringes are seen. The same remark applies also, however, to this case.

Fabry and Perrot's mirrors have already been referred to, § 243.

Position of the Interference Fringes in an Interferometer.[1]

254. Let Om and Om' be two mirrors, and p a bright point on the surface of Om. Let p' be the image of p in Om'. Let P be a point at which the interference is sought (not necessarily in the plane of the paper), and let $\angle Ap'P$ be δ.

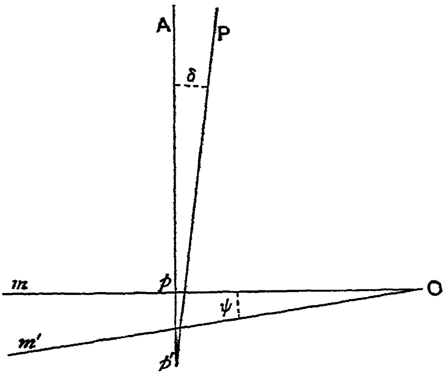


FIG. 275.

Then the difference of path is

$$\Delta = Pp' - Pp$$
$$= pp' \cos \delta \text{ (nearly)}$$
$$= 2t \cos \delta,$$

where t is the distance apart of the surfaces at p.

Let $cdef$, $c'd'e'f'$ (Fig. 276) be two plane images inclined to one another at an angle 2ϕ, and let their line of intersection be parallel to df, so that the angle between cd and $c'd'$ is 2ϕ, whilst the mirrors are at an angle ϕ with one another.

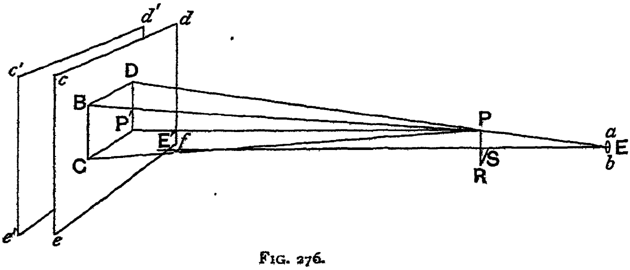


FIG. 276.

Let PP' be normal to the image and equal P.

Let the distance apart at P' be $2t_0$.

Then at B, the distance apart, $2t$ is given by

$$t = t_0 + CP' \tan \phi$$
$$= t_0 + P \tan i \tan \phi,$$

where the $\angle P'PC = i$.

[1] Michelson, *Phil. Mag.*, 1898.

$$\Delta = 2t \cos\delta = 2(t_0 + \mathrm{P}\tan i \tan\phi)\cos\delta$$
$$= 2 \cdot \frac{t_0 + \mathrm{P}\tan i \tan\phi}{\sqrt{1 + \tan^2 i + \tan^2\theta}}$$

where the $\angle\,\mathrm{DPP}' = \theta$.

Thus Δ may have all possible values for different values of i and θ. But if the beam be supposed to be limited, say, by a pupil at E, the difference in Δ can be made small. For given maximum values of i, θ, we want the position of P for which the variation of Δ shall be a minimum. Differentiating first with respect to θ, we have

$$\frac{\delta\Delta}{\delta\theta} = \frac{-2(t_0 + \mathrm{P}\tan i \tan\phi)\dfrac{\tan\theta}{\cos^2\theta}}{(1 + \tan^2 i + \tan^2\theta)^{\frac{3}{2}}}.$$

This is zero if $\theta = 0$, or if $\Delta = 0$.

Now, differentiating with respect to i, we have:

$$\frac{\delta\Delta}{\delta i} = 2\left\{\frac{(1 + \tan^2 i + \tan^2\theta)\dfrac{\mathrm{P}\tan\theta}{\cos^2 i} - (t_0 + \mathrm{P}\tan\phi\tan i)\dfrac{\tan i}{\cos^2 i}}{(1 + \tan^2 i + \tan^2\theta)^{\frac{3}{2}}}\right\}.$$

This will be zero if the numerator vanishes, or if

$$(1 + \tan^2\theta)\,\mathrm{P}\tan\phi = t_0 \tan i,$$

or if
$$\mathrm{P} = \frac{t_0}{\tan\phi} \cdot \tan i \cdot \cos^2\theta.$$

Hence both $\frac{\delta\Delta}{\delta\theta}$ and $\frac{\delta\Delta}{\delta i}$ will be zero, and therefore the fringes will be most distinct, if

$$\left.\begin{aligned} \theta &= 0, \\ \mathrm{P} &= \frac{t_0 \cdot \tan i}{\tan\phi} \end{aligned}\right\}.$$

From these we see that if $t_0 = 0$, or if $i = 0$, the fringes will coincide with the mirror. If $\phi = 0$ (*i.e.* if the surfaces are parallel), $\mathrm{P} = \infty$.

If i and ϕ have the same sign the fringes are in front of the mirrors, if of opposite sign they are behind.

Shape of the Interference Fringes.

255. To find the form of curve as seen by an eye at E. Let T be the distance between the surfaces at E′, the projection of E (Fig. 276).

Draw PR and RS parallel to *ce* and *ef* respectively, where S is the intersection of EE′ with a plane through P parallel to *cdef*. Let $EE' = s$; $SE = D$; $\tan\phi = k$. It is required to find the locus of P.

Let the coordinates of P be x and y referred to an origin at S.

Then $$\tan i = x/D,\ \tan\theta = y/D.$$

Then, as before, $$\Delta = 2\frac{T + s\tan\phi\tan i}{\sqrt{1+\tan^2 i + \tan^2\theta}}$$
$$= 2\,.\,\frac{DT + sk\,.\,x}{\sqrt{D^2+x^2+y^2}},$$

or $$\Delta^2 y^2 = (4s^2k^2 - \Delta^2)x^2 + 8TskDx + (4T^2 - \Delta^2)D^2.$$

The curve will be a hyperbola, parabola, or ellipse according as $2sk$ is greater than, equal to, or less than Δ.

If $k = 0$, this is a circle; if $\Delta = 0$ it is a straight line on the surface of the mirror (as t_0 will be $= 0$).

The focal plane varies rapidly with i, and it is in general impossible to see all parts of the fringes in focus at once; but when the surfaces are parallel and $P = \infty$ they are all distinct, and are circles of which the centre is the foot of the normal from the objective of the telescope upon the surface.

The angular semi-diameter of the n^{th} circle is given by

$$\delta_n = \sqrt{\frac{n\lambda}{t}}.$$

For let MM′ be the two surfaces, and A′ be the image in M′ of a point A on M. Then A and A′ are the origins of two parallel rays AT and A′T′ which produce the interference. Let AT make an angle δ with the axis AN of the telescope.

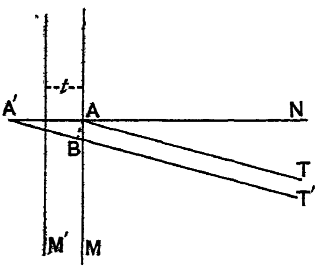


FIG. 277.

Then the difference of phase

$$\Delta = A'B = 2t\cos\delta$$
$$= 2t - n\lambda \text{ (say)}.$$

Thus, approximately, $n\lambda = \frac{\delta^2}{2}\,.\,2t$,

or the semi-diameter of the n^{th} ring,

$$\delta_n = \sqrt{\frac{n\lambda}{t}}.$$

CHAPTER XV

FURTHER EXPERIMENTS ON DIFFRACTION

The Straight Edge.

256. *Apparatus.*—The experiments are best performed with the optical bench already described. The straight edge must be mounted in the stand which carried the bi-prism, in about the same position as the bi-prism. A slit and an eye-piece will be wanted, in the same stands as before. A good light is essential.

Experiment.—The only adjustment is the parallelism of the slit and edge. This can be secured by removing the eye-piece, and observing that the slit is cut off at the same instant all along its length by a movement of the eye as directed for the bi-prism (page 259). On inserting the eye-piece the fringes will be visible. The final adjustment must then be made with the tangent screw, and the width of the slit set to produce the clearest fringes. They should be examined in white light and their appearance sketched.

Place the slit on the first, second, third, ... dark bands and take the readings. This will approximately be the distance of the successive dark bands from the edge of the geometrical shadow. If x_n be the distance of the nth dark band,

$$x_n = \sqrt{\frac{b(a+b)2n\lambda}{a}} \text{ approximately.} \quad \text{(Preston's } Light\text{, p. 215.)}$$

Thus λ is given by

$$\lambda = \frac{x_n^2}{n} \cdot \frac{a}{2b(a+b)}.$$

Enter the readings in a table thus:

No. of Band.	Reading.	Distance from M = PM.	$(PM)^2$.	$\frac{(PM)^2}{n}$.

The numbers in the last column should be constant. If they gradually increase or gradually decrease, it will be because the position of M has not been properly found. For this reason it is advisable to determine this position several times and use the mean value. The mean of the last column must then be multiplied by

$$\frac{a}{2b(a+b)}.$$

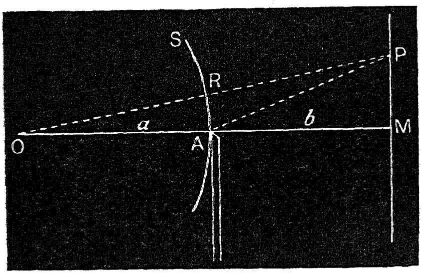


FIG. 278.

This method of course does not give good values of λ, chiefly because the diffraction bands are very indistinct, and it is very difficult to say exactly when the cross-wire is upon one, partly because they are not numerous, fading away very rapidly, and partly because of the uncertainty of the position of M, and lastly because the theory is not quite accurate.

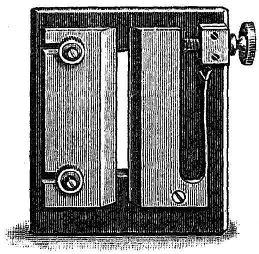


FIG. 279.—Double Edge.

Double Edge Method.

257. *Apparatus.*—The optical bench with the usual slit and micrometer eye-piece; a double straight edge (Fig. 279); sodium flame; velveteen; millimetre scale; a reading microscope.

The double straight edge has one edge, which may be clamped at any distance required from the other edge, and roughly parallel

to it The second edge can then be adjusted accurately parallel to the first by a fine screw.

The accurate theory of the straight edge involves the use of the integral calculus. The results lead to Fresnel's integrals. The theory is given in Jamin's *Cours de Physique* (vol. iii. part 3, p. 366). It is there shown that the distance of the successive dark bands from the edge of the geometrical shadow are given by the formula

$$x_n = A_n \sqrt{\frac{b(a+b)\lambda}{2a}},$$

where A_n is to be given in succession, the values in the following table:[1]

$A_1 = 1{\cdot}8726.$	$A_5 = 4{\cdot}4160.$
$A_2 = 2{\cdot}7392.$	$A_6 = 4{\cdot}8479.$
$A_3 = 3{\cdot}3913.$	$A_7 = 5{\cdot}2442.$
$A_4 = 3{\cdot}9372.$	

The determination of the true position of the edge of the geometrical shadow is avoided by using two straight edges, and thus obtaining two sets of diffraction fringes.

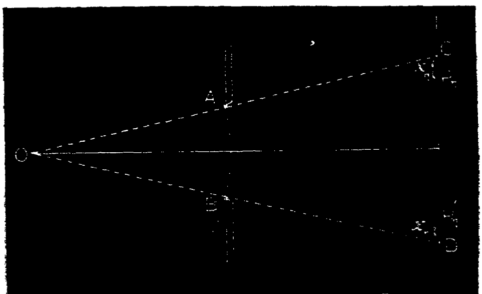


FIG. 280.

If O be the source of light and A and B two straight edges, the geometrical shadow will end at C and D. The distance CD can be

[1] For purposes of comparison, the corresponding values according to the elementary theory, are given below. They are all too large.

$A_1 = 2 \quad = 2.$	$A_5 = 2\sqrt{5} = 4{\cdot}47.$
$A_2 = 2\sqrt{2} = 2{\cdot}82.$	$A_6 = 2\sqrt{6} = 4{\cdot}89.$
$A_3 = 2\sqrt{3} = 3{\cdot}46.$	$A_7 = 2\sqrt{7} = 5{\cdot}29.$
$A_4 = 2\sqrt{4} = 4.$	

It will be found that $A_n = \sqrt{8n-1}$ gives very nearly correct value.

found by similar triangles if the width AB is known. Then if P, P′ are a pair of dark bands, CP or DP′ is the X_n of our formula. It is obvious that $$PP' = CD - 2X_n.$$

The apparatus is set up on the optical bench as before, O being the slit, and AB a screen with an aperture bounded by two straight edges about 5 mm. apart. OA is about 50 cms., and AC anything from 2 cms. upwards.

The edges of the aperture in the screen AB must be very accurately parallel to one another,[1] and to the slit O.

Take the readings of corresponding fringes. The readings must be taken in one direction only, and may be entered in columns.

Number of Fringe.	Reading.		Difference $=PP'$.	CD.	$CD - PP' = 2X_n$.	$\frac{X}{A_n}$.
	Left.	Right.				

As the readings are to be taken in one direction, the entries in the second column will be made from top to bottom, and in the third column from bottom to top. Subtracting the second column from the third gives the distance PP′. In the next column enter the distance CD, calculated from the distance AB (which must be measured directly by the reading microscope), and the distances OA and OC. Calling AB, d, obviously

$$CD = d\frac{OC}{OA} = d\frac{a+b}{a}.$$

Subtracting PP′ from CD gives $2X_n$ as already described.

The last column should be a constant, being the root of

$$\frac{(a+b)}{2a} \cdot \lambda.$$

[1] This aperture can be made by two pieces of "Printer's Rule," mounted parallel to one another upon a piece of brass or wood, in which a half-inch hole has been cut. The distance apart can be adjusted by inserting at the top and bottom two short bits of the narrow wire used for experiment § 258. If the straight edges are pressed as nearly together as these will allow, it will ensure their being parallel, as well as the right distance for the experiment.

From this, if a and b are known, λ can be calculated. For, calling the mean value of the last column M,

$$\lambda = M^2 \cdot \frac{2a}{b(a+b)}.$$

If preferred, the value of the wave-length may be assumed, and the last column calculated, and then by dividing it into X_n (determined experimentally), the series of numbers A_n can be found, and compared with those given in the table above.

The Narrow Wire.

258. *Apparatus.*—The optical bench as before with slit and eye-piece. The narrow wire can be cut from a piece of Stubbs silver steel about 1·2 mm. diameter. It must be perfectly straight, and it is necessary to be fine in order to obtain the interference bands (the central part of a sewing needle will do). A hole about half an inch in diameter can be cut in a piece of brass or wood and the wire mounted across it. It must be set parallel to the slit. A sodium flame, screw gauge, millimetre scale and velvet are required.

Experiment.—If the wire is not very fine it will have to be placed at some distance from the slit, and the eye-piece still further beyond it in order to see the fringes, but a fine wire such as that suggested above may be placed at about the position which was occupied by the bi-prism. The slit and wire must be arranged vertical and parallel to one another; the diffraction bands will be easily seen. A fine slit and a slight adjustment of the tangent screw will enable the less luminous interference bands to be seen in the dark space between the two sets of diffraction bands.

Measure the diameter c of the wire with a screw gauge, its distance d from the cross-wire, and x the width of the fringes, then λ is given by the equation:

$$\lambda = \frac{cx}{d},$$

as in the interference experiments already described; for the edges of the wire act as two interfering sources of light, and take the place of the two images of the slit formed by the bi-prism. It is impossible, however, as a rule, to obtain sufficiently bright

fringes for their width to be determined. Better readings for λ by the diffraction fringes may be obtained with this narrow wire than was possible with the single straight edge. The calculations are similar to those for the double edge, § 257 above; for if C and D are the edges of the geometrical shadow of AB,

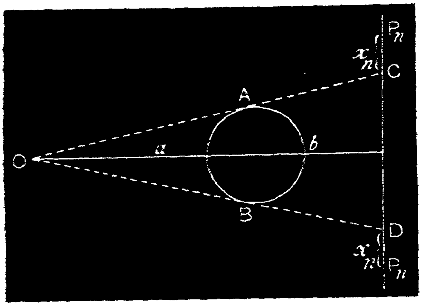


FIG. 281.

$$\frac{CD}{AB}=\frac{a+b}{a},$$

and as AB may be determined with a screw gauge, CD is easily found.

If $P_1P_2P_3P_n$ and $P_1P_2P_3P_n$ are the dark bands on each side, the distance apart of these, less CD, will give double the distances CP_1, $CP_2 \dots CP_n$. The readings should be entered in columns as before, p. 338.

Circular Aperture.

259. *Apparatus.*—An arc light if available and a good sodium flame will be wanted. With a powerful light diffraction fringes will appear everywhere (especially round specks of dust on the lenses of the eye-piece); with a poor light they may be quite invisible. The slit must be replaced by a small pin-hole, and it is important that this pin-hole should be well formed. A hole made in a piece of tin foil with a needle is not sufficiently circular. It must be drilled in a piece of very thin brass or zinc with a watchmaker's drill.[1] The circular aperture also needs to have a very small drilled hole. Several of these should be provided of diameters ranging from $\frac{1}{100}''$ to $\frac{1}{20}''$. The edges of the hole must be perfectly clean.

The aperture should be placed at some distance from the small pin-hole that replaces the slit, and at first the eye-piece must be close to the aperture. The flame must be in good condition and

[1] The plates used by jewellers for *wire-drawing* have a graduated series of pin-holes that can be used for diffraction experiments, and are quite inexpensive. I have found a drilled sapphire best of all. They can be obtained from Wood, Spencer Street, Clerkenwell, for about 2s. each.

the apparatus must be well covered in. If possible the experiment should be set up in a dark room. Having removed the eye-piece, see that the two apertures are as nearly as possible in a line with the centre of the eye-piece tube. Replace the eye-piece and a small circle of light will be perceived surrounded both within and without with fringes, if the apertures are sufficiently cleanly cut, upon which the distinctness of the rings will entirely depend. On gradually withdrawing the eye-piece the rings will appear to open out from the centre, and the central spot will be alternately dark and bright. If the sodium flame is replaced by a bright white light, the rings will be brilliantly coloured, and as the eye-piece is withdrawn to a greater distance from the aperture, they also will open out. As the centre will become dark for the different wave-lengths in turn, the colour of the centre will change from one hue to another, the colours it assumes being similar to Newton's "colours of thin plates."

260. **Approximate Theory.** Let O be the source of light and Q the position of the focal plane of the eye-piece at any moment. Then if

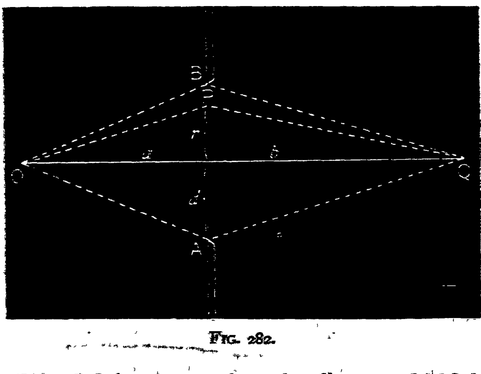


FIG. 282.

the distance OB + BQ is greater than the distance OMQ by less than half a wave-length, the light reaching Q will be all in one phase and Q will be bright. If the difference be between a half wave-length and a whole wave-length, partial interference will occur; and if it is exactly a whole wave-length, the light arriving at Q will consist of two portions of about equal intensity differing in phase by a half wave-length, which will mutually interfere.

If OM, QM and PM are a, b and r respectively,

$$\mathrm{OP}+\mathrm{PQ}-\mathrm{OQ}=(a^2+r^2)^{\frac{1}{2}}+(b^2+r^2)^{\frac{1}{2}}-(a+b)$$
$$=\frac{r^2}{2a}+\frac{r^2}{2b}.$$

If r_n be the radius of the zone for which this distance is $n\frac{\lambda}{2}$,

$$r_n^2=n\cdot\frac{\lambda}{2}\cdot\frac{2ab}{a+b};$$

thus r_n^2 is proportional to n. By giving n the values 1, 2, 3, 4, ... in succession, we shall obtain a series of zones such that the light arriving at Q from one zone will be half a wave-length ahead of that from the next zone, and the areas of the zones will all be equal. Hence, if we neglect the obliquity, the light from these zones which arrives at Q will destroy one another in pairs, and Q will be bright or dark, according as there is an odd or even number of such zones. Thus the centre Q will be bright for values of b given by the odd value of n in

$$b=\frac{ad^2}{na\lambda-d^2},$$

d being the radius of the aperture.

Opaque Disc.

261. *Apparatus.*—The slit must be replaced by a small circular hole, as in the last experiment. The disc itself must be mounted upon a piece of worked glass (or good patent plate, or a "flat" spectacle lens). It is absolutely necessary that the edge of the disc should be perfectly circular. The best discs are small bicycle balls, one of which about $\frac{1}{2}''$ in diameter may be mounted on the glass with a little electrical wax, or shellac, or sealing wax. (If desired, a flat may be made upon it with a grindstone to attach it by.) Care must be taken that the mountant does not smear the glass, or project beyond the ball. The fringes are viewed by a low-power eye-piece, held on a suitable support.

Theory shows that at any point on the axis there should be brightness, which at a distance becomes nearly equal to that at the same point when the disc is removed; for the light which reaches that point from the whole circumference of the disc will be in the same phase, and will therefore produce brightness. In fact the wave from O may be divided into half-period elements, commencing at the edge of the disc (as in the ordinary Huygens's

zones for a spherical wave), and the argument is similar to that which shows that the light should travel in straight lines. If the glass is good, the bright centre can be seen with quite large balls. It is less bright when the viewing eye-piece is only a short distance behind the ball. Stokes says he has produced the white centre with a disc the size of a sovereign. In order to see the bright

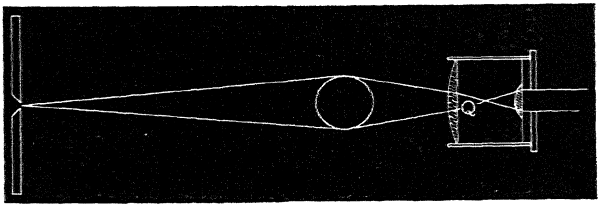


FIG. 283.

spot in the centre of the shadow of a large disc, it is best to put the disc (say a $\frac{1}{2}''$ bicycle ball) at about six feet from the aperture, and the eye-piece another four to six feet beyond, when, if the light is good and all other light is cut off, the effect is easily produced. An arc light shows it up splendidly. All diffraction experiments are much more striking with an arc light or a lime light.

The appearance with white light should be sketched, particular attention being paid to the centre and to the edge of the shadow. If the $\frac{1}{2}''$ ball is supported on the top of a wire or rod, beautiful fringes will be seen running from the centre of the shadow of the ball to the junction of the shadow of the rod and ball.

The Zone Plate, Huygens's Zones.

262. The light which arrives at Q from a bright point O may be looked upon as the effect of the interference of alternate Huygens's zones, which nearly destroy one another in pairs. It follows that if we could remove the alternate zones, the light at Q ought to be very much stronger. This can be done by what is known as a "zone plate." As shown above, in considering the circular aperture, § 260, the areas of the zones should be equal, and the

squares of their radii be proportional to the natural numbers 1, 2, 3, 4. Professor Wood in the *Phil. Mag.* published a reduced photograph of a very carefully executed series of alternately black and white zones whose radii followed this law, and described a method by which reduced copies might be produced. He pointed out that an ordinary photograph of this would be neither perfectly

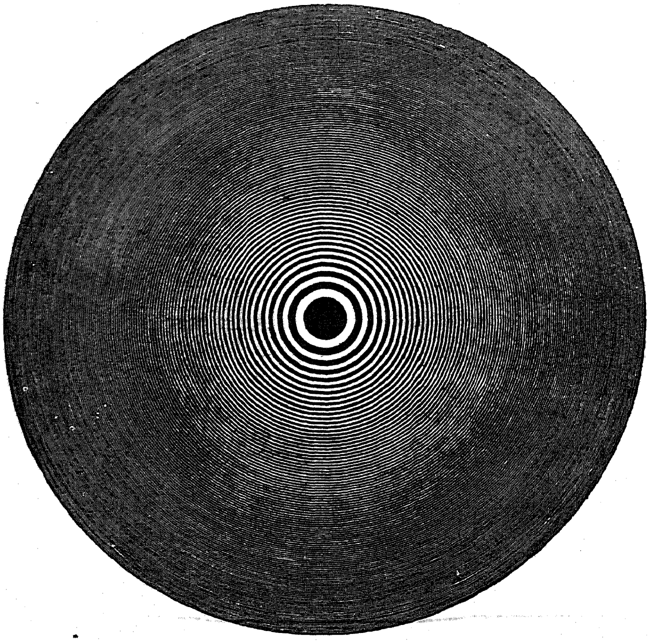


FIG. 280.—Huygens's Zones.

black nor perfectly transparent in the alternate zones. He therefore coated a glass plate first with silver, and then with bichromated gelatine. By exposing this under a micro-negative, and then washing the unexposed parts with warm water (as in the "carbon" process), he obtained a plate in which the silver was alternately protected by a gelatine film and exposed. Dipping this in nitric acid removed the silver from the exposed parts, and left a plate which was alternately opaque and transparent. With

such a plate he obtained very much more brilliant results than usual.[1]

263. *Experiment.*—To observe the images produced by the zone plate, a suitable object is the incandescent filament of an electric glow lamp, fixed at, say, two metres from the zone plate. Hold a piece of white paper on the other side of the plate; withdraw the paper gradually, and presently an image of the bright filament will be formed. Withdrawing it still further, a succession of these images will be observed if the optical bench is long enough, the largest and most distant of these is the one for which each zone on the plate corresponds to one Huygens's zone. As in the case of a convex lens, it will be seen that this image is always inverted, and that its size is to that of the object, as its distance from the zone plate is to the distance of the object. In fact, the zone plate behaves as a lens, and if r_n is the radius of the nth zone, the conjugate foci are given by

$$\frac{1}{v}+\frac{1}{u}=\frac{n\lambda}{r_n^2};$$

its focal length is therefore $\frac{r_n^2}{n\lambda}$.

This may be termed its principal focal length, call it f. It varies rapidly with the colour.

There are a series of other focal lengths equal to

$$\frac{f}{3},\ \frac{f}{5},\ \frac{f}{7},\ \text{etc.},$$

but the images corresponding to these are not so bright. A zone plate behaves equally well as a concave lens; the focal lengths are necessarily the same, namely,

$$\frac{r_n^2}{n\lambda},\ \frac{r_n^2}{3n\lambda},\ \text{etc.}$$

Phase Reversal Zone Plate.

264. Professor Wood pointed out in the article referred to, that if the alternate zones, instead of being cut out, could be delayed half

[1] These plates can be obtained from Newton & Co., of Fleet Street, from 5s. upwards.

a wave, the light from them would arrive at P in the same phase as that from the other zones, and the amplitude at P would thus be twice as great as with a zone plate consisting of opaque and transparent zones. The intensity would therefore be four times as great. Such a zone plate can be made apparently with even less trouble than the silver one previously described. A glass plate is coated with a thin coat of gelatine, and sensitised with bi-chromate of potash. It is then printed from a zone plate negative and washed (as in the carbon process of photography), and thus the alternate rings are left with a thin layer of gelatine. Out of several plates so formed, some would be found to give a very good effect,—no doubt because the layer of gelatine happens to be of the right thickness. The focal length of such a plate depends upon the size of its rings, that is, upon the extent to which the photographic copy was reduced. The smaller the copy, the shorter its focal length. By making one fairly large copy, having therefore a long focal length, and a very small copy with a short focal length, Wood was able to use the former as objective and the latter as the eye-piece of a telescope. He found that the eye-piece when at a distance equal to the sum of their focal lengths worked as a convex lens, and the telescope was equivalent to the astronomical telescope, but that when the plates were at a distance apart equal to the difference of their focal lengths, the eye-piece behaved as a concave lens, and the arrangement was equivalent to an opera-glass, producing an erect image of a distant object. On slowly altering the distance between the two plates, a succession of images, some erect and some inverted, was produced. It is easy to see from a figure that, with parallel light, a zone plate may act either as a convex or concave lens, and it may be experimentally tested.

265. **The Wave-length with the Zone Plate.**—Determine as many as possible of the focal lengths of the zone plate, using it as a convex lens.

A monochromatic light is essential, as the focal length for any colour varies inversely as the wave-length, and therefore changes extremely rapidly with a change of colour.

This is most easily done by the mirror method, the zones being placed close against the surface of a front silvered mirror, if the

images formed by the zone plate can be made sufficiently brilliant to be seen upon the screen. If not, it must be found by the ordinary conjugate focus method, an eye-piece being substituted for the screen, the focal-plane of the eye-piece being determined with the assistance of a lens. Then, if u and v are the distances from the zone plate to the object and image respectively, the focal length is given by

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=\frac{n\lambda}{r^2}.$$

Find the diameters of the rings with a reading microscope. Start the readings from about the twelfth ring on one side and read across the centre to the same ring on the other side. It will be sufficient if the readings are taken for the five outside rings only. Enter the readings thus:

No. of Ring.	Reading.		Difference $=d$.	d^2.	$\frac{d^2}{n}$.
	Left.	Right.			
8					
9					
10					
11					
12					

ADDITIONAL EXERCISES.

1. Mount two straight steel wires, each about 1 mm. diameter, crossing one another at an angle of 10° over an aperture in a piece of wood. Fill up the obtuse angles with cards, and examine the fringes produced by the acute ~~angles.—Try and find~~ the asymptotes and compare their positions with the distance apart of the points of the V's.

~~2. Mount two $\frac{1}{2}''$~~ steel balls touching one another on a sheet of plate-glass and examine the fringes; or suspend them from the pole of a magnet, one below the other.

3. Set up a parallel slit, as in § 257, and examine the fringes, as the width of the slit is varied. Explain the results by "Cornu's spiral" (Preston's *Light*, p. 280 of the third edition, and p. 225). That is, draw a spiral, cut a piece of cotton to represent the width of the aperture, place it on the spiral, making it lie on the line. The line joining its ends is the resultant amplitude.

CHAPTER XVI

RESOLVING POWER OF OPTICAL INSTRUMENTS

Definition.—By the resolving power of an instrument is understood the minimum distance by which two points or lines must be separated in order that they may be seen and recognised as *two*.

Resolving Power of a Telescope, Determination of the Wave-length of Light from the Resolving Power.

266. **Circular Aperture.**—*Apparatus.*—A simple telescope (*e.g.* the one in the spectrometer); some fine wire gauze; a reading microscope, or screw gauge; a metre scale; iris diaphragm, or some card or thin metal ones; a good monochromatic light (*e.g.* a sodium flame); a piece of ground glass.

Place a piece of fine wire gauze in front of the flame at the far end of the room. Point the telescope at it and focus the gauze.

Fig. 285.—Resolving Power of Telescope.

Cut a series of circular diaphragms out of black card, and, placing them one by one over the objective of the telescope, determine the size of the smallest one which enables the wires upon the gauze to be separated distinctly.

If an iris diaphragm can be obtained and mounted in front of the objective, the right size may of course be found much more easily.

Airy (*Camb. Phil. Trans.* 1834) showed that the angular radius of the disc, which a telescope gives as the image of a bright point, is given by

$$\theta = 1 \cdot 2917 \cdot \frac{\lambda}{2R},$$

where 2R is the radius of the clear aperture of the lens (see § 284).

Thus each bright point of flame seen through the wire gauze will produce a disc of this size, and if the diameter of the iris diaphragm be adjusted until the dark line which corresponds to the image of a wire is on the point of vanishing, θ will then be the angle subtended by the diameter of a wire of the grating.

Find the radius R, and, by measuring the thickness of the wire either with a screw gauge or a reading microscope, the wave-length can be calculated by means of this formula.

267. **Rectangular Aperture.**—*Apparatus.*—As above, except the iris diaphragm. In place of the iris an adjustable rectangular diaphragm is required. This can be easily constructed out of a piece of thin zinc (about No. 24 B.W.G) or even from *very stout* "tin-foil" (*i.e.* thin sheet, lead). Cut out a rectangular aperture about $\frac{1}{2}''$ square in a sheet, say $4''$ by $2''$. Turn down about a quarter of an inch at the top and at the bottom (Fig. 286). Now cut two pieces each about $1\frac{1}{2}''$ wide with straight edges and insert them under the turn-down edges. By sliding them in and out the width of the opening can be adjusted at pleasure.

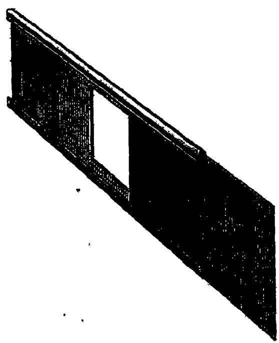


FIG. 286.—Adjustable Slit.

Point the telescope at the distant wire gauze as before, one set of wires of the latter being vertical. Hold the adjustable slit in a Bunsen clip in front of the telescope. On narrowing the slit the vertical wires will disappear.

If the width of the aperature is 2R, $\theta=\frac{c}{d}=\frac{\lambda}{2R}$ gives a dark line between two bright lines at a distance apart c, of which the distance from the telescope objective is d, which is 0·8 of the brightness of the lines themselves, and this appears to be the intensity which just enables the two bright lines to be seen as *two.*

If the width be 3R, the dark line is $\frac{0\cdot18}{1\cdot04}$ of the intensity of the bright lines.

If 4R it is quite dark.

So that, if the aperture be rectangular and of such a width that the dark lines between two bright lines or dots *can just be seen,* the formula $\theta=\frac{\lambda}{2R}$ will be an approximate value of the wave-length.

θ is the angle subtended at the objective by the dark space separating the two bright lines.

The Resolving Power of a Microscope will be found in § 284.

Resolving Power of the Eye.

268. The resolving power of the eye is the ratio of the distance apart of two neighbouring bright points to their common distance from the eye, when these points can only just be recognised as *two.* In other words, it is the circular measure of the angle subtended by two points which can just be resolved by the naked eye.

NOTE.—As the image of a distant object formed by an ordinary spectacle lens subtends exactly the same angle at the lens as the object itself, the angles subtended at the front nodal points of the eye by this image and the object will be approximately equal. Therefore a short-sighted person may use glasses in performing the following experiment without materially affecting the result.

Apparatus.—A convex mirror about $\frac{1}{2}$ inch radius of curvature (*e.g.* a spherical bulb of one inch diameter, filled with mercury), on stand; a screen with two holes each $\frac{3}{10}$ inch diameter, of which the distance apart can be varied, made as described below; an incandescent gas flame; a metre scale; a large screen to cut off the direct light of the flame from the eye; a pair of dividers.

Take a sheet of thin zinc about 6 inches square. Drill a $\frac{3}{10}$ inch hole in its centre. Cut a slot about half an inch wide and two inches long, beginning a quarter of an inch from the hole. A quarter of an inch from the end of a strip of zinc four inches long and one inch wide drill a hole $\frac{3}{10}$ inch diameter. Solder two bent strips of zinc to the large piece, so that the smaller piece may slide easily through them, with its hole over the slot. A bent card can be attached at one end of the large plate and at the other end to the moving strip, as shown in the figure, to cut off the light which comes through the slot, when the holes are more than half an inch apart. Finally the plate must be mounted on a block or in a retort stand at such a height that the holes are in front of the frame.

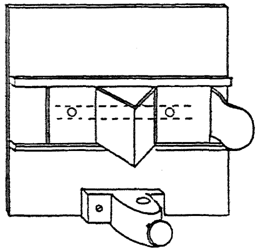


FIG. 287.

Set up the mercury bulb at about three feet from the plate with the two holes. Let the latter face the bulb, and adjust the flame behind it so that light from each hole may fall upon the bulb. An eye by the side of the zinc plate will see the images

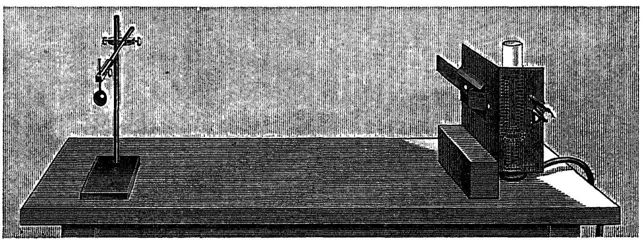


FIG. 288.—Resolving Power of Eye.

of the holes in the mirror. Interpose the large screen to shade the eye from the flame. Now vary the distance apart of the holes. When about two inches apart (the light that would escape from the end of the slot being cut off in some way) *two* very small bright points should be visible on the mirror. These images will approach one another as the holes are brought nearer together (by pushing in the sliding piece of zinc), and presently it

will become impossible to tell whether the image consists of *two* or only *one* bright point. The distance apart at which the points can just be recognised as two can be found with more accuracy than would perhaps be expected. Several independent readings should be taken—the distance a from the edge of one hole to the corresponding edge of the other (which, of course, is also the distance between their centres) being measured each time with the dividers, or of course the sliding piece may be graduated so that the reading gives the distance apart of the holes. The radius of the bulb r, the distance from the surface of the bulb to the zinc plate u, and the distance from the eye d will also be wanted.

Then if b is the distance between the images of the holes,

$$m=\frac{b}{a}=\frac{-f}{u-f}=\frac{r}{2u+r} \quad \text{(page 79)},$$

and the resolving power of the eye is given by

$$e=\frac{b}{d}=\frac{ar}{d(2u+r)}.$$

The experiment may be repeated, using vertical slits instead of pin-holes.

Resolving Power of a Grating.

269. *Definition.*—The resolving power of a spectrometer (whether a prism or a grating is used to produce the dispersion) is defined to be the inverse ratio of the difference of the wave-lengths of two lines that can just be *resolved* (*i.e.* seen as *two*) to their wave-length.

Apparatus.—Diffraction grating, and spectrometer; scale; sodium flame; the adjustable slit described in § 267.

Theory.—The deviation produced by a grating is given by

$$\sin\theta=\frac{m\lambda}{a+b},$$

where m is the order of the spectrum and a and b the widths of the space and rulings respectively. From a neighbouring wave-length $\lambda+\delta\lambda$, the increase of deviation $\delta\theta$ is given by

$$\delta\theta.\cos\theta=m.\frac{\delta\lambda}{a+b}. \quad \text{.............................(i)}$$

If the aperture of the telescope be supposed rectangular and of the width 2R, the number of lines visible through that aperture is obviously, from Fig. 289, given by

$$2R = n.(a+b)\cos\theta. \quad \text{......................(ii)}$$

Using Airy's formula for the resolution, the two lines will be just separated when

$$\delta\theta = \frac{\lambda}{2R} \text{ (see page 369).}$$

Multiplying by $\cos\theta$ we get

$$\frac{\lambda}{2R}\cos\theta = \delta\theta.\cos\theta$$

$$= m\frac{\delta\lambda}{a+b} \quad \text{(by (i))}$$

$$= nm\frac{\cos\theta.\delta\lambda}{2R} \quad \text{(by (ii))};$$

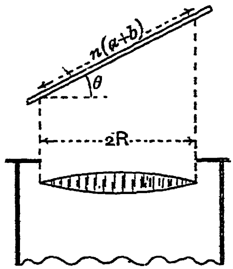


FIG. 289.

or for the *resolving power*, we have

$$P = \frac{\lambda}{\delta\lambda} = mn. \quad \text{..............................(iii)}$$

Experiment.—Adjust the grating[1] as described on page 250 to view the sodium lines; (it is necessary to use a powerful light).

See that the collimator slit is as fine as it can be made, and that the lines are in thoroughly good focus. Then place the adjustable slit over the objective of the telescope, adjust it until the lines can no longer be seen as two, and measure the width of the opening 2R.

From this, knowing the number of lines to the centimetre on the grating, and the angle which the telescope makes with the normal to the grating θ, it is easy to calculate the number of lines n_1 visible through the rectangular aperture now left.

Thus from (iii) we can find $\delta\lambda$, assuming we know λ. $\delta\lambda$ ought to be the difference between the wave-lengths of the sodium lines, *i.e.* $\cdot00005896 - \cdot00005890$. Thus $\frac{\lambda}{\delta\lambda}$ should be about 980.

The Resolving Power of a Prism.[2]

270. *Apparatus.*—Spectrometer; prism; card, stamp paper and scale.

[1] A rather coarse grating is best.

[2] Rayleigh, *Phil. Mag.*, 1897.

Theory.—The refractive index of a prism is given by

$$\mu=\frac{\sin\frac{1}{2}(\theta+A)}{\sin\frac{1}{2}A}.$$

For a neighbouring wave-length $\mu+\delta\mu$, we obtain

$$\delta\mu=\frac{\cos\frac{1}{2}(\theta+A)}{\sin\frac{1}{2}A}\,\frac{\delta\theta}{2}.$$

In Fig. 290, if PQRS be a ray passing symmetrically through the prism ABC, the angle $O\hat{Q}R$ is $\frac{1}{2}\theta$, and therefore if AN is parallel to PQ and BN perpendicular to AN, the angle ABN is $\frac{1}{2}(\theta+A)$.

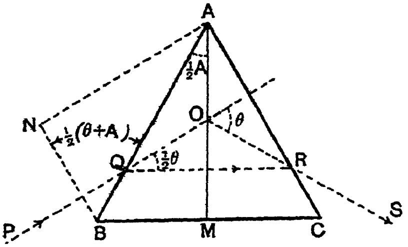


FIG. 290.

Therefore

$$\delta\mu=\frac{\cos\frac{1}{2}(\theta+A)}{\sin\frac{1}{2}A}\,\frac{\delta\theta}{2}$$
$$=\frac{BN}{BM}\cdot\frac{\delta\theta}{2}$$
$$=\frac{2R}{t}\cdot\delta\theta,$$

where 2R is the aperture BN of the beam and t is the length of the base BC of the prism.

If there is a series of prisms, the formula is still true, if t is put for the total thickness of the glass traversed at the base ends of the prisms,

$$\delta\theta=\frac{t}{2R}\,\delta\mu. \quad \text{......................(i)}$$

For resolution we have as before,

$$\delta\theta=\frac{\lambda}{2R}.$$

Therefore from (i) for resolution,

$$t=\frac{\lambda}{\delta\mu}. \quad \text{......................(ii)}$$

If we assume Cauchy's formula,

$$\mu=A+\frac{B}{\lambda^2},$$
$$\delta\mu=-2B\frac{\delta\lambda}{\lambda^3},$$

and we get from (ii),

$$t=\frac{\lambda^4}{2B\delta\lambda}. \quad \text{......................(iii)}$$

In this formula the constant B will have to be found for the particular glass used in the prism.

For Chance's extra dense flint glass, Rayleigh gives the following figures for the two sodium lines:

$$\mu_d = 1 \cdot 650388 \qquad = A + \frac{B}{\lambda_d^2},$$

$$\mu_c = 1 \cdot 644866 \qquad = A + \frac{B}{\lambda_c^2},$$

$$\mu_d - \mu_c = \cdot 005522,$$

$$\mu_d = 5 \cdot 889 \,.\, 10^{-5},$$

$$\mu_c = 6 \cdot 562 \,.\, 10^{-5};$$

from this, $$B = \cdot 984 \,.\, 10^{-10};$$

substituting, $$t = \frac{10^{10} \,.\, \lambda^4}{1 \cdot 968 \,.\, \delta\lambda};$$

for the sodium line $$\delta\lambda = \cdot 006 \,.\, 10^{-5};$$

therefore $$t = 1 \cdot 02 \text{ cms.}$$

Thus, for this glass, if the face of the prism be smaller than the objective of the telescope, and if a card is slid along the prism from the base end, gradually diminishing the width of the beam of light which it transmits to the telescope, when the sodium lines cease to be resolved, the base of the prism that is left should be found to be 1·02 cms.

Comparing a grating with a prism, for equal resolving powers t must equal $\frac{mn\lambda}{2B}$: putting this into figures as in the above case, t becomes $\frac{1 \cdot 037 mn}{1000}$. For a grating of only 3000 lines, if we substitute $n = 3000$ and $m = 4$ in the above expression, we get $t = 12\frac{1}{2}$ cms. Showing that to produce a resolution equal to that of the spectrum of the fourth order, given by a grating having only 3000 lines on its visible surface, a battery of prisms would be required, having a total length of base of $12\frac{1}{2}$ cms.

271. *Experiment.*—Adjust the prism on the spectrometer table as usual, and set the instrument to minimum deviation. If the face of the prism is smaller than the telescope objective, so that the beam is limited by the prism, and enters the telescope as a rectangular one, it will be sufficient to slide a card along the prism from the base end, and reduce the aperture until the sodium lines can just not be seen to be double. If the prism is too large, slips of stamp paper can be attached to its face so to leave a horizontal slip about half an inch broad, and on a level with the middle of

the objective. Place the prism so that the edge is within the field of view of the telescope, and slide the card along as above directed. Measure the length of the base of the prism left. This gives the value of t. Or, better, a slit similar to that described in § 267 can be used. In this case, the slit may be placed anywhere within the beam of light, and knowing the angle of the prism, the width of base that should correspond can be calculated. Determine the B in Cauchy's formula (as in § 99).

Substitute $$\lambda = 5{\cdot}889 \,.\, 10^{-5},$$
$$\delta\lambda = {\cdot}006 \,.\, 10^{-5},$$

and this value of B in the formula

$$t = \frac{\lambda^4}{2B \,.\, \delta\lambda},$$

and see if the value of t so found agrees with the one found directly.

The Echelon Grating.

272. In the *American Journal of Science*, in 1897, Michelson described an echelon grating by which dispersion of a very high order was produced.

The grating is made of a pile of glass plates of exactly equal thickness mounted on one another like a flight of steps. It is placed in the spectrometer so that the light from the collimator enters what would correspond to the base of the steps, and emerges normally from what would be the upper surface of each step. In addition to the light issuing from the steps normally, the light will be diffracted from the steps with a large difference of phase. Thus, diffracted spectra of a very high order are produced. Owing to the large difference of phase—perhaps amounting to ten thousand wave-lengths—the spectra will overlap throughout all their length, and a great number of spectra would be visible at the same time; it is necessary therefore that the light falling upon the grating should already be approximately monochromatic. This is usually attained by means of a second spectrometer furnished with an ordinary prism, the eye-piece of the telescope being removed and the light received upon the slit of the collimator supplying the echelon.

Theory.—Let m be the order of the spectrum (that is the number of wave-lengths in the difference of path of the light from the successive plates), and consider the rays, one diffracted after reaching the surface of the first, and the other after reaching the surface of the second step. The wave-front of the diffracted light will be cd, and the difference of path $m\lambda$ given by

$$m\lambda = \mu\mathrm{BD} - \mathrm{AC}$$
$$= \mu t - t \,.\, \cos\theta + s \,.\, \sin\theta,$$

where t is the thickness of a step and s the distance from A to B.

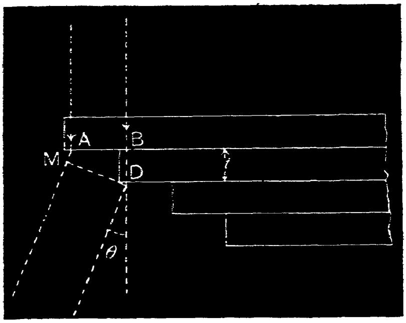


Fig. 291.

Therefore

$$\frac{d\theta_1}{d\lambda} = \frac{m - t \,.\, \frac{d\mu}{d\lambda}}{t \,.\, \sin\theta + s \,.\, \cos\theta},$$

$$\frac{d\theta_2}{dm} = \frac{\lambda}{t \,.\, \sin\theta + s \,.\, \cos\theta},$$

calling $\delta\theta_1$ the displacement corresponding to a change $\delta\lambda$ in the wave-length, and $\delta\theta_2$ the change corresponding to δm in the order of the spectrum (for consecutive spectra $\delta m = 1$)

If we assume Cauchy's formula,

$$\mu = \frac{a+b}{\lambda^2},$$

and take as a first approximation

$$m = (\mu - 1) \,.\, \frac{t}{\lambda}$$

we have $$\frac{\delta\theta_1}{\delta\theta_2} = \{(\mu - 1) + 2(\mu - a)\}\frac{t}{\lambda} \cdot \frac{\delta\lambda}{\lambda}$$

if we make $$\delta m = 1.$$

For most specimens of plate glass, the expression in the brackets is nearly equal to 1, so that if $\frac{\delta\lambda}{\lambda} = \cdot 001$ (which it would be for the two yellow sodium lines, for instance), and if $t = 5$ mm. $= 10000\lambda$, about, we should have

$$\delta\theta_1 = 10 \,.\, \delta\theta_2,$$

that is to say that the difference between two sodium lines as seen in this instrument would be ten times the distance from the sodium line in the spectrum of one order to the corresponding line in the spectrum of the next order.

If the grating is to be of any use, the optical thickness t of each of the pieces of glass must be very exactly the same; it should not vary by $\frac{1}{5}$ of a wave-length. For this it is of course necessary that the steps should be cut all from one piece of glass; but it is further necessary that that piece of glass shall be of optically constant thickness within $\frac{1}{5}$ of a wave-length for the entire surface. This constancy is attained by examining the plate as described in § 241 and polishing the thicker parts down until correct.

For this reason the cost of a grating is very high, and as the grating is only useful in very special research where the resolution of lines very close to one another is needed, very few laboratories are supplied with this instrument; but in the *Phil. Mag.* for June, 1901, Professor Wood describes the construction of a mica grating quite easily made, which is of course of no use as an optical instrument, but will demonstrate the method of using it.

273. **Wood's Mica Echelon Grating.**—The grating is made in a similar way to the Fox wedge described later on § 395, except that the thickness of mica used is greater and the mica is mounted dry without balsam. He used a thickness of about ·05 mm. $=t$, and the width of the step, $=s$, was ·5 mm. As the grating must not be mounted in balsam, he was only able to use nine plates owing to their opacity.

The steps were attached to one another with sealing wax at their edges, which was melted by a needle, and were placed in position by means of scratches made on a glass plate at the distances of ·5 mm. with a dividing engine, the plates being put n position under the microscope.

The grating was mounted on a piece of square cardboard under an opening 5 mm. wide and 2 cms. high, leaving an extra space of ·5 mm. to serve as the grating space of zero retardation. The whole number of "lines" was therefore 10.

The retardation of the film was measured with an interferometer and found to be 50 sodium waves.

The resolving power represented by the product of the number of lines (10) and the order of the spectrum, which was 50, would accordingly be 500.

The sodium lines, which require at least 1000 for resolution, were beyond the power of the instrument, but the yellow mercury lines were easily resolved.

The light from a mercury tube after passing through the collimator lens and prism was focussed on the collimator of a spectrometer,

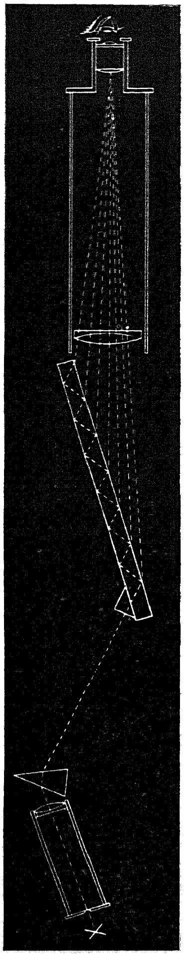


FIG. 292.—Lummer's Echelon.

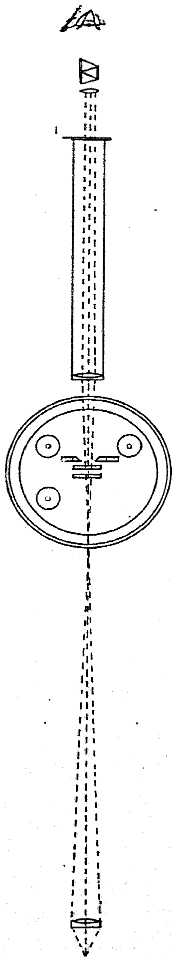


FIG. 293.—Fabry and Perrot's Plates.

the green line being brought on the slit. Placing the echelon on the table of the instrument, the spectra showed clear and sharp.

When first found, the central maxima appeared between the principal maxima owing to the small number of grating elements. By slightly shifting the position of the lens the yellow light from the tube was now focussed on the slit, when the maxima immediately doubled. The distance between the components was about $\frac{1}{3}$ of the distance between the spectra.

Using the formula above, $\frac{\delta\theta_1}{\delta\theta_2}=\frac{t}{\lambda}\cdot\frac{\delta\lambda}{\lambda}$,

where $t=\cdot 05$ mm., $\frac{t}{\lambda}=94$, $\frac{\delta\lambda}{\lambda}=\frac{1}{280}$ for the yellow mercury line, we get $\frac{\delta\theta_1}{\delta\theta_2}=\frac{94}{280}=\frac{1}{3}$, approximately.

Probably much better results could be obtained with selenite, as it is much more transparent than mica, and could therefore be made with much thicker plates.

274. **Lummer's Echelon.**—In this instrument (Fig. 292) the light from, say, a mercury lamp, after passing through a collimator and one or more prisms to disperse it, is allowed to fall obliquely on a parallel plate of glass about 5 mm. thick, in which it is reflected to and fro. The surfaces are unsilvered. A portion of the light emerges at each reflection, and there several portions enter the telescope. As their difference of path is very large, the resolution obtained is of a very high order, even with a small number of elements. To enable the light to enter the plate a small prism is cemented on to it, as shown in the figure. The theory is very similar to the Michelson Echelon, and the plate has to be equally carefully worked, but on the other hand only one narrow strip is required.

275. **The Fabry and Perrot Apparatus,** § 243, may be used as an instrument of the same type as the above, by inserting it after the second collimator of § 272 in place of the echelon, or the separation may be produced afterwards as in Fig. 293. The plates have, however, to be worked to a high degree of accuracy if it is to be used in this way; they should be separated to a greater distance. This may be done by mounting them against the opposite ends of a short tube of fused quartz, of which the temperature coefficient of expansion is very small.

CHAPTER XVII.

THE COMPOUND MICROSCOPE

276. The compound microscope may optically be supposed to consist of two independent systems of lenses, the objective and the eye-piece. The objective is arranged to produce a much enlarged image of the object at the upper end of the tube. The eye-piece is practically a magnifying lens used to view this. In other words, the eye-piece forms a *virtual image* of the image formed by the objective, and it is this virtual image that the eye of the observer looks at. As the eye has a large range of distances at which objects can be clearly seen (in a normal eye from about six inches to infinity), the virtual image may be found anywhere within this range. But as the observer knows that the object at which he is looking is near him, he will, as a rule, unconsciously focus his eye for near objects when he looks through the microscope, and then focus the microscope (namely adjust the distance between the object and the objective) to see the image distinctly. The virtual image is then, of course, formed at the distance to which his eye has been accommodated. If, however, he will try to imagine the image at a great distance (and so focus his eye for infinity) he can equally well adjust the distance of the objective from the object to obtain a clear image, which is then at infinity. *The angular* size of the image is almost the same in each case, so that obviously its linear dimensions must be very different; if the image is formed at infinity, its linear size is infinite.

Magnification.

277. From what has been said above, it is clear that the ratio of the linear size of the image to that of the object is a number

that can be made as great as we please by merely adjusting the focus. We must therefore either consider the angular magnification (as in a telescope) or adopt some convention as to the distance at which the final—virtual—image is to be formed. It is agreed to assume the final image to be ten inches (or 250 mms. when using the metric system) from the eye, and then the magnification is the ratio of the linear size of this image to that of the object.[1] When a microscope is used to project a real image—whether on to a sensitive plate in photo-micrography or on to a screen with the lantern-microscope—there is no ambiguity in defining the magnification as the ratio of the linear size of the image to that of the object.

The experimental determination of the magnification depends upon the use of a screw—either used in a dividing engine to rule a scale or used to traverse the stage of the microscope itself.

278. **Scale method.**—*Apparatus.*—A glass slip upon which a millimetre is divided into tenths and hundredths; the microscope of which the magnification is to be determined—a half-inch objective is a suitable power to practise upon; two white cards, which must be supported upon blocks so as to be in a plane normal to the axis of the microscope, and 25 cms. from its eye-piece. Each card is to have a single thick black line ruled across it; lamp; millimetre scale.

Adjust the microscope and lamp; place the scale upon the stage and focus it. Put one of the cards at the right distance—25 cms. from the eye-piece—and to one side of the stage. Look through the microscope with one eye, and observe the card with the other. So place the card that the image of one of the rulings, seen with the other eye, appears to coincide with the line on the card seen with the other eye. Move the head slightly to and fro sideways, and see if the image appears stationary on the card.

[1] Even if the magnification were to be expressed in terms of the angles subtended by the object and image respectfully, a convention with respect to the distance of the object would have to be made. It would not do, for instance, to consider the object to be at its actual distance from the eye; for the angle it subtends at the eye would then depend upon the length of the microscope tube; it would be twice as great for a 6″ tube as for a 12″ tube; and thus, if the final image subtended the same angle in each case, the "magnification" would be twice as great in the one case as in the other (an absurd result).

If the image moves relatively to the card with every movement of the head, it shows that it is not formed at the right distance, and the focus must be adjusted until the image and the card keep together. Then arrange the second card by the side of the first in such a position that the line upon it may coincide with another ruling of the image. Measure the distance apart of the lines on the cards, and knowing the actual distance apart of the rulings on the slip, the magnification is obtained. If difficulty is experienced owing to the variation in the convergence of the eyes during the experiment, it can be avoided by standing at the *side* of the microscope so that the image of the scale is vertical on the retina (or—if the stand of the microscope will allow it—turning the scale itself until its image is vertical). Or some form of "camera lucida" may be employed so that the image and the cards are both seen by the one eye.

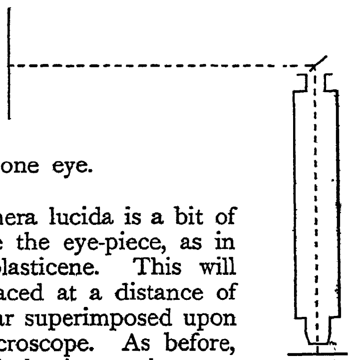


FIG. 294.

279. The simplest form of camera lucida is a bit of micro-cover glass attached above the eye-piece, as in the figure, with a pellet of plasticene. This will reflect an image of the card placed at a distance of ten inches, and cause it to appear superimposed upon the image seen through the microscope. As before, the focus must be adjusted until the images have no relative motion when the eye is moved slightly to and fro across the eye-lens. Either the divided scale or the traversing stage can be used, and the magnification found as above directed. It is necessary that the card should not be too brightly illuminated or it will not be possible to see the image through the tube properly. A black card with white lines, or simply a circular black card of known diameter, is preferable to a white one.

When a camera lucida is to be used to draw the object seen in a microscope it is best to look directly at the paper, and to observe the image formed by the microscope in the prism or mirror, as the case may be; for it is very difficult to draw when the hand is seen in a mirror. But in the above case it is not required to draw, and the simpler arrangement in the figure is sufficient.

(*Exercise.*—Draw a curve giving the magnification produced by a given objective and eye-piece, at all positions of the draw tube.)

280. *Definition.*—The names **entrance pupil** and **exit pupil** are given to the images in the initial and final media of the stop that limits the aperture of the system; *e.g.* in a photo-objective they will be the images of the diaphragm as seen from the back and front respectively; in the microscope the entrance pupil is the image of the stop at the back of the objective that is formed by the objective; the exit pupil is the image of the same stop that is formed by the eye-piece, usually called the "eye-ring"; it is where the eye should be placed

281. A method of defining magnification has been suggested by Abbé, and may be called

Definition.—**The Abbé Magnification.**—It is the ratio of the tangent of the angle subtended by the image at the exit pupil to the linear size of the object

$$=\frac{\tan\alpha'}{y}=\text{V (say)}.$$

Definition.—**The Angular Magnification.**—This is defined to be the ratio of the tangents subtended at the exit and entrance pupils by the image and object respectively,

or
$$\frac{\tan\alpha'}{\tan\alpha}=\frac{\frac{y'}{e'}}{\frac{y}{e}}=\frac{y'}{y}\cdot\frac{e}{e'}=\text{A (say)}.$$

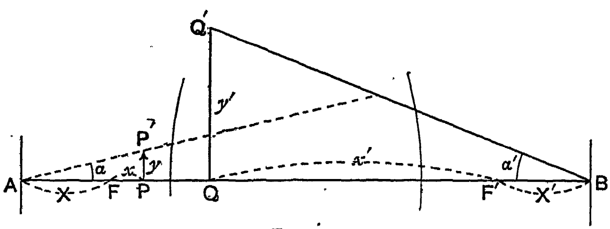


Fig. 295.

Let $\quad \text{AF}=\text{X} \quad \text{BF}'=\text{X}',$

$\text{PF}=x \quad \text{QF}'=x',$

$\text{AP}=e \quad \text{BQ}=e'.$

Then $$ff' = xx' = XX'.$$

And $$A = +\frac{x(x'-f')}{x(x-f)} \cdot \frac{x(x+X)}{x(x'+X')}$$

$$= +\frac{ff'-xf'}{x(x-f)} \cdot \frac{x(x+X)}{XX'+xX'} = -\frac{f'}{X'} = -\frac{X}{f},$$

Thus $$A = -\frac{f'}{X'} = -\frac{X}{f}. \quad \text{.................................(i)}$$

Also $$V = \frac{\tan\alpha'}{y} = -\frac{\tan\alpha}{y}\frac{X}{f}$$

$$= -\frac{X}{f(x+X)} = -\frac{1}{f} \cdot \frac{1}{1+\frac{x}{X}}$$

$$= -\frac{1}{f}\left(1-\frac{x}{X}\right)\left(\text{if } \frac{x}{X} \text{ is small}\right) = -\frac{1}{f}\left(1-\frac{X'}{x'}\right). \quad \text{........(ii)}$$

Thus V depends upon the *power* $\frac{1}{f}$, and the distances x' and X' of the exit pupil and the image from the back focal point.

If the system is to be orthoscopic for the points AB, then $\frac{\tan\alpha'}{\tan\alpha}$ is to be constant for all values of α.

Aperture.

282. **Hockin's Proof of the "sine law."**—Let Q and Q′ be the images, supposed aplanatic, of the points P and P′. Let ARR′ be the

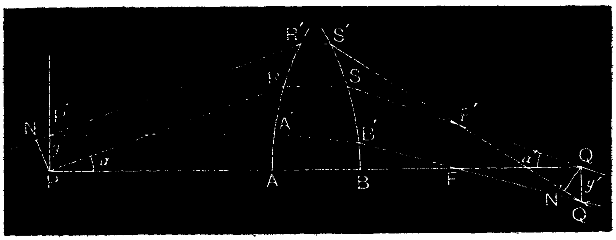


Fig. 296.

first surface, and BSS′ be the last surface of the combination, and let the refractive indices at P and Q be μ and μ' respectively. Draw a pair of rays from P and P′ parallel to the axis, which after

passing the principal focus F will arrive at Q and Q′, also a pair of parallel rays PR and P′R′, inclined to the axis at an angle α, and these will pass through the corresponding focus F′ and also arrive at Q and Q′ respectively.

Draw PN, QN′ at right angles to P′R′ and Q′S′ respectively.

Then as SQ and S′Q′ are parallel in the limit when y and y' are infinitely small, Q′QN is the angle α'.

As Q and Q′ are by hypothesis aplanatic images of P and P′, the "optical paths" from P to Q must be equal to one another, and so also must those from P′ to Q′ be equal to one another.

By "optical path" is to be understood the distances through the various media multiplied by the refractive indices of the media, for the light travels in any medium with a velocity inversely proportional to the refractive index of the medium.

Thus

$$\left.\begin{array}{l}\text{the optical path } (P'R'S'F'Q') = \text{optical path } (P'A'B'FQ') = \chi \text{ (say)},\\ \quad ,, \quad ,, \quad (PRSF'Q) = \quad ,, \quad ,, \quad (PABFQ) = \psi \text{ (say)}.\end{array}\right\}$$

$$\left.\begin{array}{l}\text{But the optical path } (P'A'B'F) = \text{optical path } (PABF),\\ \text{and (if } QQ' \text{ is small)} \qquad FQ = FQ'.\end{array}\right\}$$

Thus $\qquad \chi = \psi.$

Thus the optical path (PRSF′Q) = (P′R′S′F′Q′) = (P′R′S′F′N′Q′).

The left side of this equation = (N′P′R′S′F′ . F′N′), since F′ is the focus of the wave PN.

Thus the optical path (NP′ . P′R′S′F′N′) = (P′R′S′F′N′ . N′Q′).

Subtracting the common part,

$$\text{The optical path } (NP') = (N'Q'),$$

$$i.e. \ \mu NP' = \mu' N'Q',$$

or
$$\mu y \sin\alpha = \mu' y' \sin\alpha',$$

or
$$\frac{\mu \sin\alpha}{\mu' \sin\alpha'} = \frac{y}{y'}$$

$$= m \text{ (say)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

283. *Definition.*—**Numerical Aperture** or N.A. is defined to be the product of the refractive index of the medium on the object side of the system into the sine of the angular semi-aperture as seen from the object. $\qquad$ N.A. $= \mu \sin\alpha = a$.

As μ' is generally 1 and α' is small

$$\alpha' = \frac{a}{m}.$$

For we have just proved that

$$\frac{\mu' \sin \alpha'}{\mu \sin \alpha} = \frac{1}{m}.$$

If e is the semi-aperture of the exit-pupil of the objective, and l the distance from the exit-pupil to the image, $\alpha' = \frac{e}{l}$ from the figure, and $\therefore\ e = l.\frac{a}{m}$.

The same is true for the complete system if m be taken to be the final magnification.

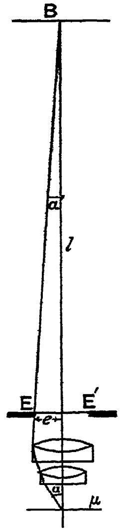


FIG. 297.

Resolving Power.

284. Assume the system aplanatic for the two points P and Q. Then a wave starting from P after passing through the system emerges as a spherical wave of which A is the centre of curvature. Only a portion of the wave emerges from the system, namely a cap AB having an angular radius α' as seen from Q. A neighbouring

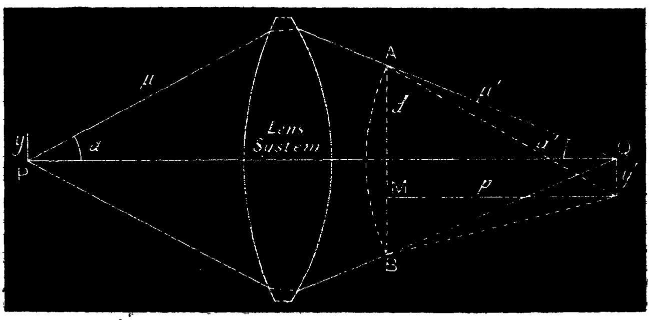


FIG. 298.

point Q' will receive a certain amount of light unless Q' is so situated that the light from one part of the cap AB interferes with the light from the other part. This will occur for the first

time when $Q'A - Q'B$ is rather less than a wave-length.[1] Up to a distance from Q such that $Q'A - Q'B$ is about a half wave-length, the illumination of Q′ is almost as great as that of Q; thus even with an aplanatic system, the image of a point P will be a disc at Q of finite radius.

$$Q'A - Q'B = \sqrt{(d+y')^2+p^2} - \sqrt{(d-y')^2+p^2}$$
$$= \sqrt{p^2+d^2}\left\{1 + \frac{2dy'}{2(p^2+d^2)} - 1 - \frac{2dy'}{2(p^2+d^2)}\right\}$$
$$= \frac{2dy'}{\sqrt{p^2+d^2}} = 2y' \sin \alpha',$$

and Q′ will be dark when this is about ·8 of a wave-length in the medium at Q.

So that the radius of the disc y' is $y' = \dfrac{\cdot 8\lambda}{2\mu' \sin \alpha'}$.

Thus, if y is the radius of an object which should produce a disc of this size,

$$y = \frac{\cdot 8\lambda}{2a},$$

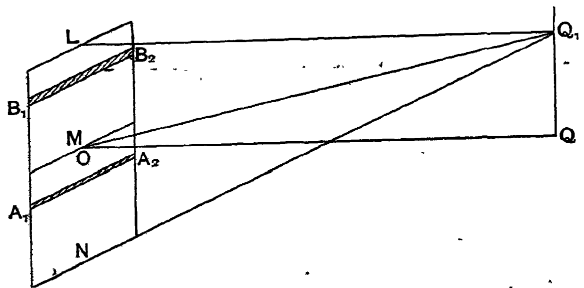


FIG. 299

[1] It would be exactly a wave-length if the aperture of the lens were square; for if the aperture be supposed to be divided into a large number of equal strips, parallel to a side of the square, the light from each strip B_1B_2 in one half of the aperture would exactly neutralize that from the corresponding strip A_1A_2 in the other half of the aperture (Fig. 299), as it would arrive at Q′ half a period ahead very approximately, since Q is the centre of curvature of the wave front, and therefore the difference of path at Q′ will increase nearly uniformly from L to N. With a circular aperture the strips will be of unequal length, and one near the centre,

where a is the numerical aperture,

since $$\frac{y'}{y}=\frac{\mu \sin \alpha}{\mu' \sin \alpha'}=\frac{a}{\mu' \sin \alpha'}.$$

Thus two bright points on an object which are $2y$ apart would give rise to discs each of radius y', which would just meet, surrounded by one or more rings, and if the illumination was constant up to the edge of these discs there would be no dark line between the images of the bright points, and they would appear as a short, bright line, instead of two separate bright points. But the illumination of the image of a *point* falls off according to the curve shown where

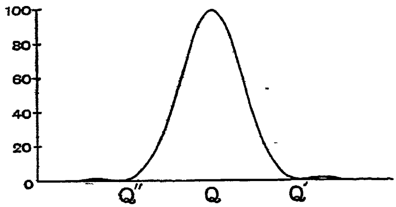


FIG. 301.—Point Source, Circular Aperture.

$$QQ'=\frac{\cdot 8\lambda}{2\,\mu' \sin \alpha'}.$$

Owing to the rapid fall of illumination the least distance apart at which separation is possible is less than this, and occurs very approximately when $Q'A - Q'B$ is *half* a wave-length.

If Q is a point on a *line* of light, the curve (Fig. 302) is very similar. The least distance for which the two bright lines can be separated is, according to Rayleigh, when the first maximum of

such as M_1M_2, will take more than one of those near the edge, such as N_1N_2, together with B_1B_2, to neutralize it. (Fig. 300.)

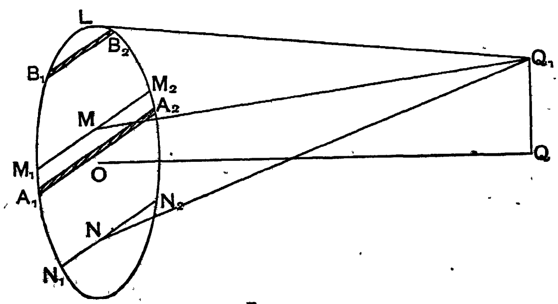


FIG. 300.

the image of one line coincides with the first minimum of the other. The total illumination of the two lines is shown in

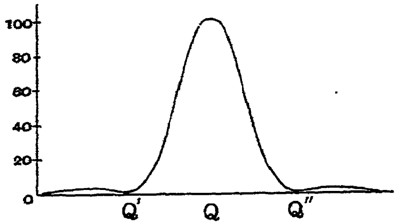


FIG. 302.—Line Source, Circular Aperture.

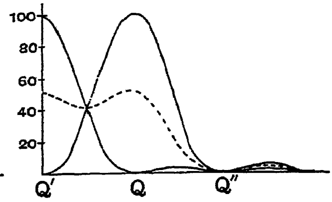


FIG. 303.—Two Line Sources.

Fig. 303. There is a drop between the two maxima to ·8 of the maximum.

The least distance apart of the lines is given by

$$z = 2y = \frac{\lambda}{2a}. \quad \text{..............................(iv)}$$

The reciprocal of this distance is the *resolving power* of the system. The resolving power is therefore $\frac{2a}{\lambda}$, where λ is the wave-length of the light in air and a is the numerical aperture of the system.

At this distance there will not be a *black* line between the images of the two points, but only a very slight decrease in brightness. As the distance apart is increased a darker interval will separate the images, and they can of course be still more easily seen as two.

The resolving power depends therefore upon the N.A. of the objective, and not (as one would at first think) upon the focal length or *power* of the objective, of which it is now seen to be independent. It can be increased by using light of short wave-length or by increasing the N.A. or both.

285. **Diffraction of a Grating in a Medium of Refractive Index μ, illuminated by Oblique Light.**—Let the incident light make an angle β with the normal, and the diffracted angle α with the normal on the opposite side of it.

Then if AM is an incident and AN a diffracted wave, the difference of phase of the light from apertures of the grating z apart is

$$MB + NB = z \sin \beta + z \sin \alpha.$$

The first bright line is given by making this difference a wave-length in the medium or $\frac{1}{\mu}$ of the wave-length in air. If the direct beam can just enter one edge C of the lens, and the diffracted beam just enter the other D, so that the lens can just embrace both, making $\beta = \alpha$ and each equal the angular aperture of the system, we have

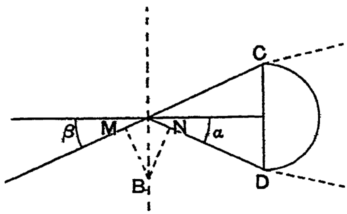


FIG. 304.

$$z 2 \sin \alpha = \frac{\lambda}{\mu},$$

so that

$$z = \frac{\lambda}{2 . \mu \sin \alpha} = \frac{\lambda}{2a}.$$

We have seen above that this is the least distance apart that bright points (or lines) may be in order that the lens system may just resolve them. Thus we arrive at the practical test for resolution.

"*If a given lens cannot embrace at least the direct beam and one diffracted beam emitted by a grating illuminated by an oblique parallel beam of light, it cannot* resolve *that grating, however it is illuminated.*"

Since parallel light will be focussed by the objective at its back focal plane, these spectra will be formed there. They can be seen by removing the eye-piece and looking down the tube; then if a ruled grating or other similar periodic structure is upon the stage, illuminated by the light from a distant lamp, or by the condenser with a slit below it, the spectra are easily seen in its posterior focal plane as little coloured bands in the disc of light at the back of the objective. They are formed again of course at the "eye-ring," but are too small to be easily seen. They may be observed by adding *another* eye-piece above the ordinary eye-piece.

If the incident light is normal to the plate, the first diffraction spectrum will not be embraced by the lens unless z is twice this, *i.e.* $z=\frac{\lambda}{a}$.

In this case (if the eye-piece is replaced) the grating also cannot be resolved unless $z=\frac{\lambda}{a}$; for the beams of light issuing from the grating spaces are no longer independent, but have a phase relation. Thus the effect in the image plane cannot be found by merely adding intensities, but the amplitudes of the images due to the individual spaces will have to be compounded as in ordinary interference phenomena.

286. **The Effect of Aperture on Defining Power.**—The larger the number of diffracted rays admitted into the objective the greater is the similarity between the image and the object.

(1) Perfect similarity depends on the admission to, and utilisation by, the optical combination of the whole of the diffracted rays which the structure is competent to emit.

(2) When a portion of the total diffraction fan is lost the image will be more or less incomplete and dissimilar to the object.

The smaller the object the greater the width of the diffraction fan.

The numerical aperture $\mu \sin \alpha$ measures the number of rays an objective can admit.

The diffracted fan has a greater angle than 180° if the object is reduced in size to a small number of wave-lengths. Then either a very highly refractive medium, or a light of very short wave-length is necessary to reduce the angle.

If in the case of a regular periodic structure the central maximum and the first diffracted band on one side are included, the distance apart of the striae (or the number per inch) will be correctly given; but the breadth of each single stria will not always be shown correctly. If only this one fan in addition to the central maximum is included by the objective, the light and dark intervals will appear of equal breadths and gradually increasing and decreasing brightness.

Note that $\mu \sin \alpha$ measures the minuteness of structural details that can be *separated* by the objective, not the minuteness of details that can be *seen*.

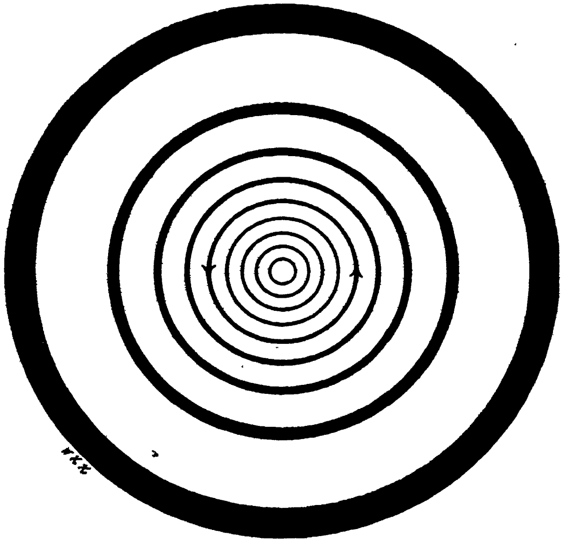


FIG. 305.—Cheshire's Apertometer.

287. **Measurement of N.A. of a Low-power Objective by the use of an Object ruled with Concentric Circles.**—Let P and Q be the aplanatic foci of an ordinary micro-objective working in air.

Then the numerical aperture is sin α, if α is the largest angular cone the objective can transmit, and $\dfrac{\sin \alpha}{\sin \alpha'}=m.$ Let the objective be racked up a distance PA so that the image of the object AA′ is at BB′. If PA is some known amount—say 1 cm.—then a scale could be ruled, the divisions upon which seen from P subtended angles APA′, of which the sines were ·1, ·2, ·3, ... The aperture would obviously be the largest of these circles of which the image was formed at B′, which could be seen from Q when the eye-piece was removed, or by the use of an additional eye-piece above the ordinary one. To obtain accurate results in the case of low power objective (with

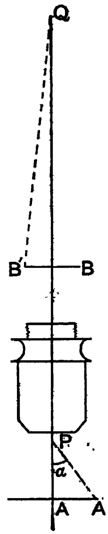


FIG. 306.

which the distance QB is small), a pinhole should be placed at Q, *i.e.* at the focal plane of the eye-piece. Focus the microscope on the chart, rack it up the amount PA (1 cm. suppose), remove the eye-piece, look through a pinhole at Q, and count the number of circles visible in the back of the objective.

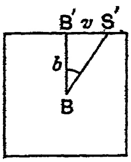


FIG. 307.

288. **Geometrical Shape of the Object of which a Lens obeying the "sine law" shall yield a Square as Image.**—Let BB′S be a section of the image at B of Fig. 307, and AA′S′ be a section of the object at A of Fig. 308. Let S′ be a point on the image, and S the corresponding point on the object.

Let the lines SP and S′Q′ make angles α and α' at P and Q respectively with the axis PQ.

Then $$\sin\alpha=\frac{AS}{SP};\quad \sin\alpha'=\frac{BS'}{S'Q};$$

$$\frac{b}{v}=\frac{y}{x}\text{ (since BS}'\text{ is in the same plane as AS),}$$

and $$\sin^2\alpha=m^2\sin^2\alpha'.$$

Let the coordinates of S be x, y; and those of S′ (which is to be a point on a straight line) be b, v.

$$\therefore\quad \frac{x^2+y^2}{x^2+y^2+t^2}=\frac{m^2(b^2+v^2)}{b^2+v^2+s^2}=\frac{m^2(x^2+y^2)\dfrac{b^2}{y^2}}{(x^2+y^2)\dfrac{b^2}{y^2}+s^2},$$

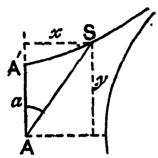


FIG. 308.

or $$m^2(x^2+y^2+t^2)=x^2+y^2+\frac{s^2y^2}{b^2},$$

or $$x^2(m^2-1)-y^2\left(\frac{s^2}{b^2}+1-m^2\right)+m^2t^2=0,$$

which is the equation for the locus of S. In the case of the microscope, as α' is small,

$$\frac{s}{b}=\frac{1}{\alpha'},\text{ nearly,}$$

$$=\frac{m}{\sin\alpha},$$

and therefore is greater than m, unless $\alpha=90°$. So these curves are hyperbolae.

The equation may be written

$$x^2-y^2\left\{\frac{1}{\alpha'^2(m^2-1)}-1\right\}+\frac{m^2}{m^2-1}t^2=0.$$

As m is large, this is practically

$$x^2-y^2\left\{\frac{1}{\sin^2\alpha}-1\right\}+t^2=0,$$

and if t is fixed (say 1 cm.), the equation represents a set of hyperbolae which will become straight lines when imaged by an objective, of which the principal focus is 1 cm. above the plane containing the hyperbolae.

If $x=0$, and $y=AA'=a$ (Fig. 308)

$$\frac{1}{\sin^2\alpha}=1+\frac{t^2}{a^2},$$

or

$$\sin\alpha=\frac{a}{\sqrt{a^2+t^2}}.$$

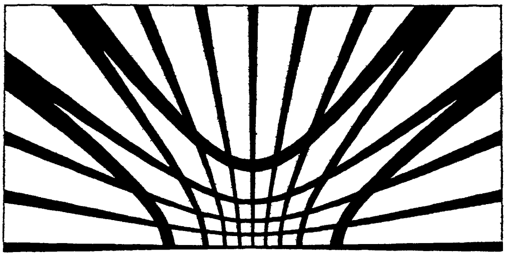


FIG. 309.—Abbé's Hyperbolae.

Thus if t be taken as 1 cm., and a series of hyperbolae be drawn with $\sin\alpha=\cdot 1$, $\cdot 2$, $\cdot 3$, etc., successively, these hyperbolae will be imaged as squares when the aplanatic focus of the objective is 1 cm. above the plane of the hyperbolae, and thus this position can be easily found. At the same time the number of these hyperbolae of which images are included by the aperture of the objective gives at once the N.A. (numerical aperture) of the objective.

Further Notes on a System which obeys the "Sine Condition."

289. **The Equivalent Refracting Surface.**—Let S, S′, be the first and last surfaces of a lens system, and let Q be the image of P formed by the system, supposed aplanatic. Let PS and S′Q be any entrant and emergent ray.

Produce PS and QS′ to meet at R, and drop RM perpendicular to PQ.

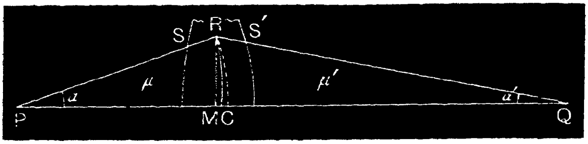


FIG. 310.

Then $$\frac{\mu \sin \alpha}{\mu \sin \alpha'} = m = \frac{\mu}{\mu'}\,\frac{QR}{PR} = \frac{\mu}{\mu'}\,\frac{QC}{PC}.$$

∴ the locus of R is a circle of which the radius is $\dfrac{d\,.\,\frac{\mu'}{\mu}m}{\left(\frac{\mu'}{\mu}m\right)^2 - 1}$,

where m is the linear magnification, d the exact distance from P to Q and μ' and μ the refractive indices of the media at Q and P respectively.

Thus in any wide-angle optical system, which satisfies the sine-condition for a pair of conjugate foci, the equivalent refracting surface for these foci is a part of a sphere.[1]

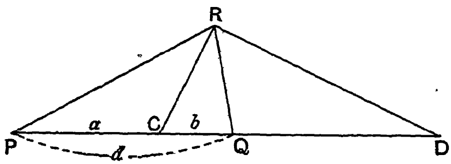


FIG. 311.

If R move so that the ratio PR : QR is constant, the locus of R is a circle.

For bisect PRQ by RC internally, and externally by RD.

Then $$\frac{PR}{QR} = \frac{PC}{QC} = \frac{PD}{QD} = n \text{ (say)} = \frac{d - QC}{QC} = \frac{d + QD}{Qd}.$$

∴ pts. C and D are fixed, and as CRD is a right angle, R is on a circle of which CD is the diameter.

$$2R = CD = QC + QD = \frac{d}{n+1} + \frac{d}{n-1}.$$

$$R = \frac{d\,.\,n}{n^2 - 1} = \frac{(a+b)\,\frac{a}{b}}{\frac{a^2}{b^2} - 1} = \frac{ab}{a - b}.$$

$$\therefore \text{ if } m = 1, \text{ R is } \infty.$$

[1] *Cheshire Journ. Querhill Club*, 1904.

290. **Condition of Orthoscopy.**—Suppose the field to be approximately flat, and let the linear magnification for a very small area in the centre of the field be m as above.

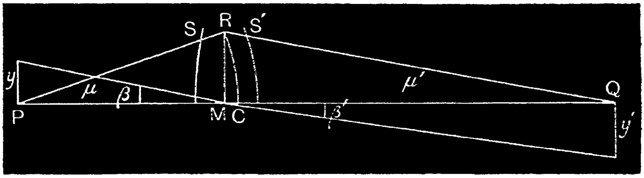


FIG. 312.

Then
$$\frac{\mu \sin \alpha}{\mu \sin \alpha'} = m.$$

If the magnification is to remain constant for a larger distance from the axis, (*i.e.* if the lens is *orthoscopic*), we must have $\frac{y'}{y} = m$ also.

But
$$m = \frac{\mu}{\mu'} \frac{QC}{PC}.$$

$\therefore$ joining C to the edge of the object and of the image

$$m = \frac{\mu}{\mu'} \frac{QC}{PC} = \frac{y'}{y}.$$

$$\therefore \frac{\mu}{\mu'} = \frac{\frac{y}{QC}}{\frac{y}{PC}} = \frac{\tan \beta'}{\tan \beta}.$$

If $\mu = \mu'$, the lines joining C to the edges of the object and the image will be in one straight line.

NOTE.—These lines are not *rays*, they are geometrical only.

291. **The two Principal Planes** (of the Gauss theory for narrow pencils) **become portions of spheres through the Principal Points.**—

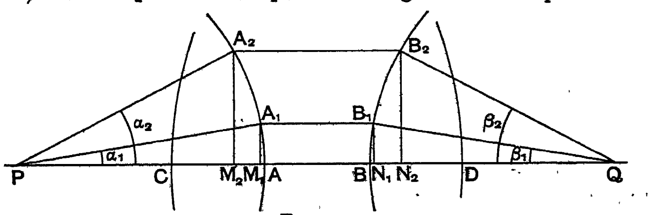


FIG. 313.

For draw spheres through the Principal Points with P and Q as centres, and let them be AA_1A_2 and BB_1B_2. Then these, by definition, are loci of unit magnification, when the angles α_1 and α_2 are small.

Therefore a ray PA will emerge as B_1Q, where $M_1A_1 = N_1B_1$.

$$\therefore \frac{\sin \alpha_1}{\sin \beta_1} = \frac{BQ}{AP} = \text{constant},$$

if the angles are small, from the ordinary Gauss theory. From the figure it is obvious that if it obeys the "sine condition," or if

$$\frac{\sin \alpha_2}{\sin \beta_2} = \frac{\sin \alpha_1}{\sin \beta_1},$$

then $$A_2M_2 = B_2N_2;$$

and $\therefore$ the loci of unit magnification are spheres.

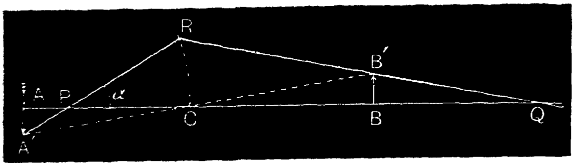


FIG. 314.

292. **Method of drawing images.**—Let P and Q be the aplanatic foci of a system of which the equivalent reflecting surface is the sphere RC, and suppose it is required to find the image conjugate to an object AA′. Through A′ draw a ray A′PR to meet the surface at R. Join RA. Join AC and produce it to meet RQ at B.′ Then B′ is the image of A′. Thus the image of AA′ is BB′.

The Abbé Apertometer.

293. The above method of measuring the N.A. only applies to an objective which is used when "dry." The Abbé apertometer will measure the apertures of immersion objectives as well as of dry ones. The instrument consists of a semicircular piece of plate glass, of which the straight edge is bevelled at an angle of 45°. The circular edge of the glass is graduated in degrees, and two metal shades slide along it. The sharp edge is a diameter of the circle, and on a radius at right angles to this, and about a tenth of an inch from it, is a small circular clear hole in a film of silver, upon which the objective is focussed.

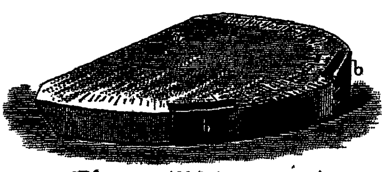


FIG. 315.—Abbé Apertometer.

The principle of the apertometer is most easily followed by

reference to Fig. 316, in which a plate similar to the apertometer, except that the edge is not bevelled, is supposed to be set up in a vertical plane. Suppose that the most oblique rays that can enter the objective come from H and K, making at O on the axis of the cylinder, of which the plate is a slice, an angle of $2\alpha''$ with one another.

In the air the angle will be 2α, where $\mu'' \sin\alpha'' = \sin\alpha$; μ'' being the refractive index of the glass.

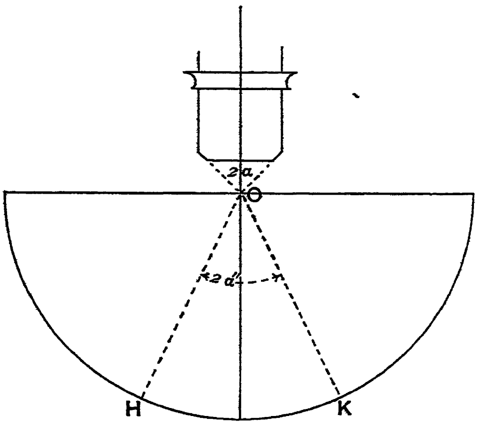


FIG. 316.

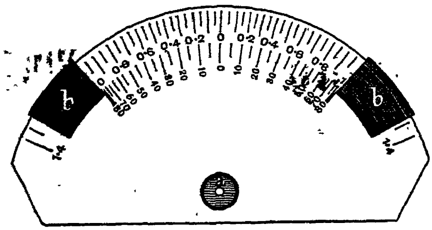
The actual apertometer is optically identical with this one, for the aperture upon which the objective is focussed may be considered to be the image in the bevelled edge, which acts as a mirror, of the point O above.

The refractive index of the glass used is 1·615.

To use the instrument, adjust the microscope so that aperture in the silvering is in focus. Remove the eye-piece and look down the tube at the back of the objective. Place a flame level with the stage, so that the light from it passes in through the circular edge of the apertometer, and is reflected up the tube by the bevelled edge. Rotate the stage until the light begins to be cut off, and the circle of light at the back of the objective begins to be dimmed at one edge. Rotate it back a little until the field is again well lighted; if the stage does not rotate, the flame must be removed instead. Now move one of the shades along the edge of the glass until it just begins to be visible on the edge of the disc of light. Repeat with the opposite edge of the disc of light. The angle between the internal edges of the two shades is the aperture of the objective in glass. Then, knowing the refractive index of the glass, the corresponding angle in air can

be calculated. The apertometer is usually graduated so as to give directly the numerical aperture, and the equivalent angle in air, instead of the angle in the glass.

As an example, suppose the angle in the glass was 60°. Then α'' is half this and is 30°, and μ, as already said, is 1·615.



View from above

FIG. 317.

$\therefore$ The angle α in air is given by

$$\sin \alpha = 1{\cdot}615 \,.\, \sin 30^\circ$$
$$= {\cdot}8 = \text{N.A.}$$

Thus the angle α is 53°, and $2\,\alpha = 106°$.

As a further example, suppose the objective is an oil immersion one, and that the refractive index of the oil is 1·5, and suppose $\alpha'' = 45°$.

Then $$1{\cdot}5 \sin \alpha' = 1{\cdot}615 \sin 45^\circ$$
$$= 1{\cdot}4 = \text{N.A.}$$

and $$\alpha' = 51^\circ,\ 2\,\alpha' = 102^\circ.$$

Testing the Objective.

294. The best way to test an objective is to examine the image formed by it with a deep eye-piece, but of course this mode of testing depends upon the judgment of the observer.

Magnifying power, *resolving power*, *penetration power* are numerical.

Definition, *brilliancy of image*, *centring* have no numerical expression.

295. **Zone Test.**—A test object is first sharply focussed, and the flame is also to be focussed upon it by the condenser. The object is moved just out of the field (not removed entirely, as this would alter the illumination). A card with a series of holes of varying sizes is placed at the lower focal plane of the condenser, and the light transmitted by the objective examined at its back focal plane (either by removing the eye-piece, or by an extra eye-piece temporarily mounted above the ordinary one and focussed upon the

eye-ring). The hole which transmits a beam of the correct size is thus found, namely one having a diameter $\frac{1}{4}$ the diameter of the back lens.

A new card is pencilled with two such holes at a distance apart equal to two diameters from centre to centre. The card is then fixed in its place at the lower focal plane of the condenser, so that the beams shall be transmitted through the outer zone of the lens. The test object and eye-piece are replaced; the condenser and card, or the card alone, is then revolved round the axis of the instrument, and the appearance of the test object is noted as the beams sweep the field of the objective.

The spherical aberration of an uncorrected condenser will spoil the results of the test. If no good condenser is at hand, a disc may be put at the back of the focal plane of the objective, with holes of the corresponding size, and rotated. The objective is of course not to be rotated.

296. **Abbé's Test Plate.**—This consists of a series of cover-glasses ranging in thickness from ·09 mm. to ·24 mm., silvered on the under surface, and cemented side by side on a slide, the thickness of each being marked upon the silver film. Groups of parallel lines are cut through the films, and these are so exactly ruled that they are easily resolved by the lowest powers; yet from the extreme thinness of the silver they also form a very delicate test for objectives of even the highest power and widest apertures.

To examine an objective of large aperture the discs must be examined in succession, observing in each case the quality of the image in the centre of the field, and the variation produced by using alternately central and very oblique illumination. When the objective is perfectly correct for spherical aberration for the thickness of cover-glass under examination, the images of the lines in the centre of the field will be perfectly sharp by oblique illumination, and without any nebulous doubling or indistinctness of the minute irregularities of the edges. It should have the same focus for central as for oblique light. If with any one of the discs it fulfils these conditions, it is free from spherical aberration for cover glasses of that thickness.

Nebulous doubling with oblique illumination indicates over-correction of the marginal zone; indistinctness of the edges

without marked nebulosity indicates under-correction of the zone.

The test of chromatic aberration is the colour seen with oblique illumination. The edges of the lines in the centre of the field should show only narrow coloured bands in the complementary colours of the secondary spectrum, namely on one side yellow-green to apple-green, and on the other violet to rose. The better the spherical correction, the clearer these colours.

An achromatic condenser should be used, and for wide apertures cedar oil must be put between its under surface and the test plate. A movement of the diaphragm is the best way to change from central to oblique illumination.

The illumination must be brilliant, and a high-power eye-piece is necessary.

Indistinctness of outline towards the borders of the field results as a rule from unequal magnification of the different zones of the objective; colour bands on the margins are caused by unequal magnification of the images of different colour.

A test for low powers up to $\frac{1}{8}''$, of N.A. ·65, is an object on a dark ground, one of the delicate polycistinae for the lowest powers, for medium powers a coarse diatom. Unless the lens is well corrected the image will be fringed with scattered light. The aberration produced by the cover glass is plainly manifest, and by accurate adjustment can be done away.

297. **Error of Centring.**—Place a sensitive object in a certain direction, and when the adjustments have given the best image, rotate it 90°. If it does not alter, the lens is good. Dallinger recommends a hair of Polyaenus lagurus in balsam for medium powers, and a Podura scale for higher ones. It should be strongly marked, and in optical contact with the cover glass.

Wide aperture objectives can be tested with a homogeneous condenser of wide aplanatic area, *e.g.* a semi-apochromatic oil-immersion condenser of 1·3 N.A. It must be a good objective that does not show signs of breaking down under the strain. If it will stand a cone filling three-quarters of the back of the objective, it is an excellent lens.

298. **Testing for Spherical Aberration.**—A glass slide is coated with an opaque layer of Chinese ink in which fine holes or lines are

made with a needle, or if the layer of ink is allowed to dry cracks will form naturally. An Abbé test plate may be used.

The objective must be filled with light.

If there is a fog surrounding the image, there is spherical aberration.

Let AB be the objective, p the focus of the central rays, p' the focus of the marginal rays.

Then the objective is over-corrected.

The real image will be most clearly seen on a screen at f_1, drawn through p. It is, however, bounded by a fog caused by the marginal rays. At f_2 the image is dim and the fog greater.

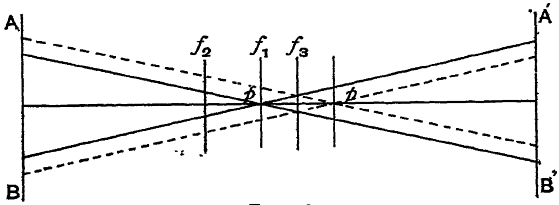


FIG. 318.

At f_3 the cross-section of the cone is least and the fog disappears. The screen at f_3 is illuminated by a sharply bounded circle of light. It remains sharp even further to the right.

With an under-corrected lens A'B' the phenomena are reversed. Therefore the approach or withdrawal of the eye-piece will decide whether the lens is under- or over-corrected.

For testing spherical aberration:

(1) Use a full cone.

(2) Focus best, and see if there is a (bluish) fog.

(3) If the fog rapidly increases or disappears by reducing the cone, it is due to marginal rays. If reducing the cone does not improve the definition, try a central stop.

(4) Alter the focus or the eye-piece distance to ascertain in the case of marginal rays if it is under- or over-corrected.

In the case of a micro-objective:

The resolving power is directly proportional to the N.A.
The penetrating " inversely " "
The illuminating " directly proportional to the (N.A.^{2}).

The Condenser.

299. In critical work with the microscope, it is a matter of practical experience, that the best results can only be obtained when the image of the flame, formed by a good condenser, is made to coincide with the object. The condenser must be free from spherical aberration, and approximately free from chromatic aberration; it must also have a large aperture. The aperture should, however, be capable of being reduced; sometimes it has to be reduced to about three-quarters of that of the objective. Thus the condenser is a lens which produces a point image of each point of the flame, practically free from aberration. The phase of each point of this image should therefore depend entirely and solely upon that of the point of the flame of which it is the image. As the individual points of the flame have absolutely independent phase relations, so should the individual points of the image produced by the condenser. This is very important, for it shows that there *should* be no *interference* of the light from any given pair of points of this image. If, for instance, this image is made to coincide with an ordinary transparent grating, so that the image of the flame is periodically obstructed, the light transmitted by the spaces of the grating *should* not be in a condition to interfere; if any coloured spectrum is produced at the back of the objective, it can only be because the condenser is imperfect, and two or more grating spaces are illuminated by light emanating from a single point of the flame, namely, the condenser does not converge the light from a point of the flame to a point again. As shown in § 284, it is indeed impossible for a condenser to converge the light from a given point of the flame to an absolute point. It can only produce a disc, and this disc will be larger, the smaller the aperture of the condenser. If the condenser were perfect the light from each space would be independent of that from the neighbouring ones, and the grating could be considered to be self-luminous. The nearer the approximation to this, the better is the practical result.

300. All condensers are under-corrected for spherical aberration. The aperture of a condenser can be measured with Abbé's

apertometer, but its effective aperture or *aplanatic* aperture cannot be so measured.

To measure the ordinary aperture, place the condenser on the substage and an objective on the nosepiece, and focus *both* on an object. Use the edge of the flame, and let its images be central. Now move the object just out of the field, remove the eye-piece and examine the back of the objective. If the aperture of the aplanatic illuminating cone is greater than that of the objective it will show the back lens full of light. Therefore, if the aperture of the objective is ·5, we know that of the condenser is at least ·5.

Exchange the objective for one with wider aperture. It will perhaps be found that before the back of the objective can be filled with light by racking up the condenser, two black spots will be formed on either side of the middle of the discs. The last point before the appearance of the black spots indicated the largest aplanatic aperture of the condenser, and is the limit of the condenser for critical work. (E. M. Nelson, *Eng. Mech.*, No. 1234, 1888.)

Microscopic Relief.

301. If a solid object be placed in front of a lens supposed free from aberration, the images of all the points in a plane perpendicular to the axis of the lens will be formed again in the conjugate plane, and the image formed by any one part of the lens (for instance, one half of the lens) must coincide exactly with that formed by any other part (the other half, for instance). The points in any other plane, in the same way, will have their images again in the corresponding plane. Thus if the object is not all in one plane, its image will also have depth. The image may be looked upon as being formed on a series of parallel planes, all perpendicular to the axis, the image in each plane being formed in exactly the same place by any part of the lens. Thus the image produced by a lens is formed in relief (but in a different sense from the ordinary stereoscopic relief).

On viewing this image with an eye-piece, the images formed on the retina by the different parts of the eye-piece will not be similar; if the image formed by the right-hand half of the lens could be viewed by the right eye, and that formed by the left-

hand half by the left eye, an appearance of stereoscopic relief would result. Thus, it is not necessary that the images formed by the right and the left-hand halves of the *objective* should enter different eyes, but that those formed by the two halves of the *eye-piece* should do so.

Stereoscopic relief was obtained by Wenham, who introduced a prism, above the objective, which received the light from the right-hand half of the objective, and after two internal reflections threw it across the direct beam and up a side tube, an eye-piece being placed in each of these beams. But as a real image of the back of the objective is formed just above the eye-

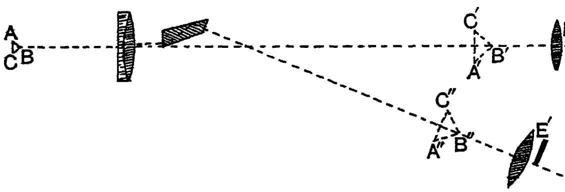


FIG. 319.—Wenham's Binocular Microscope.

piece (at the eye-ring), stopping half the field at the back of the objective is equivalent to stopping half this eye-ring; thus as the prism cuts away the light from half the back of the objective, its effect on the direct beam is equivalent to a stop at the eye-ring, cutting off the light which issues from the left half of the eye-piece. In the same way it cuts off the light which issues from the right of the eye-piece which receives the reflected beam. If E represents the eye-piece, a real image of the back of the objective will be formed by E at a short distance from the lens, and a stop placed to cover half the objective and one to cover half the aperture of the eye-piece will be equivalent. If the objective is fully corrected the image as viewed by the two eye-pieces will be identical. Although the images A′B′C′, and A″B″C″ viewed by the eye-piece are identical, yet as they are not in one plane, and as the rays allowed to enter the right and left eye respectively are those issuing from the left and right sides of the respective eye-pieces, the images those eyes view will not be identical, and stereoscopic relief will result.

True stereoscopic relief was also obtained by Abbé, who divided the rays emerging from the objective at the eye-piece end of the tube, by partial reflection at a thin stratum of air between two glass prisms; the reflected beam being reflected up the second tube by a right-angled prism. The stereoscopic relief is then produced by covering the eye-ring of each eye-piece with a semi-circular stop.

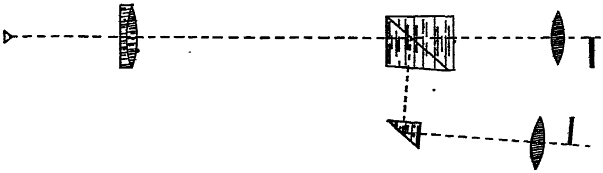


FIG. 320.—Abbé's Binocular Microscope.

If the inner half of the aperture of each eye-piece be covered, we shall obtain stereoscopic relief. By reversing this, the relief becomes pseudoscopic.

The instrument without the stops gives binocular vision without stereoscopic relief.

The tube containing the second prism and eye-piece can be moved nearer to or farther from the direct tube to suit the distance between the observer's eyes.

It will be noticed that the penetration appears to be much greater when stereoscopic relief is obtained. This is probably because the observer is assisted by the feeling that the image is at different distances to make use of his accommodation, and is able to use the whole range of his accommodation. When viewing the image with one eye alone, if a portion of the image is much out of focus, without an indication of the direction in which the accommodation must be varied to see it, the observer fails to effect the necessary change.

CHAPTER XVIII

PHOTOMETRY

THE basis of the photometric comparison of two lights is the law that the intensity varies inversely as the square of the distance. This is true only if the screen upon which the light is to be received is illuminated solely by the direct lights and receives no scattered or reflected light from surrounding objects in the room itself. It is obvious, for instance, that the light transmitted along a perfectly reflecting tube would remain constant at all distances. It is therefore necessary in photometric work to have a room in which the reflections can be eliminated. The walls of the room should be painted a dull black, and the table covered with a black cloth. The instrument should also be blackened wherever reflections are likely to occur. Some photometers are much more liable to error in this respect than others. The grease-spot photometer is perhaps most sensitive to scattered light. A photometer such as Rumford's, in which the ordinary scattered light from a room will equally affect the two shadows that are to be compared, will only give false readings to the extent in which either of the lights is reflected upon the screen,—for instance, if it has a polished base.

Simple Photometers.

302. **The Grease-spot Photometer.**—*Apparatus.*—Materials for constructing a grease-spot photometer as enumerated below.

Let AB be a piece of white paper, and CD be the grease spot. Let S and S_1 be two light sources. It is usually stated that the light from S which falls upon AB will be chiefly transmitted through the semi-transparent grease spot CD, and will be scattered from the remainder of the paper; so with the light S_1

which falls upon the other side. If, therefore, the distances of S and S_1 are so chosen that the light on each side is equally intense, that part of the light of S which passes through CD, and is, therefore, lost as scattered light to an eye placed at E, will be made up by the light which goes through the same spot CD from S_1. No doubt the amount of light leaving the area CD from the two sources together must be the same as that leaving the remainder of the screen, but with an ordinary grease spot the amount which reaches the eye placed at E will not be the same. The light which passes through the grease spot is not equally scattered in all directions, but is much more intense along the axis, and falls off rapidly away from the axis, whereas

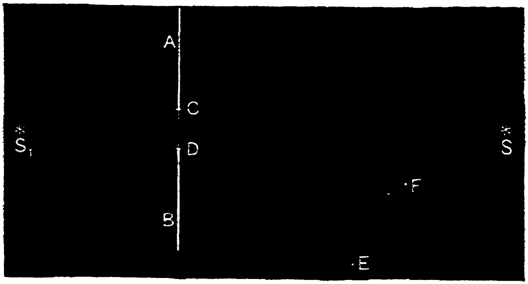


FIG. 321.—Grease-spot Photometer.

the light scattered from the remainder of the paper is scattered far more equally in all directions, so that the brightness of the spot varies very considerably according to the position of the eye. If, for instance, S_1 and S_2 are equally strong lights, and the screen be placed half-way between them, the spot will appear comparatively dark, when the eye is at E, whilst at F it would appear very much brighter than the remainder of the paper. Any grease spot is, therefore, untrustworthy and practically useless. The fault, of course, lies in the incomplete scattering of the spot itself.

303. A modification of this, invented I believe by Sir W. Abney, gives much better results. A screen, AB, has a hole, CD, $\frac{1}{4}$ inch high by $\frac{1}{8}$ inch broad. On one side of this screen, a piece of opal glass $\frac{1}{4}$ inch square is placed so that one half of it covers the hole in the screen. A movable pillar, P, is placed to cut off

the light from S_2 from this opal, as shown in the figure. The opal is viewed in a mirror, M, by an eye placed at E. It will be seen that the half CD of the opal is lighted by S_1 and the other half CF by S_2. The thickness of the glass is reduced until the

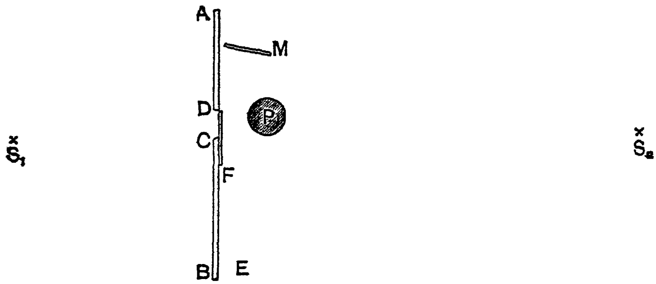


FIG. 322.—Improved Grease-spot Photometer.

opal resembles a microscopic rock-section. The scattering is then very perfect, and very concordant readings are possible. The intensities of the lights will be proportional to the inverse square of their distances from the opal.

304. **The Rumford Photometer.**—*Apparatus.*—This consists of a board covered with black velvet upon which a small cube about 1 inch side is mounted. The front face of the cube must be perfectly white, the other faces black. A movable vertical pillar, about $\frac{3}{4}$ inch diameter, is placed in front of the screen.

The two lights to be compared, S_1 and S_2, are on the same side of the screen, and the pillar is adjusted until its shadows formed by S_1 and S_2 respectively just meet. The distances of S_1 and S_2 are adjusted, usually, by allowing one of them to move to and fro on a guide, until the two shadows are equally dark. When this is the case, the square ought to appear evenly illuminated, and indeed the dividing line will disappear if the two lights are of the same colour. When the pillar P is too close to the screen, a black line divides the shadows down the centre, and when too far away a bright line takes its place, but when the shadows just meet, there is no dividing line visible. The intensities of the lights will be proportional to the inverse square of their distance from the cube.

305. **The Paraffin-wax Photometer.**—This is one of the best of the simple photometers.

Apparatus.—Two pieces of paraffin wax, each about 1 inch square and $\frac{1}{4}$ inch thick, are carefully made of the same uniform thickness, a little over $\frac{1}{4}$ inch. A piece of tinfoil is placed between them, and they are fastened to the centre of a piece of wood or metal some 4 to 6 inches square. An aperture is cut in this $\frac{3}{8}$ inch square, as indicated by the dotted lines in Fig. 323, so that the edges of the blocks are visible through this aperture with the tinfoil as dividing line. The lights S_1 and S_2 to be compared.

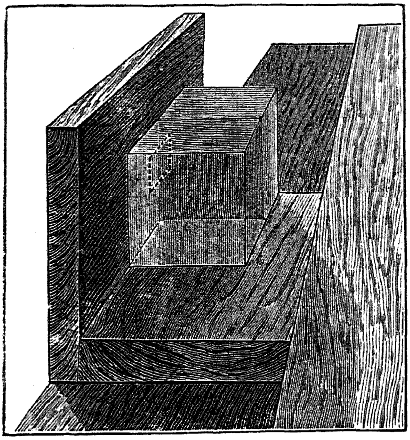


FIG. 323.—Paraffin-wax Photometer.

The lights to be compared are placed at the ends of a long guide, this photometer is moved to and fro upon the guide, until the halves of the paraffin wax seen through the aperture appear equally bright. The position is then read off on the scale attached to the base. It is an advantage to have the photometer at the bottom of a block of cardboard or cloth box, that the eyes

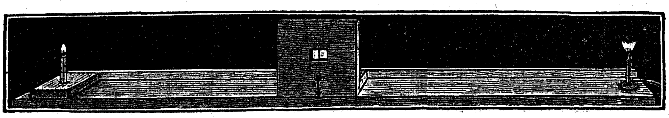


FIG. 324.—Paraffin-wax Photometer.

may not be dazzled by the direct light from the lamps. As an exercise, the light given by one wax candle may be compared with that given by 1, 2, 3, 4, etc.

The total length of the bench should be as great as can be conveniently arranged, so that when extended flames are being

used, we may be able to consider them as points for purposes of calculation, and also that in comparing lights and their different intensity the distance from the weaker light to the screen may not have to be too small for measurement. It should be 12 to 20 feet long.

ADDITIONAL EXERCISES.

1. Measure the brightness of an incandescent gas (or electric light) in terms of that of a wax candle, with each of the photometers, and compare the results.

2. Place sheets of white card, in the neighbourhood of each of the photometers in turn, so as to reflect the scattered light on to the instrument, and again measure the brightness of the incandescent light. Hence determine upon which photometer scattered light has the greatest effect.

The Swan, or Lummer and Brodhun Photometer.

306. The principal feature of this photometer is the pair of right-angled prisms, P_1, P_2, placed with their hypotenuses in contact,

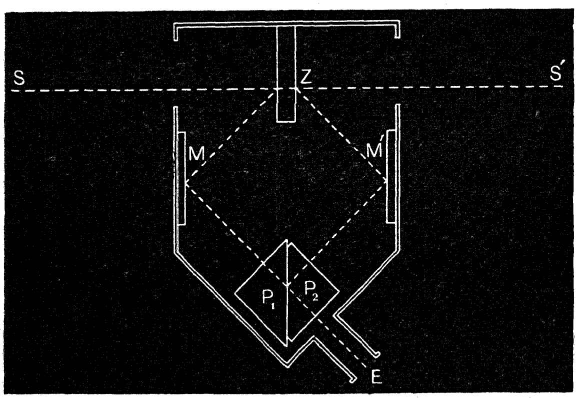


FIG. 325.—Swan Photometer.

and forming a cubical block. The hypotenuse of P_2 is first silvered, and then the silver is removed over some portion, such as a circular ring. The two prisms are then cemented together with Canada balsam, with the result that an eye placed at E can look through the block where the silver is removed, whilst the block will act as a mirror for the rest of its surface. The light

from the lamp S_1 to be tested falls upon one face of an opaque white screen Z. After scattering at Z, it is reflected by M′ through the transparent portion of the block to an eye at E. The light from the standard lamp S, after being scattered from the other surface of Z, is reflected by M and then by the silvered surface of the block to the eye. Thus the dividing surface of the block will appear as a series of rings, lighted alternately by S_1 and S. If S_1 and S are of the same colour and placed so as to illuminate the surfaces of Z equally, these rings will appear equally bright, and the surface uniform. The block, screen, and mirrors, and a short tube for the eye to look through, are mounted in a box, and replace the paraffin-wax photometer above described.

The box is placed on the baseboard, and moved nearer to or farther from S_1, until the adjustment is perfect; the distances from the lights to Z are used to determine their relative luminosity. The baseboard of the photometer for commercial work is divided so that the reading gives directly the ratio of the luminosities of S_1 and S, that is, the ratio of the inverse squares of the distances of S_1 and S from Z.

In another form of the instrument, two pieces of plane glass are inserted between P and the mirror, each covering a small portion of the field. As the light in passing through these pieces of glass is very slightly reduced in intensity, when the balance is secured the field will not be uniformly illuminated, but will consist of alternately lighter and darker bands. It is claimed that this enables a more exact adjustment to be formed; for without them, when the balance is nearly perfect the dividing line disappears and there is no longer anything for the eye to fix its attention upon; thus it may become focussed at the wrong distance and not perceive any slight variation in the brightness of the fields. But with this faint shadow remaining, the dividing lines between the fields will not vanish, and at the same time the inequality is so small, that it can be balanced with great accuracy. See also the remarks on page 402.

Measurement of Opacity.

307. A distinction has to be drawn in the measurement of the opacity of a semi-transparent screen—*e.g.* of a photographic

plate,—according as the light after passing through it proceeds for some distance without obstruction, or at once falls upon a paper (as in ordinary photographic printing). In the latter case the light after striking the paper will be largely scattered, and returning will fall upon the film once more, and be reflected back. In the former case, none of this to and fro reflection will occur. In photographic enlarging or projection, it will be the opacity in the former case that will be required.

Hürter and Driffield Photometer.

308. This instrument will measure the opacity when there is no to and fro reflection. It consists of a box which should be about 2 feet long (H. and D. used one half this size, but the larger size is

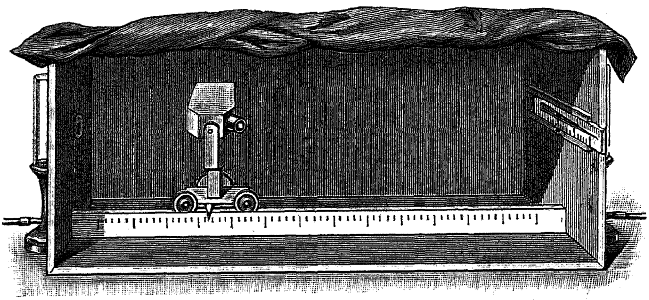


FIG. 326.—"H. and D." Photometer.

more reliable), with an aperture at each end, each illuminated by a duplex paraffin lamp, placed with the flame flatwise outside the box.[1] Inside the box is a bar upon which one of the photometers previously described, preferably, perhaps, Abney's, may be moved to and fro, the opal being in a line with the two apertures. There is a scale and pointer. The photometer should run easily upon wheels so that it may be rocked to and fro past the point at which the balance occurs; in this way the true position can be found with considerably increased accuracy. The inside of the box must be blackened everywhere. The aperture at the left-hand end is about $\frac{1}{4}''$ in diameter, and the plate to be measured is placed at this end, and held there by springs. The aperture at the other end is $\frac{1}{4}''$ wide and $\frac{1}{2}''$ high, and there is a long sliding

[1] An incandescent gas lamp would do as well as, if not better than, the duplex.

taper diaphragm which can be moved to and fro across this, reducing its height from $\frac{1}{2}''$ to zero.

Suppose the lamp on the left gives light of intensity I_1, and that on the right intensity I_2, and suppose the photometer to be at a distance x from the centre when the illumination is the same, then

$$\frac{I_1}{(l-x)^2}=\frac{I_2}{(l+x)^2},$$

$$\frac{I_1}{I_2}=\left(\frac{l-x}{l+x}\right)^2.$$

If a plate be now inserted, it reduces the light of I_1 to an intensity I_3. Then the photometer will have to be moved still further from the centre, say to a distance y, and then

$$\frac{I_3}{(l-y)^2}=\frac{I_2}{(l+y)^2},$$

or
$$\frac{I_2}{I_3}=\frac{(l+y)^2}{(l-y)^2}.$$

Thus
$$\frac{I_1}{I_3}=\left(\frac{l-x}{l+x}\right)^2\left(\frac{l+y}{l-y}\right)^2,$$

and this is the "opacity" of the plate.

H. and D. called the log. of this the "Density" D, thus

$$D=\log\left(\frac{l+y}{l-y}\right)^2-\log\left(\frac{l+x}{l-x}\right)^2.$$

If, therefore, at the distance x and y were marked the values of the $\log\left(\frac{l+x}{l-x}\right)^2$ and $\log\left(\frac{l+y}{l-y}\right)^2$, these logs. could be simply read off.[1]

[1] The following table for the graduation of this scale is given by them in their paper (*The Journal of the Society of Chemical Industry*):

$\log\left(\frac{l+x}{l-x}\right)^2$.	Distance from Centre of Instrument.	$\log\left(\frac{l+x}{l-x}\right)^2$.	Distance from Centre of Instrument.
.000	$l\times.000$	.900	$l\times.476$
.100	$l\times.057$	1.000	$l\times.519$
.200	$l\times.114$	1.100	$l\times.560$
.300	$l\times.171$	1.2	$l\times.599$
.400	$l\times.226$	1.3	$l\times.634$
.500	$l\times.280$	1.4	$l\times.667$
.600	$l\times.332$	1.5	$l\times.698$
.700	$l\times.382$	1.6	$l\times.726$
.800	$l\times.430$	1.7	$l\times.752$

There is a movable scale attached to the edge of the diaphragm which is used to reduce the amount of light through the narrow rectangular opening. This taper diaphragm is made of sheet metal, about 12″ long and 2″ wide, out of which is cut a triangular opening $10\frac{1}{2}$″ in length from base to apex, the width of the base being $\frac{1}{2}$″. A zero is marked 10″ from the apex. The other points of the scale are marked to read the densities directly, that is, the log. of the fraction $\frac{10}{x}$.

An index is fixed to the inside of the box over the scale of the taper diaphragm, pointing to the number to be read.[1]

The following example will indicate the use of these scales. For a small density, with the sliding scale at zero, balance the lights by moving the photometer to and fro and take the reading. Insert the plate and again balance, the difference of the readings will give the density. In the case of a high density, place the sliding scale at the 0, and by placing a piece of opal glass outside the box between it and the lamp, reduce the light at the right end until the photometer has to be brought nearly up to that end to secure balance. Insert the plate to be measured and again obtain a balance. The difference of readings will give the density. With, however, a very large density this will not be sufficient, and it will be necessary to move the taper diaphragm in addition, and the density will then be obtained by adding the reading of this movable scale to the difference of the reading on the photometer scale. If, for instance, the difference of readings

[1] The following is the scale they give:

MOVABLE SCALE.

Value of log $\frac{10}{x}$.	Distance from Apex.	Value of log $\frac{10}{x}$.	Distance from Apex.
.00	10	.5	3.16
.05	8.91	.6	2.51
.10	7.94	.7	2.0
.20	6.31	.8	1.58
.3	5.01	.9	1.28
.4	4	1	1.0

It is obvious that the photometer scale of the instrument, having been marked at the places given by their table, might be *numbered* straight through from one end to the other, instead of from each one to the centre.

of the photometer scale was $2 \cdot 5$ and the movable scale was shifted from the zero to $\cdot 7$, the density would be $2 \cdot 5 + \cdot 7 = 3 \cdot 2$.

The opacity of the plate is the number whose log. is $3 \cdot 2$, and is therefore 1585, that is to say, this plate will transmit only $\frac{1}{1585}$ of the light it receives.

Chapman Jones Opacity Balance.

309. In this instrument the source of light is a Welsbach incandescent burner W, and by an arrangement of mirrors M_1, M_2, M_3, this is used to illuminate both apertures. The plate of which the opacity is to be determined is placed against the back of an Abney photometer, as indicated in the figure, where N is the

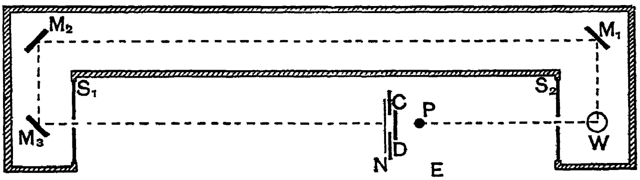


FIG. 327.—Opacity Balance.

plate of which the density is to be determined, CD the opal, P the pillar, and S_1 and S_2 the two apertures. The photometer slides to and fro on a bar divided similarly to that of the Hürter and Driffield described already, and the bar may be graduated in the same way. The screen is observed from E. It will be seen that this time, the plate being close to the opal, we are measuring its opacity for *printing*, as the light will be reflected to and fro between the plate and opal.

Experiment with the Hürter and Driffield Photometer.—The following examples are given by Dr. Hürter in his paper (*loc. cit.*).

i. **Experiment with Indian Ink.**—An Indian Ink solution was mixed with water in known proportions, and the density of one solution being known, that of the other was calculated. The following table shows the observed and calculated densities. The calculated density is simply proportional to the amount of Indian Ink employed.

The greatest error made does not reach 4 per cent. of the total amount, and even better results can be obtained if more than one

reading be taken. But this accuracy is quite sufficient for photographic purposes, where, from other causes, still greater errors are liable to arise.

Indian Ink employed to 100 c.c. of water.	Density calculated.	Density found.	c.c. of Indian Ink found.
c.c.			
5	·240	·240	5·00
10	·480	·500	10·42
15	·720	·750	15·62
20	·960	·950	19·80
25	1·200	1·245	25·90
30	1·440	1·440	30·0
35	1·680	1·665	34·7
40	1·920	1·885	39·3

Sometimes, when using the instrument for analysing solutions of coloured salts, a difficulty arises from the different colours of the two images of the Bunsen disc. This is easily overcome by viewing the disc through appropriately coloured glass—red, green, and blue glasses being the most useful.

ii. The following **experiment with indigo** solution is representative of one of the most difficult, since a dark-blue glass was used to view the disc :

Indigo Solution employed.	Indigo found.	Density calculated.	Density found.
c.c.	c.c.		
100	96·0	1·554	1·487
50	50·6	·777	·787
25	24·1	·388	·375
10	10·0	·155	·155

It will be seen again that the results are only accurate within five per cent. of their value.

With regard to the lamps, they should be powerful petroleum lamps with duplex burners. The flame should be in planes at right angles to the axis of the instrument. Very erroneous results are obtained if Argand burners are used. The lamps should be placed close to the diaphragms, and it is advisable to provide a small stage outside the diaphragm to hold coloured glasses, when a substance requires investigation in light of a particular colour.

CHAPTER XIX

COLOUR MEASUREMENT

The Tintometer.

310. *Apparatus.*—This instrument was invented by W. Lovibond for the purpose of registering colours, and forms a good means of illustrating three-colour vision. It consists merely of a box with apertures at one end, at the same distance apart as the eyes; at

FIG. 328.—Tintometer.

the other end are two apertures each about ½″ square. In front of one of the latter are a series of vertical grooves, in which coloured glasses may be placed, the colour to be matched is placed either within or opposite to the aperture. The glasses themselves are blue, red and yellow; they are graduated from the faintest tint up to a fairly deep colour, and the graduations are such that a combination of any three glasses red, blue and yellow of the same number will produce a neutral tint, and also that the numbers are additive, so that for instance, a 2·4 is equivalent to a 1·8 plus a ·6. These effects may be readily tested with the instrument itself, by placing on one side any glass and matching it by two or more other glasses on the other side, when it will be found that the sum of the numbers attached to the glasses on one side equals the similar sum on the other.

With this instrument a large number of colours (though not quite all) may be matched. For instance, a solution of bichromate of potash may be placed in a suitable vessel in front of one aperture and matched by a glass in the other aperture, or some strips of coloured glass may be obtained from a glazier and matched. The colours selected by the makers for the tintometer are, however, not quite good enough for matching every colour. They should be "double-colour" glasses; that is to say, they should be (1) a *yellow* which transmits all the spectrum except the violet end, (2) a *greenish* blue which transmits all but the red end, and (3) a *carmine* transmitting all but the middle.

For experimental work, a tintometer may be made from a cigar box, and the coloured glasses may be replaced by films of collodion on clear glass plates. These colours will not possess the permanency that the actual stained glass has, but a better approach to the theoretical colours is possible with them.

311. To make the coloured glass, clean a piece of thin white glass thoroughly and warm it to remove all traces of moisture. Coat it with collodion as described on p. 446.

If the plate is completely free from moisture, the collodion will be perfectly transparent when dry; if not, it will have a milky appearance. Make a strong solution of naphthol yellow in hot water and stain one plate in that; stain the others with rhodamine and methyline blue respectively, and set them aside in a room free from dust to dry. They must be dipped in the staining solution before the collodion has perfectly dried, that is, in from one or two minutes after pouring the collodion upon their surfaces. The greater part of the surface of the plate should appear of a uniform colour, and if this is the case, having removed those parts which are uneven (generally the margin of the plate), cut up the remainder into pieces each about $\frac{5}{8}''$ by $1\frac{1}{4}''$. This will form a number of plates of equal density, and of each of the three colours. Two or three plates of each colour should be dyed in this way in differing intensities. For instance, with a few trials, it is possible to obtain a very faint blue, a medium, and a deep blue, so that one deep blue is equivalent to four medium, and one of the latter is equivalent to five faint blues; so also with the red and yellow. It is very essential, especially with the red and yellow, that the glass used shall be free from colour, as otherwise when four or five pieces are placed behind one another the glass itself will produce a greenish tinge.

If these are stored carefully in the dark, they will keep their colour for a long time. They may be compared at intervals with some standard coloured glasses. With these glasses nearly any colour may be matched, even including most of the aniline dyes.

312. *Experiment.*—The instrument must be used in daylight or with an arc lamp. Place it near a window and point it down at a white surface that is well illuminated. Place the colour to be matched, if it is a transparent one, opposite one of the apertures.

An opaque tint—for instance a colour printed on paper—is placed on and in contact with the white surface, in such a position that whilst it is seen through one aperture, the white surface not covered by the paper is seen through the other aperture.

Insert coloured glasses in the other aperture, and vary them until a match is made. Write out the results thus:

	Matched by						Equivalent to		
	Yellow.		Red.		Blue.		Yel.	Red.	Blue.
	Light.	Dark.	Light.	Dark.	Light.	Dark.			
Signal Green Glass	1	2	1	—	—	3	9	1	12

More accurate results would be possible if the two patches to be compared could be adjacent, for instance if the tinted glasses took the place of one (or both) of the vessels B, D of Fig. 329.

Comparison of Absorption of Liquids.

313. A simple apparatus for the purpose is shown in Fig. 329. C, D are right-angle prisms about $\frac{1}{2}''$ face. A is a compound prism made of two isosceles right angle prisms of which the smaller has been silvered over its hypotenuse, and then a straight narrow strip of the silvering has been removed from the middle, parallel to the long side of the surface. The two are then cemented together with Canada balsam. Thus the light can pass straight through the prism where the silvering has been removed, and is totally reflected from the remainder. B and E are two cylindrical glass vessels with plane glass bottoms. F, G is a white screen.

Then placing the eye at O (which is about 10″ above A), the unsilvered strip will not match the rest of the field unless the light coming from B is as bright and of the same hue as that coming from E.

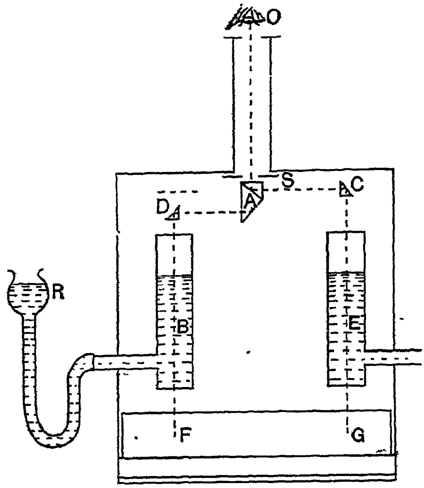


FIG. 329.—Absorption Photometer.

If monochromatic light is used (say a sodium flame), and B and E contain absorbing liquids, the balance can be obtained by varying the depths of the liquids in B and E. This is most easily done by connecting reservoirs R to B and E with rubber tube; raising or lowering the reservoirs will cause the liquid to flow into or out of B and E.

In all such photometric work it is much better, if possible, to have the field divided into three parts *a*, *b*, *c* (Fig. 330), of which *a* and *c* are illuminated by one light and *b* by the other. For suppose *b* brighter than the rest. If the attention is directed to the line dividing *a* from *b*, the part of the retina on which the image of *b* falls will become fatigued as compared with that upon which the image of *a* falls. Then directing the attention to the line separating *b* from *c*, the weaker light from *c* falls upon the fatigued part of the retina, and the contrast between *b* and *c* is therefore accentuated. In this way "fixing" the dividing lines alternately, more accurate readings are obtained than are possible with a field which has only a single dividing line.

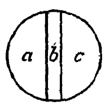


FIG. 330.

314. To compare the absorption in different parts of the spectrum, insert a small pocket spectroscope with the slit removed, and the viewing lens downwards in the tube above the double prism A, as in Fig. 331. This will produce three spectra side by

side from the three parts of the field *a*, *b*, *c*, at the focus H of the lens L_1, conjugate to the prism. These spectra can be observed by a lens L_2 placed at a suitable height above the spectroscope. To obtain the different regions of the spectrum in the field, the spectroscope can be hinged at H (or mounted in a short cork which will allow of a little play), and inclined slightly to one side or the other.

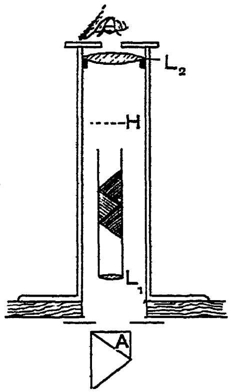


FIG. 331.

Luminosity Curves of Inks and Pigments.

315. **Luminosity of Coloured Surfaces—Abney's Method.**—*Apparatus.*—Rotating machine; black, white and coloured discs of three sizes,—say 4″, 6″ and 8″ diameter, see Fig. 334.

Take two yellow discs, an 8″ and a 4″, and mount between them a pair of interlaced black and white discs, 6″ diameter. Rotate the four fast enough to overcome the flicker. Adjust the proportion of black to white to form a grey matching the luminosity of the yellow. A very exact match can be made by observing the discs through the black of a photographic plate developed with metol. Measure the angles of white and black, and correct the white for the light reflected by the black.

Repeat with two green discs in place of the yellow ones. Now interlace discs of yellow, green and blue; adjust to form grey, and match it with interlaced black and white. From this the luminosity of the blue can be found. Repeat with red, green and blue, forming a grey, and matching it, and so find the luminosity of the red.

Various combinations may be made to match, and the results used as a check, *e.g.* black and yellow can be made to match red and green.

Having found the luminosities of red, green and blue, other colours can be measured by substituting them for one of these, and making a grey, and matching it with black and white.

Abney's Colour-Patch.

316. *Apparatus.*—The *collimator* and *projecting lens* mounted in the square boxes already described on page 108 will do. The spectrum should be produced by good white glass *prisms*, P_1, P_2, or a grating of large area and lenses of correspondingly large aperture. A replica of a Rowland's grating by Thorpe with about 14,000 lines to the inch, and an area of $3\frac{1}{4} \times 2\frac{1}{2}$ will do, but prisms give more light and are freer from spurious reflections. The *slit* should open fairly wide so that a width of $\frac{3}{16}''$ may be had if desired, and must be limited vertically to about $\frac{3}{16}''$. The light must be either daylight (thrown on with a heliostat) or an arc light concentrated by a *lens* C. The position of this lens should be such that its diameter subtends the same angle at the slit that the lens of the collimator subtends, so that the cone of rays after passing the slit may just fall on the lens of the collimator and not spread inside the box. This will mean that the condenser will be at a distance of perhaps two feet from the slit. Between it and the slit another box one foot square should be placed to enclose the light, and the lantern itself should be enclosed as far as possible, so that no scattered light shall be visible in the room (which ought to be perfectly dark).

Abney placed a *double image prism* D over the projecting lens; this breaks up the original spectrum into two, and it must be rotated until these spectra are vertically over one another. A *vertical slit* V long enough to include both these spectra is mounted in a sliding frame, and placed in the focal plane of the lens so that it can be placed upon any colour of the spectra. A *lens* L_1 five or six inches in diameter and about twenty inches focus, is placed beyond the spectra to receive the light from this slit and form with it an image of the double image prism on the screen W. This image should be a patch of light of uniform brightness. A difficulty will be experienced in focussing, owing to this lens not being achromatic. The distance of the lens should be adjusted until the horizontal edges are sharp, and then it should be rotated slightly round a vertical axis until the patch remains stationary as the slit is moved across the spectrum. This movement of the slit will, of course, alter the colour of the patch.

In front of one of the slits at the level of one of the spectra, a *right-angled prism* R_1 of about 1″ face is attached to the sliding frame, to reflect the light of that spectrum to one side. At a distance of about eight inches is a second *right-angled prism* R_2 to reflect the light forward once more. The light passes through a similar combining lens L_2 about 6″ diameter, but of a little longer

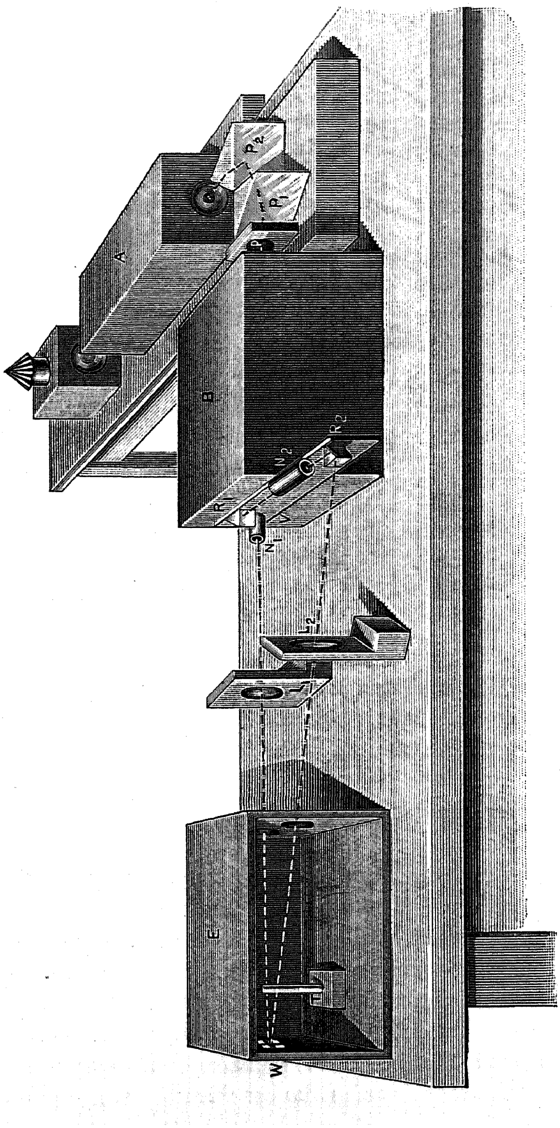


FIG. 332.—Abney's Colour-patch Apparatus.

A. Collimator.
P_1, P_2. Prisms.
B. Projecting-lens Box.
R_1, R_2. Right-angle Prisms.
N_1, N_2. Nicol Prisms.
D. Double-image Prism.
E. Dark Box.
L_1, L_2. Lenses.
T. Rumford Pillar.
W. Patch.

focal length. The prisms and lens must be adjusted to form, with this reflected beam, a patch on the screen coincident with the first patch. A blackened *pillar* T must be placed in front of the patch, as in the Rumford photometer, to obtain a sharp dividing line, so that one half of the patch may be lighted by the direct, and the other by the reflected beam. The distance from the spectrum to the patch will be about four or five feet.

To avoid stray light the patch should be formed at the end of a *box* E about a foot square, blackened inside, and received upon a square W formed by cutting a window about 1″ square in a card covered with black velvet and placing a piece of white paper at the back. An image of the double image prism, a little larger than this window, must be focussed on this white paper; it should appear evenly illuminated all over.

317. If a pigment is to be measured, it may be rolled on a piece of paper with a roller such as that used in photographic mounting. Cut a piece out with a straight edge and slip it in between the white paper W, upon which the patch is to be received, and the card covered with black velvet. The piece of white paper referred to is pasted to this card round three sides only, so that the coloured patch can be slipped in at the fourth side, and the patch must be adjusted until its edge just coincides with the edge of one of the shadows formed by the black rod. To observe the patch and to be able to adjust the rod, a part of the side of the box, at the end of which the black card is placed, is removed. To further reduce scattered light, the mouth of this box should be covered in with a black paper in which apertures are cut, just large enough to admit the beams. When the observations are being taken, the head must be placed under a large *black cloth* which is spread over this box.

The direct beam will therefore fall upon one half of the patch, and the reflected beam will fall upon the other half. If the coloured paper is on the half which receives the reflected beam, the two halves of the patch will appear unequally bright. They are illuminated by the light coming through the vertical slit in the spectrum, that is practically by monochromatic light. They are therefore of the same *colour*, but the coloured patch will absorb some of this light and will therefore appear darker than the white patch.

318. To make the two patches equally bright, the light falling

upon the white patch will have to be reduced. To do this Abney uses a *rotating sector* run by a motor. This sector consists of two discs each of about 9" diameter, each having two quadrants removed, so that when the discs are coincident, half of the light incident upon them will be transmitted; on rotating these discs relatively to one another, the aperture may be reduced to zero, and thus the beam can be reduced to any desired degree.

The discs may be moved relatively to one another while the sector is running by means of a sleeve on the axis which works in a spiral groove, so that moving the sleeve to and fro lengthwise on the axis causes it to rotate relatively to the axis, and to rotate one of the discs which is connected with it, relatively to the other disc which is fixed to the axis.

If such a sector is not available, as the double image prism polarises each of the beams into which it divides the original spectrum, the direct light may be reduced by means of a nicol placed in the beam. This nicol must be carried by the sliding frame to which the right-angled prisms were attached, and if it is mounted with a divided circle, the amount of light it transmits may be found by using the law of Malus, that the intensity of the light is given by $I \cos^2 \alpha$, where α is the inclination of the plane of polarisation of the nicol to that of incident light.

As the sector will reduce the light of the direct beam to one half, it will not be equal to the light in the reflected beam, and unless the reflected beam can also be reduced it will be impossible to make them equal; therefore it is necessary to put a second nicol in this beam, which can conveniently be placed between the two right-angled prisms.

The double image prism, as already stated, will have to be rotated until the spectra are vertically over one another. This adjustment is very important, therefore the double image prism should be mounted in a frame fitted with a tangent screw, by which it may be very slowly rotated in its own plane. The whole experiment must be conducted in a room absolutely dark, and preferably with darkened walls.

319. *Adjustment.—Firstly.* See that the collimator lens is evenly illuminated. Place a sheet of paper just beyond the lens, and observe the patch of light upon it. The lantern must be adjusted until this patch is uniform. If the lantern condenser, the slit and the collimating lens are all in a line, it will only be necessary to adjust the arc. The crater will have to be focussed very nearly

upon the slit. As the carbons burn, if the arc is not a self-feeding one, it will be necessary to have an attendant to keep the light in adjustment, and the correctness of the readings will depend largely upon the care with which this is done.

Secondly. The prisms should be set for minimum deviation and must be so placed that the light shall fall centrally upon their surfaces, so as to waste as little as possible. By breathing upon the face of the prism it will be easy to see where the light strikes it. When adjusted they must be firmly clamped so that they shall not be moved accidentally. The collimator must be adjusted for parallel light; I prefer to do this by actually measuring the focal length of the lens, using the mirror method, p. 61, and then putting the slit exactly at this distance from it.

Thirdly. The spectrum will have to be "scaled." By opening the arc and focussing the spaces between the carbons upon the slit the bright line spectrum will be produced, and potassium, lithium, magnesium and strontium may be introduced in turn. Also a hydrogen tube may be put against the slit and the positions of the hydrogen lines observed. The spectrum itself can be received upon a piece of celluloid, placed between a glass plate and a glass Zeiss scale. If this celluloid is placed in the focal plane in place of the slit V, the bright lines will be easily seen through the celluloid, and their positions may be read on the scale to $\frac{1}{10}$ mm. with ease.

Plot a curve from this with wave length, or better, with squares of wave numbers, page 130, as ordinates, and scale readings as abscissae. Consider the D line as the zero of the scale. The wave length of any colour of the spectrum in future experiments can then be at once determined, if the position of the D line in that experiment is also known. Replace the sliding frame carrying the right-angled prisms (but without the nicols) and slide it along until the D line is seen in the slit. If the slit is not too narrow this is not difficult to do. A piece of celluloid may be placed against the slit to receive it if required. The collimator slit should be rather fine. When the D line is at the slit of the sliding frame, observe the reading of the scale attached to the frame. Then for any other position of the slit measured upon the scale, by reference to the curve, the colour transmitted is known.

Fourthly. See that the direct beam is focussed upon the patch by its combining lens L_1, and adjust the right-angled prisms attached to the sliding frame until the reflected beam is formed upon the same patch, and focus it with its combining lens L_2. Place the pillar in position. This pillar when too near the patch will form a black line down the centre of the patch, and when too far away there will be a bright line there. Its distance must be adjusted until there is no line visible, black or bright.

Fifthly. Insert the nicol's prisms in the sliding frame, and setting the one in the direct beam to transmit as much of the light of that beam as possible, adjust the one N_2 in the reflected beam until the two patches are as nearly as possible equal. If this is done in the yellow light, the patches will probably appear slightly different in hue, one being slightly orange and the other blue. This will be because the two spectra are not quite vertically over one another, and the double image prism must be rotated until the two patches match in hue. In doing this the eye should be turned aside for a short time now and then as the unequal tiring of the nerves, produced by the difference of colour, will otherwise cause the patches when correct to appear tinged with the complementary hues. When the match is perfect as regards hue, if the nicol prism be also properly adjusted, the patch should appear absolutely uniform and the dividing line quite indistinguishable.

Read the position of the nicol on the circle, and the position of the scale. Slide the frame along to another position, again adjust the nicol, and read the circle and the scale. Repeat throughout the spectrum. Enter the results in a table, and plot a curve.

Measurement of a Coloured Pigment.

320. Introduce the coloured paper into the half patch lighted by the reflected beam, taking care that the edge of the paper coincides exactly with the edge of the shadow. If the edge is vertical, the final adjustment may be made by moving the pillar. Place the slit at one of the positions previously read when the whole patch was white, and adjust the nicol to make the two halves identical. Read the nicol and scale. Repeat this at each of the previous

positions of the slit. It will be found that as the slit is moved, the pillar will also have to be slightly adjusted in order to keep the edge of the shadow upon the edge of the coloured patch. At the extreme ends of the spectrum, unless scattered light is very perfectly eliminated, it will be found impossible to make the two halves match; this is because the scattered light will be reflected as white light from the white patch, and as coloured light from the coloured patch, and this additional light will be combined with the monochromatic beam coming through the slit. As the luminosity of the ends of the spectrum is small, and is probably still further reduced in order to make the match, it will be obvious that a very small amount of scattered light will be sufficient to produce a very perceptible effect. Dust or moisture on the surfaces of the prisms or lenses will produce scattering in addition to any light which may escape from the lantern, or be reflected internally from the apparatus. The presence of scattered light is indicated at once unmistakably by a difference in *hue* of the two halves of the patch.

Enter the result thus:

Scale Reading.	Wave Length.	Reading of nicol when the whole patch is white.	Reading on nicol when matching the colour.	Percentage of light reflected by the colour.

If the divisions on the circle attached to the nicol's prism read the squares of the cosines, the last column will be obtained by dividing the 4th column by the 3rd and then moving the decimal two places. (The division is easily performed with a slide-rule.)

The Luminosity of the Spectrum.

321. *Apparatus.*—The spectrum is produced as before, and a patch formed with a combining lens, but the double image prism is removed so that the whole light now goes into the direct beam. A mirror is placed to reflect on to the same patch the light that is reflected from the surface of the first prism, and this is focussed by a large lens of long focal length which is adjusted to form an image of the surface of the prism upon the patch. This beam is

of course white, thus a white patch is produced upon the coloured one coming through the slit in the sliding frame. A rod is set up to cast a shadow as before, so that one half of the patch shall be lighted with the white light and the other half with the coloured beam. As the light is not polarised this time, it cannot be conveniently reduced by nicol's prisms, and a rotating sector should be employed; but there is no necessity to have one which can be adjusted while running. The sector can be made of two cardboard discs about 10 inches diameter, a sector is cut away from each, leaving rather more than a semicircle, and they are so mounted on a motor that they may be clamped to overlap one another to any desired extent. This sector must be placed in the reflected beam.

Experiment.—The apparatus is adjusted and the spectrum scaled as already described, the sector being set so that it transmits say, a quarter of the light. The sliding frame is moved so that the slit is carried along the spectrum with it. When the yellow light is allowed to pass through the slit the coloured patch will probably appear brighter than the white patch; and although the two patches are not of the same hue, it is quite easy to see that this is the case. If the yellow patch is not brighter than the white one, the slit in the sliding frame should be made a little wider so as to increase the amount of light it transmits. On moving the slit into the blue, it will be obvious that the white patch is brighter than the blue one. Thus by moving the slit to and fro, the coloured patch will be seen in one place darker and in the other lighter than the white one and by rocking it to and fro it is possible to find the position in which the two, although not matching in colour, will seem equally bright. In the same way a place can be found between the yellow and the extreme red at which the two patches seem equally bright. Take the scale readings at each of these two points and also read the aperture of the sector with a protractor. Slightly alter this aperture and again find the points at which the patches seem equally bright.

In this way the comparative luminosity of each part of the spectrum can be obtained. The result should be plotted in the form of a curve. The shape of this curve depends upon first, the source of light,—for the proportion of each colour differs with each source of light; and secondly, upon the kind of glass of the

prisms, partly because of absorption, but also because, owing to anomalous dispersion, the areas over which the different parts of the spectrum are dispersed vary with the kind of glass. A spectrum formed with a diffraction grating would be free from this variation.

The Luminosity of a Coloured Pigment in White Light.

322. *Apparatus.*—An arc light A and Abney sector S_1 ; a silvered mirror M_1. As the Abney sector will reduce the beam even when fully open to one half, an additional sector S_2 will be required to reduce the other beam by the same amount. This need only

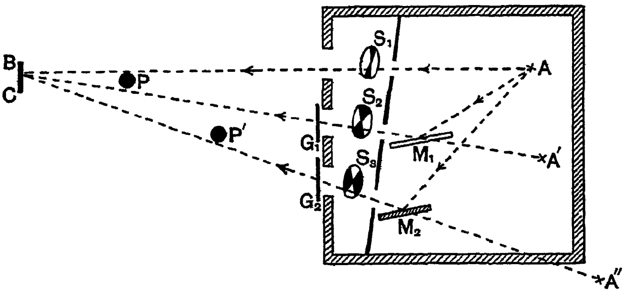


FIG. 333.—Measurement of Absorption of Coloured Papers or Glasses.

be a card disc mounted on a small motor. The mirror is arranged to reflect at oblique incidence a patch of light from the arc upon a screen BC upon which the direct light is also allowed to fall, and a Rumford post P is set up to shade the two halves of the screen, one from the direct light of the arc, and the other one from the reflected beam. A sector is placed in each beam. The arc, the mirror and the two sectors must be inclosed in a large box as in Fig. 333, so that no light may reach the screen except that which is transmitted through the sectors, for which apertures in the box are to be cut.

Experiment.—Adjust the Abney sector to make the two patches equal. Suppose it is open 90 divisions. The coloured paper is then put to cover the half of the patch which is illuminated by the light coming through the other sector. The white half of the patch is then obviously brighter. By reducing the aperture of the sector, this patch can be made much darker. By

opening and closing the sector alternately, the white half will be seen to grow alternately lighter and darker than the coloured half. By making the oscillations smaller and smaller, the position in which the two halves seem equally bright can be determined. If the aperture be now 45, then the luminosity of the pigment is $\frac{45}{90} = \cdot 5$, as compared with white. In the same way the luminosity of any other pigment can be determined. The incidence upon the mirror should be as oblique as possible, so that the light which forms the direct and the light which forms the reflected beam, shall each be emitted from the arc as nearly as possible in the same direction, as the intensity of the light emitted in different directions is not necessarily proportional. To avoid this source of error, a semi-silvered mirror could be inserted in the direct beam to reflect the light on to the silvered mirror, which would be adjusted to reflect it on to the screen and form its patch with this light; the two patches would now be equally affected by any variations in the arc passed through the two sectors. If the arc and mirror are not enclosed in a box the readings will not be accurate; as in addition to the light supposed to be falling on the patches, namely, that going through the sectors, the two halves will be illuminated by the scattered light in the room, and this will increase the brightness of the white patch to a greater extent than that of the coloured patch.

The Luminosity of the Light transmitted by a Coloured Glass Plate.

323. *Apparatus.*—As before.

Experiment.—Adjust the Abney sector to equalise the beams, read it, and introduce the glass plate G_1 (Fig. 333) into the other beam. The patches will now be unequally bright, and in the same way as in the last experiment, the sector must be adjusted to make them equal by oscillating it to and fro. It will be found that unless the glass plate is made of worked glass the patch produced will be very uneven; an ordinary piece of glass placed in the path of the beam from an arc light produces a blotchy appearance upon the screen.

It is instructive to find the luminosity of a series of coloured plates which as nearly as possible match the colours of the

spectrum. Instead of coloured glasses, pieces of patent plate upon which films of stained collodion have been formed (see p. 446), or even coloured gelatines, may be examined.

Add a second mirror M_2 so placed that the light from it shall go through the cardboard sector on the opposite side of the axis, as in the figure, so that two beams of light may be thrown upon the one patch. Match the luminosities of each of these beams in turn (the other being for the time covered up) and place two pieces of coloured glass one in each beam—whose luminosities have already been determined. There ought now to be on the screen a patch of which the luminosity is the sum of the luminosities of the separate colours. Measure it and see if the readings agree with the calculated ones.

Maxwell's Discs.

324. *Apparatus.*—Cards of various colours, cut to circles of two sizes, say 3″ and 5″ diameter. Three cards of one size should be carmine, yellow and blue respectively, the colours being similar

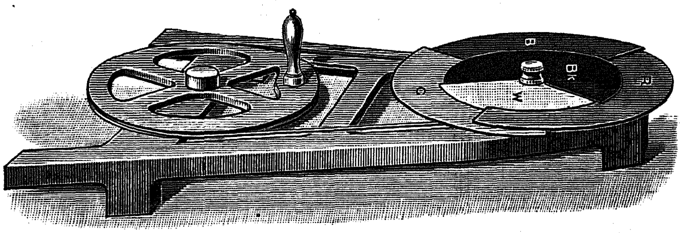


FIG. 334.—Whirling Table with Maxwell's Discs.

to those used in three-colour printing, a black and a white of each size, and some discs of other colours, such as orange, green, buff, vermilion, will be required; a small motor, or a whirling table, or a large top will be required, on which the cards can be mounted to rotate at least 20 times a second. Each card must have a hole at the centre of the circle to fit the axis of the whirling table, top or motor, as the case may be; it must also be slit radially from this hole to the edge. The axis upon which the cards are to rotate must have a nut which can be screwed up to clamp the cards when they have been adjusted. The cards can be threaded

into one another through the radial slit, and caused to overlap to any desired extent.

Thread a black and white disc together of the same size, and the carmine, blue and yellow discs together of the other size. Put the two sets on one spindle and rotate. The black and white will, of course, form a grey; the other three discs will, as a rule, produce some other colour, but it is possible to so proportion them that they also produce a grey, and then by varying the proportions of the black and white pair match the grey they make with the other. When this match has been produced measure the angular width of all the sectors exposed, and so obtain a colour equation, *e.g.* $246\,B + 114\,W = 142\,B + 113\,Y + 105\,C$. In the same way match the green, buff and other discs with the set of blue, yellow and carmine ones (it may be necessary to add a little black to one of them).

Curve of Sensitiveness of the Eye.

325. *Apparatus.*—Circular white cards about 12″ diameter; whirling table; some dead black, chinese white, and a paint brush; compasses and dividers; protractor; squared paper.

Draw three diameters on a card at 60° to one another, and put the card on the whirling table. Paint the card somewhat as shown in the figure in such a way that when rapidly rotated it shall appear to shade *regularly* and evenly from black at the centre to white at the edges; this will of course have to be done by trial and error. Keep the figure as symmetrical as possible. Draw a series of concentric circles such as ABCD ... N cutting the arms of the star (the circles must be drawn very faintly in pencil, or they will spoil the apparent gradation of the light). With a pair of dividers measure BP equal to CD, PQ equal to EF, QR equal to GH, and so on, and

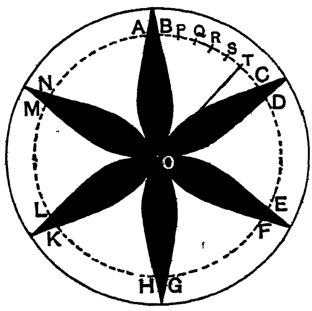


FIG. 335.

thus find the length of the arc AT which is equal to the sum of the arcs AB, CD, EF, ... MN. With a protractor measure the angular width of the arc AOT.

Measure as in § 320 or 322 the amount of light reflected by the black that has been used. Suppose, for instance, it reflects 4 % of the light, and that the arc AT subtends 60° at O. Then the arc on this circle which is left white subtends 300°, and reflects 300/360ths of the total light falling on the disc; but the black also reflects 4 % of the light, and therefore reflects 4/100 of 60/360, or 2·4/360ths of the light. Thus the total light reflected at this radius is 302·4/360ths of the light.

On a piece of squared paper take a length OX to represent the radius of the disc (say equal to half the radius), and OY to represent 360°. Then plot the radius of the circle OA along OX, and the angular width 302·4 along OY (= AB), and so find the point P. Repeat this for each of the series of circles, and so obtain the curve OPQ.

Also tabulate the results so:

Radius of circle = S.	Angular width of				log I.	$\frac{\log 100 - \log I}{10 - S}$
	white.		white = eq. of the black.	corrected white = I.		
0	0		2	2	·301	·170
1	·8		1·8	2·6	·415	·175
2	2·1		1·6	3·7	·569	·179
3	4·5		1·4	5·9	·771	·176
4	7·4		1·2	8·6	·934	·177
⋮						
9	66·7		·2	66·9	1·825	·175
10	100		0	100	2 mean	= M ·1758

Take the logs of the angular widths and write them in column 3. Plot the log I against the radii of the circles (*i.e.* the "sensation"). It will be found that the points lie on a straight line, which, however, does not go through the origin. Thus the sensation is connected with the stimulus by an equation

$$S = S_0 + M \log I.$$

(The constants are, of course, arbitrary.) This shows that the stimulus has to increase in geometrical progression if the sensation is to increase in arithmetical progression.

326. One curious result, pointed out by Professor Lloyd Morgan, is that in a poor light the gradation on the card will no longer appear correct. For instance, a sensation of 50 % of the maximum is produced by a stimulus of 12 % of the maximum in a full light. In a poor light—say 65 % of the maximum with a sensation of 90 %—the middle gradation should be 45 % of the maximum. From the curve at the point it is seen that the intensity should be 9·5 %. But if the card is whirled, the intensity at the 50 radius

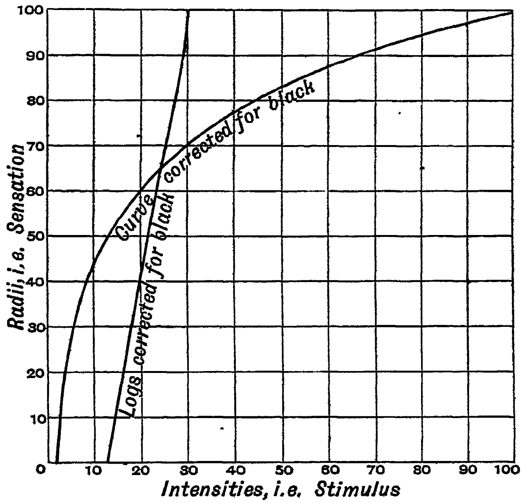


FIG. 336.

will now, of course, be $\frac{65}{100}$ of 12 or 7·38. Hence, the intensity will be too small, and the disc will not appear to be uniformly gradated. If, instead of whirling the disc in a poor light, the card itself is grey or coloured, and therefore the intensity of the light reflected is reduced, the same results follow as if the light had been reduced. But by measuring the luminosity of the grey paper (as in § 322), and, after correcting for the luminosity of the black, plotting the points against sensation numbers reduced proportionally, the correct curve will be again obtained. If, for instance, the luminosity of the grey is 28 % of that of the white

card, corresponding to 70 % sensations, the sectorial angles are to be plotted against sensation numbers ranging from 0 to 70 instead of from 0 to 100. So if the star is blue on a red ground the luminosity of the red and blue are to be measured, and then the sectorial angles plotted from the sensation number corresponding to the blue luminosity to that corresponding to the red luminosity, instead of against sensation numbers from 0 to 100.

The Colour Sensation Curves of the Eye.

327. The simplest apparatus is the one invented by Maxwell, known as his Colour Box. In forming an ordinary spectrum, light starts from the collimator slit, and after passing through the prisms

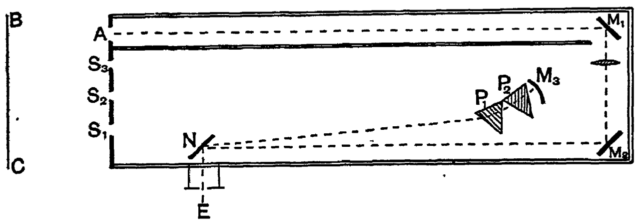


FIG. 337.—Maxwell's Colour Box.

it is focussed on the screen. If from any point on this screen, light of the same colour as that which falls at the point were sent back through the apparatus, it would emerge at the collimator slit. If, for instance, a concave mirror was placed at this screen, so that the light from the projecting lens fell everywhere normally and was reflected back along its path, it would all return to the slit of the collimator. If the surface of this mirror were covered by a card with one or two slits in it, then the colours which pass through these slits only would be returned. And, lastly, if the mirror were removed and each of these slits illuminated by a white light, only that colour which originally went through any slit would be so deviated as to reach the slit of the collimator,—the other colours being deviated more or less would fall to one side. This is the principle of the Maxwell Colour Box.

Let E be supposed to be the collimator slit. Light passing through E falls upon a mirror N, from there it is reflected and passes through the two prisms P_1 and P_2 to fall upon a concave

mirror M_3. From this it is again reflected through the prisms and forms a spectrum on the screen $S_1S_2S_3$. The concave mirror has a focal length, such that this screen is the conjugate focus of the slit E. Suppose slits $S_1S_2S_3$ in this screen to be arranged in red, green and blue respectively; then the red light which starts at E is deviated just enough to arrive at S_1. If, therefore, white light started from the slit S_1, it would form a spectrum of which the red rays pass through E, any other coloured light starting from S_1 would be deviated either too much or too little. In the same way white light at S_2 and S_3 will produce at E green and violet respectively. Therefore an eye placed at E will receive at the same time red light from S_1, green from S_2 and violet from S_3. The slits can be very conveniently made by threading six pieces of brass with straight bevelled edges, on a screw as shown in Fig. 338. A nut in each piece of brass which works in a slot causes it to travel along the screw and enables it to be fixed in any desired position A pair of such pieces will form one slit, and it can be adjusted to any part of the spectrum, and to any width. The spaces between the slits can be stopped up with pieces of thin black card, or thick lead foil. As the light will arrive at E from the whole of the face of the prism P_1, this area will appear uniformly illuminated with a mixture of the three coloured lights. By adjusting the widths of the slits, the proportions of these colours can be varied at will. At the same time light from the slit A after reflection from the mirror M_1 is seen in the mirror M_2, and thus two illuminated areas are seen touching one another, the one formed by the combination of the three colours from $S_1S_2S_3$, and the other illuminated by the light from A. The white screen BC is placed beyond the slits to act as the source of light. The card should be illuminated by a light of constant colour composition, for which purpose the arc light is to be preferred to sunlight, as the latter varies with the altitude of the sun and the state of the atmosphere.

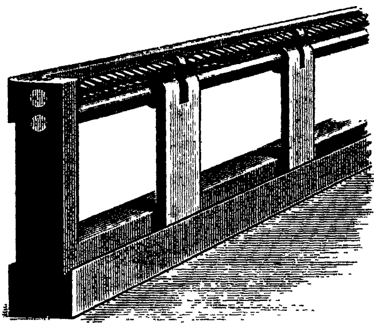


Fig. 338.—Method of Mounting Slits.

First obtain the positions of the principal lines in the spectrum, so that the colour which reaches the eye from the various slits may be known. Illuminating the slit at E with the arc light, obtain on a celluloid or glass screen at S the bright line spectrum, and take readings as previously described.

The frame carrying the slits should have a millimetre scale engraved upon it, so that the position of the slits can be at once read off. The width of a slit may be found with a reading microscope pointing vertically downwards, using a low-power objective and observing the image of the slit formed in a strip of mirror glass placed along the front of the slits at an angle of 45°, which must be removed during the experiment and replaced after each match has been formed.

The Curve of Absorption of a Coloured Solution. Glazebrook's Spectro-photometer.[1]

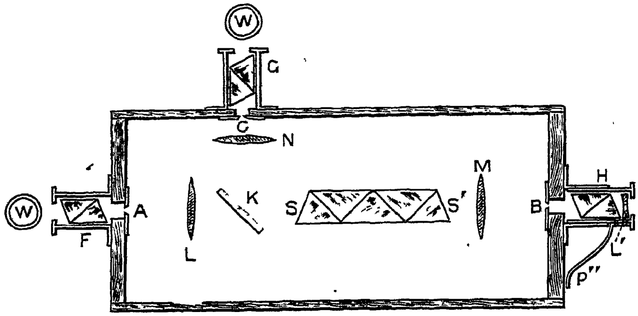


FIG. 339.—Glazebrook's Spectro-photometer.

328. The construction of the instrument will be easily followed from the above diagram.

A and C are fixed slits. B is a slit which can be moved to and fro horizontally. F, G, H are nicols, F has its plane horizontal, G has its plane vertical and H can be rotated on a divided circle, its rotation being observed by a pointer P″. L, M, N are lenses each at their respective focal lengths from A, B and C. SS′ is a direct vision prism. At L′ is a concave lens fixed on

[1] *Proc. Camb. Phil. Soc.*, 1883.

a silvered mirror K, which is so placed as to cover half of the lens L, so that in the lower half the light reflected from N is seen, and above that the direct light from L.

Let the amplitudes of the vibrations from A and C be a and c, and suppose the plane of polarisation of A to make an angle θ with the plane of F, that is with the horizontal.

Place the slit B in any part of the spectrum and turn the nicol H until the two halves of the field are equally bright.

Then, as the beams are equal with the nicol in this position,

$$a \cos \theta = c \sin \theta.$$

Therefore
$$\frac{I_a}{I_c} = \frac{a^2}{c^2} = \tan^2\theta.$$

Now place the cell containing the coloured liquid between L and K and once more balance the light.

If I'_a be the intensity of the light of this colour transmitted by this solution,

$$\frac{I'_a}{I_c} = \tan^2\theta'.$$

Therefore
$$\frac{I'_a}{I_a} = \frac{\tan^2\theta'}{\tan^2\theta}.$$

If k is the proportion of light lost by absorption,

$$I'_a = I_a(1 - k).$$

Therefore
$$k = 1 - \frac{\tan^2\theta'}{\tan^2\theta}.$$

It will be found that there are four positions of the nicol H at which the balance occurs. The mean of the four readings must be taken to avoid index errors.

There will be an error caused by the loss of light by reflection from the surface of the vessel and absorption by the glass. Let I_1 be the intensity when using the cell filled with a colourless liquid, and I_2 the intensity with the coloured liquid. Then

$$I_1 = I_a(1 - k_1)$$
$$I_2 = I_a(1 - k_2),$$

and k is approximately

$$k_1 - k_2 = \frac{I_2 - I_2}{I_a} = \frac{\tan^2\theta_2 - \tan^2\theta_1}{\tan^2\theta}.$$

Glazebrook calls the extinction coefficient e the reciprocal of the thickness which reduces the intensity of the incident light to $\frac{1}{10}$.

If the thickness of our cell be m, then,[1] $me=\log_{10}\frac{I}{I'}$.

By graduating the scale over which the pointer moves in $\tan^2\theta$ instead of degrees, the calculations would be much simpler. See also § 313.

The Brace Spectro-photometer.

329. *Apparatus.*—The instrument consists essentially of a spectrometer mounted with two collimators T, T′ (Fig. 340), and a telescope R, one of the collimators having a slit with a micrometer screw; and a special prism. This prism was made of the Jena glass 0·102. The prism is an equilateral one, made of two prisms each 30° angle, put together with alpha-mono-bromo-naphthaline,[2] which is cemented in round its edges by gelatine or shellac.

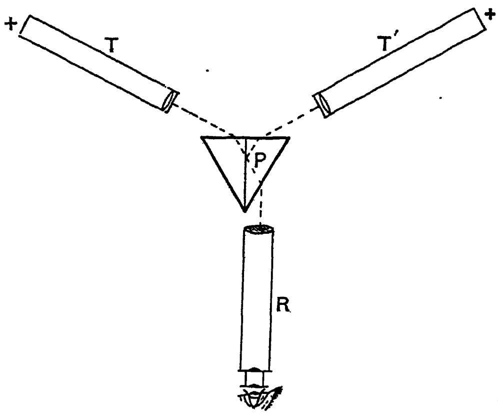


FIG. 340.—Brace Spectro-photometer.

[1] Let
$$\frac{I'}{I}=e^{-am},$$
where a is a constant depending upon the absorption,

and therefore
$$\log_e\frac{I'}{I}=-am.$$

Then
$$\frac{1}{10}=e^{-\frac{a}{e}},$$

and
$$\log_e\frac{1}{10}=-\frac{a}{e};$$

therefore
$$me=\frac{\log_e\frac{I'}{I}}{\log_e\frac{1}{10}}=-\log_{10}\frac{I'}{I}$$
$$=\log_{10}\frac{I}{I'}.$$

[2] This is used because its refractive index is nearly the same as that of the glass.

One half, ADC, of this prism (Fig. 341) is silvered on the surface AD along a horizontal strip SS', the edges of the silvering being sharp and straight.

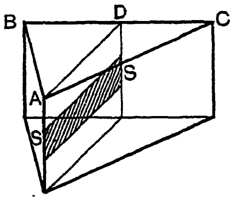


FIG. 341.– Brace Prism.

The prism is to be mounted on the spectrometer as usual, the light from the collimator T (which may have a unilateral slit) is transmitted through the unsilvered portion, and the prism and telescope are adjusted for minimum deviation with sodium light. The cross wire of the eye-piece of the telescope R is placed on a sodium line. Then T is screened, the telescope clamped, and the collimator T' (which must have a bi-lateral slit) is adjusted until the sodium line from it, reflected from the silvered strip, is also focussed on the cross wire. The two spectra will then be superimposed throughout their length, if the two halves of the prism ABC are similar.

Remove the eye-piece; put instead an ocular slit, 2 mm. high and from 0·5 to 1 mm. broad, in its principal focal plane and place an incandescent burner in front of each collimator, with a piece of ground glass between it and the collimator silt. Look through the ocular slit. A circular field with a horizontal band is seen, the field and the band will each appear uniformly illuminated, but their intensities will in general not be equal to one another.

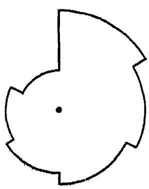


FIG. 342.

In addition to the above, a sector is required cut in a series of steps, which can be rotated by a small motor, and adjusted in front of the slit T', so that the light may pass by either step at pleasure.

Experiment.—It is first necessary to calibrate the micrometer screw of the slit of T'.

Open the slit T' to about 1 mm. and place the sector so that its lowest step is opposite this slit. The strip will now appear fairly dark, as the light only passes through the small angular aperture of this lowest step,—an aperture of perhaps $11\frac{1}{4}°$.

Adjust the slit T until the rest of the field is equally dark

Take the micrometer reading of T′. Now move the sector until the next step, which may have an aperture of $22\frac{1}{2}°$, is opposite the slit T′, and adjust this slit T′ to again balance the rest of the field. Its width should now be about $\frac{1}{2}$ mm. Take its reading. Move the sector again to the third step having an aperture of 45°, and again adjust T′.

Continue this until the full aperture of the sector is reached, namely 360°. T′ will now have an aperture of about $\frac{1}{64}$ mm.

Replace the sector so that the step $11\frac{1}{4}°$ is again opposite T′, and reduce the aperture of T to balance the field. Move the sector until the next step $22\frac{1}{2}°$ is in front, and adjust T to balance once more.

Its aperture should now be $\frac{1}{128}$ mm.

In this way the calibration of the screw may be continued.

The results may be entered in columns thus:

Sector aperture.	Reading of micrometer screw of T′.	Ratio of light transmitted.	Ratio of sector apertures.

If we call the light transmitted by the slip T when it was opposite the first step 1, the last column will begin with 1 and the other succeeding numbers will be 2, 4, 8, 16, etc.

It may be found that the continuations of the graduations from $\frac{1}{64}$ can be better effected by putting the sector to some intermediate step such as 45, instead of to the $11\frac{1}{4}$, which would perhaps cause too great a reduction of the light.

This calibration once effected, the instrument may be used for finding the absorption of a coloured medium. The method is the same as described for Glazebrook's spectro-photometer, substituting the numbers in the third column for the $\tan^2\theta$ in his formula.

Differential Spectro-photometers.

330. The photometer just described has one very serious fault—it depends upon *two* light sources, and if these are not frequently compared with one another their variation in brightness may

entirely upset the readings. In the differential spectro-photometers two portions of the slit of the same collimator are illuminated from one light-source and brought into apparent contact by some prism device. The intensity of one portion of the slit is diminished by the absorption to be measured, that of the other is varied either by varying the width of its slit (as in the Brace instrument) or by polarising prisms (as in Glazebrook's instrument).

One such arrangement is shown in Fig. 343. The light from an incandescent burner is parallelised by a lens placed at about its focal length from the flame, from which a part goes through the cell, the nicol prism, and after two reflections enters the upper portion of the slit. Another portion passes directly through the block and through the unsilvered portion of the interface to the lower portion of the slit. A block of colourless glass is inserted in the cell, so that the path in the liquid may be reduced as much as required.

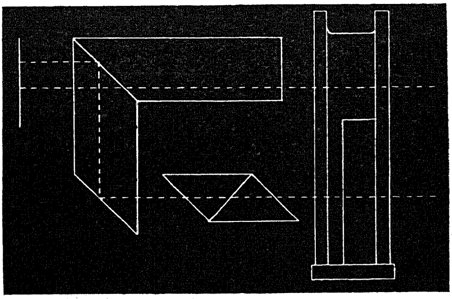


FIG. 343.

The method of experiment is similar to that described for the Glazebrook spectro-photometer.

A symmetrical arrangement is shown in Fig. 344.

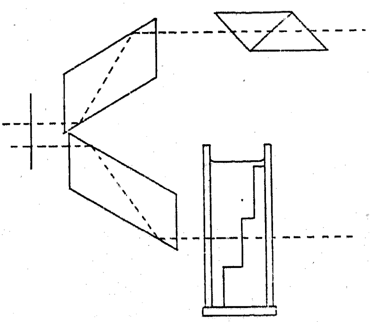


FIG. 344.

There is, however, no real advantage in a symmetrical arrangement. Its only object is to interpose the same absorptions in both beams. But as it is necessary to compare the beams for every colour whenever a polarising device is used to reduce the

intensity of one beam (see page 409) owing to the unequal proportion of the lights of different colours reflected by the first surface of the dispersing prism, the advantage is illusory, and the simple arrangement of Figs. 343 or 345 is really just as good. As the polarising prisms reduce the light to less than half, an absorbent wedge may be interposed in the other beam. The wedge is made either by photographing a card which is shaded uniformly from one end to the other, or by a glass plate smoked so that the density of the deposit increases gradually from one end to the other. The wedge can be moved to and fro in front of the slit until the beam is reduced to the required amount and clamped there.

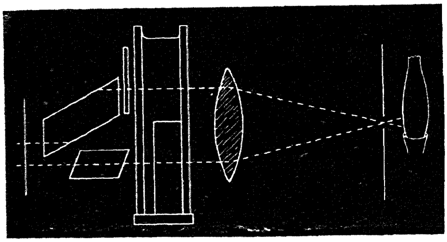


Fig. 345.

When in proper adjustment, by either of the above devices a field is obtained in which the dividing line between the two portions has no breadth. In this way the sensitiveness of the eye to the contrast of intensity is used to the utmost, for this sensitiveness is greatly reduced if the dividing line has any apparent breadth.[1]

The rest of the instrument may be an ordinary spectroscope with the addition of an analysing prism, which must be mounted on a divided scale, by which the intensity of the comparison beam is measured, as already described (§ 328). But as the instrument is not required for obtaining refractive indices, it is best to fix the telescope and use a constant deviation prism; this enables the instrument to be made more simply, and at the same time the telescope with its nicol and divided circle can be mounted more rigidly.

331. The principle of the constant deviation prism can be easily illustrated, for draw the path of a ray PQRS through a prism at

[1] See also the remarks on p. 402.

minimum deviation on tracing paper (Fig. 346). Then fold the paper along any line DOE passing through O, the mid point of QR.

If one imagines the part of the prism to the right of DE to be reflected in DE, the ray will now follow the path PQOR′S′. Also, as PQ makes the same angle with BQ that R′S′ makes with B′R′, it is obvious that PQ and R′S′ will make the same angle with one another as BA does with B′A′. If the direction of DE be so chosen, for instance, that B′A′ comes at right angles to AB, R′S′ will also be perpendicular to PQ.

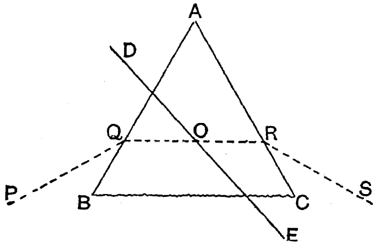


FIG. 346.

If DE bisects the vertical angle of the prism CAB, the ray R′S′ will coincide with QP, and we have the ordinary auto-collimating prism. The angle between PQ and R′S′ can thus be varied at pleasure, and all that is necessary in any case is to determine the angle between PQ and R′S′, and to mount the telescope and collimator at the same angle with one another.

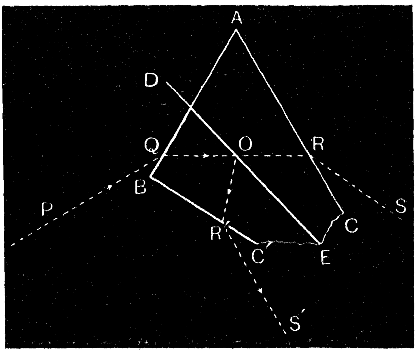


FIG. 347.

The prism must be mounted on a turn-table, with a circle which need only be divided for a short distance, and in any arbitrary manner; it can be read with a single vernier. The scale must be calibrated by determining the positions of known lines as in § 98.

Instead of a divided circle a screw can be used to actuate the turn-table, and the wave-lengths can then be marked on a drum attached to the head of the screw. The readings of the drum may give directly the wave-length of the light in the centre of the field of the telescope.

Flicker Spectro-photometer.

332. The principle of the flicker-photometer could be applied to spectro-photometry as in the following instrument, which is an auto-collimating one.

The light from an inverted incandescent burner is parallelised by a lens and after passing through a slightly ground screen (not shown), one beam traverses the reflecting prism P and the slit S. The other beam passes through the two glass plates on the lower ends

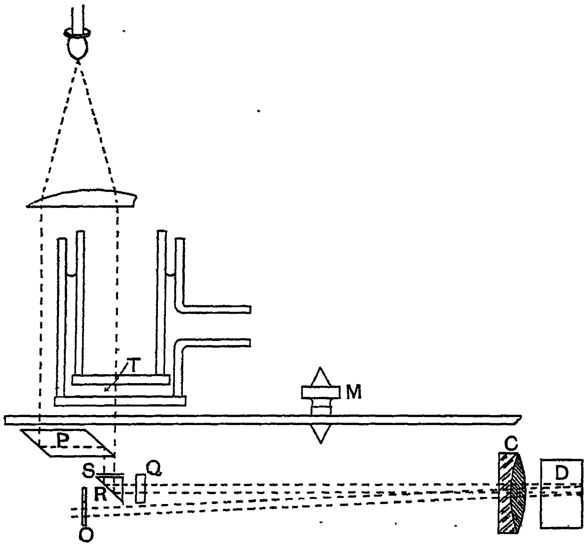


FIG. 348.

of the concentric tubes T and the slit S. From the slit the beams are reflected forward by the right-angle prism R. They then traverse a very weak double-image prism Q and emerge from the lens C of the collimator as parallel light, enter the prism D, from the back silvered surface of which they are reflected again through the prism D, the double-image prism, and focussed by the lens C on the ocular slit O, immediately behind which is the pupil of the eye. The images of the upper and lower portions of the slit S are drawn out to form two spectra, each of which is doubled by the double-image prism, and the strength of this prism is just sufficient to superpose one of each at the short ocular slit

O.[1] The pupil of the eye is put close to the slit, and will see the field uniformly illuminated with light of a colour that is determined by the position of the slit. A brass disc mounted on nearly frictionless bearings rotates just above the prism P as in the figure. This plate has a row of spaces concentric with its axis, the angular breadths of the spaces being all equal, and is so mounted that each beam has to go through the ring of spaces, and that as a tooth cuts off one beam, a space allows the other to go through. Thus, if the disc is rotated, the eye receives the light from the one beam and the other alternately. The eye therefore sees the field of the lens C uniformly illuminated with mono-chromatic light, first by the light of one beam, then by that of the other, then by the first again, and so on as the disc is rotated. If the beams are not quite equal in intensity, there will be a *flicker* when the disc is rotated, which will only disappear when the light is exactly balanced. The intensity of the comparison beam (the one that does not pass through the coloured solution) is reduced by reducing the width of that part of the slit S through which it passes by a pair of jaws which move symmetrically and independently of the jaws forming the portion of the slit through which the direct beam passes, or by interposing a nicol prism. By moving the ocular slit to different parts of the spectrum, the comparison can be made with light of any colour.

As the number of the teeth on the disc is large, it has only to be rotated slowly, and this can be easily done with the milled head M, by twirling it occasionally with the thumb and finger. Any spectrum colour can be brought into the field by rotating the prism.

The solution to be measured occupies the space between the two tubes T. One of these tubes is fixed and the other is moved up and down vertically by a rack. Thus the distance between the glass plates is easily varied, and so the thickness of the layer of coloured liquid can be varied at will. The side tube communicates with a larger vessel to maintain the liquid in the annular space between the tubes T at an approximately constant level.

A reading must first be taken with clear water in the space between the tubes T, and this reading should be repeated in different parts of the spectrum, for as polarised light is used, the intensities will vary from colour to colour in the spectrum (see p. 426).

[1] The other spectrum of one pair will be produced above, and of the other pair below, the short ocular slit, by which they are thus cut off.

CHAPTER XX

THE EYE AND VISION

Illustration of the Behaviour of the Muscles which move the Eye as a whole.

333. *Apparatus.*—Stereoscope;[1] a stereoscopic picture; pieces of ordinary red glass and green signal glass.

The axis of the eye may be pointed up or down, and may be moved right or left, the eye can also be rotated on its axis. The muscles which control the two eyes act together, as is well known. To a small extent they can be adjusted independently of one another.

i. Take two similar pictures: for instance, similar advertisement or concert tickets. Place one in the stereoscope opposite each eye, and adjust them until they are seen as one. It will be found that keeping one still, the other may be moved right or left for half an inch without any perceptible discomfort, and that coincidence may still be maintained at greater distances than this, but with a certain amount of pain. Do not attempt to carry it too far, as it is not advisable to run any risk of injuring the eyesight.

ii. In the same way it will be found that one of the pictures may be moved up and down to a slight extent and coincidence still maintained, showing that to a slight extent the muscle which moves one eye in that direction can be used independently of the other. As in ordinary circumstances we do not require to move the eye independently in this direction, we

[1] A cheap form of this instrument is made, that will answer the purpose perfectly, costing less than 2s.

have not exercised these muscles independently, and it is probably for that reason that this motion is so small. It is conceivable that with exercise it might be possible to make these motions quite independent.

iii. Lastly, keep one card at rest, it will be found that the other may be rotated slightly without becoming blurred. This can only mean that the corresponding eye has been rotated on its own axis to correspond.

Note.—These experiments can be shown to a class by using two lanterns and projecting two similar diagrams on the screen at the same time, one with red and the other with green glass, the members of the class being each supplied with a card having two holes the distance apart of the eyes, over one of which is a red and the other a green glass. By holding these in front of the eyes only a red picture will be seen by the eye which looks through the red glass, and similarly with the green. Then move the slide in one of the lanterns to and fro, up and down, and rotate it.

The Blind Spot.

334. That part of the retina where the optic nerve enters is insensitive to light, being devoid of the minute rods and cones necessary to vision. It is called the Blind Spot, and is situated a little nearer the nose than the optic axis.

Apparatus.—Cardboard, red wafers or drawing-pins.

This may be proved by sticking a red wafer or a drawing-pin in a piece of card and making a cross some 6 or 8 inches from it. Holding this in the right hand, with the wafer to the right of the cross and in the same horizontal line, and closing the left eye, it will be found that at a certain distance, if the attention is fixed upon the cross, the wafer becomes invisible. By bringing the card nearer, or moving the wafer further from the eye, it will disappear. If the card be inverted, so that the wafer is to the left of the cross, and the experiment repeated with the left eye, the position can again be found at which the disappearance occurs. By fixing attention on the cross, the axis of the eye is pointed

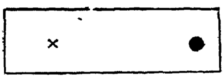


FIG. 349.

at it, and the light from the wafer, after passing through the lens, proceeds to a point on the retina to one side of the centre, at a certain distance this point coincides with the blind spot and the wafer becomes invisible.

Stereoscopic Vision.

335. If, without moving the head, first one eye then the other be closed, whilst objects at different distances are observed, it will be noticed that there is an apparent displacement of a near object relatively to a further one. For instance, if a position be taken up such that a lamp or a retort stand in the room is between the observer and the window, and the eyes be alternately closed, the lamp will appear relatively displaced to the window bars. The images, therefore, of objects at different distances do not exactly coincide on the retinas of the two eyes. If attention be fixed on an object at a certain distance, objects at other distances will be differently situated relatively to it. It is this displacement which we associate in our minds with "relief." An object appears in relief when this relative displacement is produced. If an object outside the window be observed in relation to the windows, it will be seen that its relative motion across the bars is in the opposite direction to that of an object in the room.

By taking photographs of natural objects, either with two cameras side by side, or with a single camera which is moved a few inches laterally between the two exposures, two pictures are produced in which a similar relative displacement of the objects at the varying distances occurs; and if prints of these pictures can be seen, one with one eye and the other with the other eye, similar displacements are produced on the retina as are produced when the actual objects are seen, and there will be the feeling of relief.

The stereoscope is designed to produce this effect.

The Stereoscope.

336. *Apparatus.*—In the usual form of instrument a pair of prisms, with curved surfaces which both bend the rays and enlarge the image, are placed at the distance apart of the eyes. The prisms

should have such an angle that the two pictures to be viewed placed side by side are caused apparently to coincide with one another. Owing to the fact that the eyes are not always the same distance apart, it is an advantage if the distance between these prismatic lenses can be slightly varied. If an ordinary 8-inch lens be cut in halves, and the halves placed one in front of each eye with their thin edges towards one another, a pair of stereoscopic pictures, not too widely separated, can be viewed.

It will be easily seen from Fig. 350 that the light from the pictures A and B after passing through these two half lenses will appear to come from a point C.

When viewing an object at a short distance, the axes of the eyes are automatically converged to meet at that distance, so that to

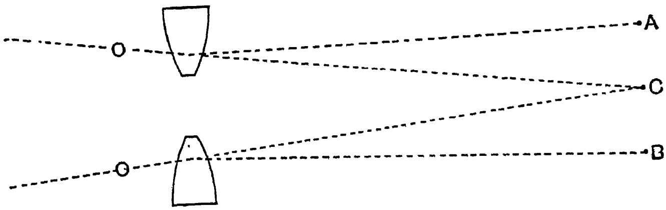


FIG. 350.—Principle of Stereoscope.

view these pictures at a distance of 8 or 10 inches, one adjusts the axes to meet at that distance; it is a strain upon the eyes to keep the axis parallel, and in viewing the pictures A and B, the axes will be pointed together and will meet at C. The distance apart of the pictures A and B, or the deviation of the rays by the lens, must be adjusted so that the light entering the eyes appears to come from C. As the lenses become more prismatic the more nearly the light passes through their edges, it is obvious that by altering the distance apart of the lenses so that the light entering the pupil of the eye (which is a pencil only 1 or 2 mm. in diameter) may pass through the lens nearer or farther from its edge, the amount of the convergence can be controlled. It is also obvious that, if the lenses are at such a distance that this convergence shall be correct for a person whose eyes are $2\frac{1}{4}$ inches apart, it would not remain correct for a person whose eyes are $2\frac{1}{2}$ inches apart.

If a pair of simple stereoscopic pictures be drawn on two cards—for instance, one square within another, the inner square in one picture being to the right and in the other to the left of the centre of the larger square, and the corners of these squares connected as in Fig. 351, then if the cards are put in a stereoscope so that one is viewed with the left eye and the other with the right eye, it is easy to see that the appearance must be that of a prismatic block, the inner square being nearer the observer than the outer. By interchanging the cards so that the left-hand one is seen by the right eye, and *vice versa*, the appearance becomes that of a hollow tube.

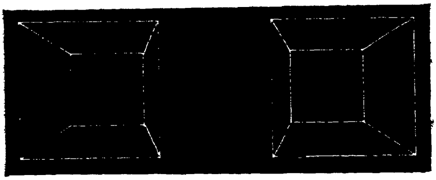


FIG. 351.—Stereoscopic Pair of Pictures of a Block. By holding a card vertically between the pictures, and adjusting the convergence of the eyes until the two appear superposed, an appearance of solidity is produced.

337. If a stereoscopic photograph is taken of a piece of carving in relief, and viewed with the pictures correctly placed, the appearance of relief, of course, will be produced. But if the pictures are interchanged, so that the picture taken in the right-hand camera is viewed by the left eye and *vice versa*, an appearance of intaglio will result. This reversed relief is known as *pseudoscopic*.

Wheatstone's pseudoscope was constructed by mounting two "erecting prisms," one before each eye, as in Fig. 352, each to cause the left side of the object viewed to appear to the right. Then, on looking at any geometrical figure—such as a cone or a spiral viewed from a point on the axis, or a medal in high relief—the relief was reversed.

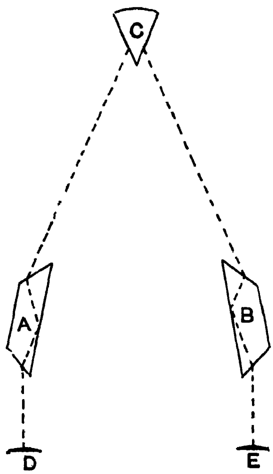


FIG. 352.—Pseudoscope.

This experiment can be so easily repeated, that it is well worth setting up.

Apparatus.—Two right-angle prisms[1] (or erecting prisms); some brass wire about No. 16 gauge, and nippers to cut and bend it with; some plain paper; a support for the prisms, *e.g.* a box about 18″ high to put on the table; some simple objects, *e.g.* a metal card tray, a photo frame, a ball, an egg-cup.

For the pseudoscope it is only necessary to put the two right-angle prisms on the box on the table, with their hypotenuses about two inches apart, and approximately parallel. Now look through them at some such object as a pencil held up vertically at a distance of about a foot (the pencil must be seen by reflection); adjust the distance apart, and the inclination of the prisms until the pencil is seen comfortably as a single object. The objects to be viewed must all be held in about the same position.

A very effective object is a hollow paper cone, about two inches diameter at the base, and three or four inches high. It is best held on a wire handle, as the illusion is spoilt if the object is held in the hand. Make a ring of wire two inches diameter, and leave two or three inches of the wire attached to form a handle. Attach the paper cone to the ring with gummed paper. If this is looked at through the prisms, it will appear hollow when the point is towards the observer, and *vice versa*. If it is slowly rotated, it will appear in some positions as though the paper is transparent. Geometrical models of wire are also easily seen reversed. They should have wire handles, so that the observer's hand does not appear.

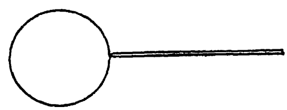


FIG. 353.

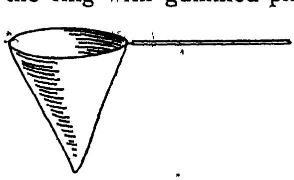


FIG. 354.

A ball will appear to be a cup, and a cup will appear to be a ball.

But the most effective objects are probably stamped metal trays (sold for waiters, cigar trays, etc.). As these are nearly

[1] Gallenkamp sell some that will do fairly well at 3d. each.

the same back and front, the imagination has no difficulty in realising the pseudoscopic image, and every one at once sees the tray reversed.

The Fovea Centralis.

338. The light after passing the crystalline lens and falling on the retina of an eye has first to pass a network of nerves and blood-vessels before it reaches the rods and cones. That is to say, the surface of the retina is placed the reverse of what would naturally be expected, the sensitive ends of the nerve fibres, the rods and cones, being below the nerves leading from them to the optic nerves as well as below the network of blood-vessels which feed them. But in the axis of the eye there is a spot which is free from this layer, and contains rods only and no cones, and for which the vision is much more acute than it is on the rest of the surface. That is to say, the defining power of this portion of the eye is greater. Its sensitiveness to light is not quite so good, and Abney has shown that its relative sensitiveness to the three primary colour sensations is not the same as that of the rest of the eye.

This spot may be observed if the light from a candle is reflected from the observer into another person's eye, who sits facing him at a short distance. If both observer and patient can fix their eyes at infinity, no lenses will be required, but as the observer expects to see this spot at the distance of the retina, he only focusses his eye at that distance, and possibly the patient also focusses his eye at a short distance, and thus a clear image of the spot is impossible. In this case a concave lens will be needed. It is very difficult to see the fovea unless the pupil of the eye is dilated.

Apparatus.—A special instrument called an ophthalmoscope is used by oculists for this examination, which constitutes an important branch of ophthalmic work. It consists of a concave mirror with an aperture in the centre.

The patient is placed with his back to a candle flame which is reflected by the mirror and focussed on the eye. A series of lenses fitted in a rotating diaphragm of varying focal length can be brought into position over the hole, and the observer rotates this until he finds one to suit his sight. The back of the eye will then appear as a pink background traversed by blood-vessels and the centre spot is the Fovea Centralis.

The fact that the central spot is not so sensitive to light as the surrounding part of the retina has probably been noticed by

every one watching a faint star on a clear night. If a star is so faint that it can hardly be seen, it will generally be seen much more easily if the attention is turned—that is to say, the axis of the eye directed, to a star in its neighbourhood. It has frequently been noticed that a poster which could not be read on the

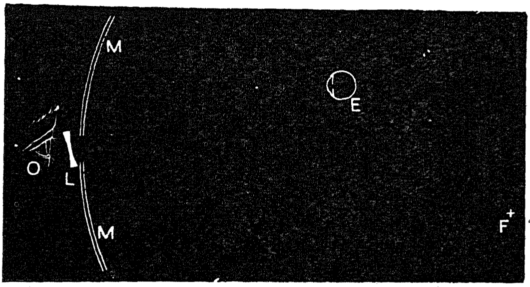


FIG. 355.—Examination of the Fovea Centralis of the Eye, E. F, candle; MM, concave mirror with a small central hole; L, lens; O, observer.

opposite side of a railway station owing to insufficient light, could be made out by looking slightly to the side.

The increased distinctness of vision over this spot of the retina is a matter of everyday observation.

Chromatic Error.

339. The eye is by no means achromatic.

1. If a window is observed by the right eye, the left eye being closed, and a finger is moved across the eye from right to left, one side of each window bar will appear distinctly coloured blue, and the opposite side yellow. If the finger is moved in the opposite direction, the colours will be reversed.

2. If a spectrum is thrown upon a screen by a lantern using an achromatic lens, and the screen being square with the lens so that the width is uniform from end to end, the spectrum will always appear to get wider towards the violet end. If a good picture—a good diagram, for instance—is thrown upon the screen with a violet light, it will be found impossible to focus it when standing a few feet away; the violet light is converged so strongly by the eye, that it is impossible to accommodate the eye to view violet by it at any distance. A concave lens will at once allow it

to be focussed sharply, showing that it was not due to insufficient light merely.

3. If a distant object is viewed through a combination of coloured films which allows only the violet end of the spectrum to pass, it will be found that however brilliantly illuminated the object may be it cannot be distinctly seen.

4. A red spot on a blue ground seems to be raised above the surface of the background, whilst a blue spot on a red ground seems depressed. This is due to the necessity for altering the curvatures of the crystalline lens in adjusting its focus for the red and blue respectively.

As we estimate distances partly by the convergence of the axes of the two eyes, that is stereoscopically, and partly by the curvature which we have to impress, by the ciliary muscle, upon the lens, it is obvious that the necessity for altering the focus of this lens must give the idea of a change in distance.

5. By projecting a spectrum on to a screen formed from a V-slit so that the image on the screen consists of a brilliant V having each limb drawn out horizontally into a spectrum, this image will seem to be in high relief, the red nearer and the violet farther from the observer.

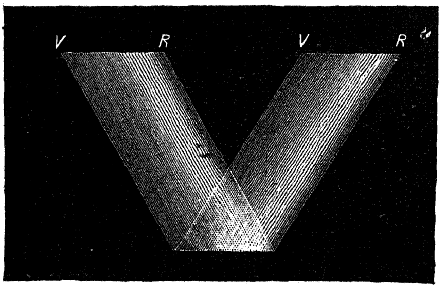


FIG. 356.—The Spectrum formed from a V-slit seems in relief.

6. Look at the filament of an incandescent lamp through cobalt blue glass. This glass transmits both red and blue ends of the spectrum. When the observer is so near that he can focus the blue distinctly, but that his accommodation is not sufficient to enable him to focus the red (the eye being, as these experiments show, of less power for red rays than for violet), he will find the filament appears blue with a red border. At a distance at which he can still focus the red, but at which his accommodation is too

small to enable him also to focus the violet, the converse will appear. At a greater distance still, say 20 feet, where it becomes impossible even approximately to focus the violet, he will still be able to obtain a distinct, red image of the filament; but each point of the filament, through this error, will form a large diffusion circle of violet light; and these circles will be so large that the whole bulb will appear filled with a violet glow.

This experiment illustrates the fact that it is impossible at short distances, even, for instance, the distance at which we nominally place distinct vision—10 inches—to clearly focus an object in red light.

7. If the filament is looked at through a deep red glass, this impossibility will be even more strongly illustrated.

Several other experiments illustrating this defect are given in Bidwell's *Curiosities of Light and Vision*. Foster's *Text-Book of Physiology* (Macmillan) may be consulted for experiments upon complementary colours, optical illusions, after-images, and other physiological matters in connection with the eye.

Images are seen inverted by the Eye.

340. *Apparatus.*—Thin card; pins; paper.

1. That images are seen inverted by the eye is most easily shown by holding a piece of card with a pin-hole in it at a

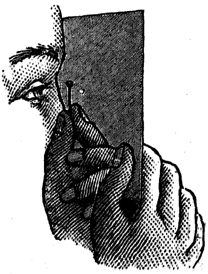


FIG. 357.—Inversion of Image on the Retina.

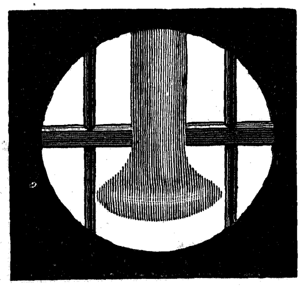


FIG 358.—Window seen through Pin-hole, showing the shadow of the Pin, which seems inverted.

distance of half an inch or an inch from the eye with one hand in front of a window, and passing a pin between the card and

the eye with the other hand. The light from the hole in the card will spread on to the retina and the shadow of the pin will be cast upon the retina as it moves across. This shadow of course moves across the retina in the same direction as the pin, but it appears to move in the opposite direction. When the shadow of the pin is cast on the retina, if the pin be gradually moved until its head appears, the shadow will appear upside down. It must, of course, be erect on the retina. Thus, a shadow which is really erect on the retina appears inverted. Evidently then, as on the ground glass of an ordinary camera, the images of the objects are always formed on the retina in an inverted position, and we have learnt to interpret them accordingly.

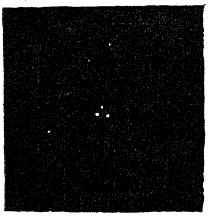


FIG. 359.—Triangle of Pinholes to be substituted for the Pin.

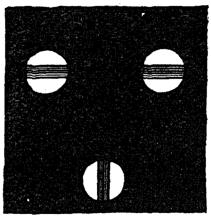


FIG. 360.—The Triangle formed by the three spots of light on the Retina seems inverted.

2. The same experiment may be performed by making three pin-holes forming an equilateral triangle in a piece of paper, each side of which is about 1 mm. If this be held between the eye and the card with the single pin-hole, the light will be allowed to pass through in three rays and form three spots on the retina, which will be in the same relative position as the triangle itself. If, for instance, the triangle is held with its vertex uppermost, these spots will be similarly arranged. But they will appear inverted.

Accommodation—Long and Short Sight.

341. The greater part of the refraction in the eye takes place at the first entry of the light through the cornea. This outer surface can easily be seen to be more convex than the general surface of the eye by looking sideways at some one's eye. The light then passes through the iris, which has the power of expanding or contracting so as to keep the intensity of the light falling on the retina as nearly constant as possible, then through the crystalline lens. This lens is of the nature of a stiff

jelly, and is naturally kept flat by muscles connected to its rim, and to the side of the eye. Surrounding the lens is a ciliary muscle, by the contraction of which the circumference is diminished and the lens caused to bulge, thus increasing its power. When looking at a near object, this increased power enables the more divergent wave from the near object to be sufficiently converged to focus on the retina. When the object observed is at a greater distance, the wave reaching the eye is flatter, and a lens of less power is needed to produce the right convergence still to focus it upon the retina. This is managed by relaxing the ciliary muscle.

When, therefore, a person is reading for any considerable time, this ciliary muscle is kept in a state of tension for that time, and if this is persisted in for many years, it is easily seen that the eyesight is likely to suffer.

As age advances, this muscle loses its power of contraction and the "accommodation" diminishes. By "accommodation" is understood this power of focussing the eye for objects at varying distances.

342. **To Measure the Accommodation.**

Apparatus.—Optical bench; convex lens of 10 inches focus; screen with small print on stand.

Place the eye *close* to a lens of about 10 inches focus, supported on the optical bench, and put a card, on which is some small print, in a stand. Look through the lens, and draw the card forward until the position at which it last ceases to be seen distinctly is attained. Note this position. Repeat the determination of this position some half a dozen times, withdrawing it from the lens and bringing it up again, so that the readings shall be as perfectly independent as possible, and take its mean position. Now withdraw the card to the greatest distance at which it can be distinctly seen, and find that position in the same way.

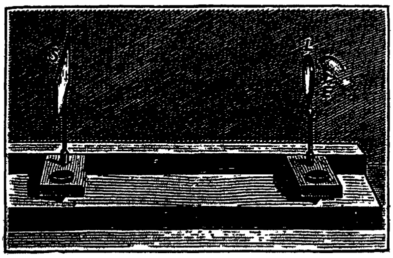


FIG. 361.—Accommodation.

Note the position of the lens, and having found the index error, obtain the two distances from the lens. Take the reciprocals of

these in metres. The difference between these results is the accommodation.

343. i. **Short Sight.**—If a short-sighted person tries this experiment, he will find that the two distances from the lens of nearest and furthest vision are each smaller than those found by a person of normal sight, but the accommodation will be the same as for a normal-sighted person of the same age.

It follows, therefore, that if a short-sighted person be given a lens which will make the farthest of these distances the same as the farthest distance for a normal-sighted person, he will have the same range of vision.

The farthest distance at which the card can be distinctly read by a normal-sighted person will be generally a little greater than the focal length of the lens. This shows that the eye is able to focus clearly not only the light from an object at an infinite distance, that is, a plane wave, but also to focus a wave that is slightly converging. If, however, a short-sighted person is given a glass that will enable him to focus a plane wave, it will be quite sufficient.

If the power of the lens used in the experiment is supposed to be 5 dioptres, and the reciprocal of the greatest distance of the card from the lens at which the short-sighted person could see it clearly were 7·5 dioptres, he would require a concave lens of 2·5 dioptres to make his sight normal.

ii. **Old Age.**—If an old man tries this experiment, he will find that the difference between the reciprocals of the distance of the card from the lens—that is, the difference between the curvature of the light entering his eye for farthest and nearest vision—is much smaller. The accommodation is less. No lens, therefore, will make his eyesight entirely normal. He may require two lenses. (If his accommodation is half the normal accommodation, this would be sufficient to enable him to see at all distances.) His accommodation, however, will finally almost disappear, and then he will require a different lens for each different distance. Usually he will be content with two lenses, one to enable him to see distinctly at about 15 inches, for reading purposes, and another to see distinctly at about as many feet, for use out of doors.

iii. **Long Sight.**—Some persons suffer from the opposite defect to short sight—namely, hypermetropia. This must not be confused

with the loss of accommodation caused by old age. With these patients, the distances for nearest and longest vision will be each longer than the distances for a person of normal eyesight. The accommodation will be the same. This error in vision may be perfectly corrected by the use of a convex lens of such a power that the longest distance of distinct vision is made normal; that is, the power of the lens must be the difference between the curvature of the light-wave from this greatest distance when it reaches the lens, and the power of the lens used in the experiment.

Astigmatism.

344. Nearly every eye is slightly astigmatic; that is to say, very few eyes indeed are perfectly symmetrical about their axes. This can be shown by taking a cylindrical lens of very low power, say 0·25 dioptre (see p. 57), and holding it before the eye, rotate it slowly while observing some distant small print. It will, perhaps, seem more indistinct in every position than when viewed with the naked eye alone, but as the lens is rotated, in certain positions the indistinctness will be more marked than in others. Were the eye perfect, the orientation of this cylindrical lens could make no difference. If the print is distinctly clearer in one position of the lens than it is without the lens, a stronger lens of 0·75 dioptre should be tried. If with this lens it is more distinct than with the naked eye, especially if the student suffers at all from headache, there is no doubt that his eyes are seriously at fault, and he should consult a specialist without delay.

Astigmatism is due to the power of the lens being a little greater in one direction than in another. Thus, for instance, if the lens is slightly flatter in a horizontal section than in a vertical direction, vertical lines, which are focussed at a distance depending upon the horizontal section, will be focussed at a greater distance than horizontal ones. Both lines of a cross drawn on a card would not be focussed perfectly at the same time—but either the vertical or the horizontal line could be focussed at will.

Apparatus.—Convex lens of short focal length (say, six inches), stand for the same; white card with a series of parallel lines on it, and stand; scale or optical bench.

345. **Examination for Astigmatism.**—If the convex lens is placed close in front of the eye (as in Fig. 361), and a well-illuminated card on which are a series of horizontal lines is gradually

approached to the lens, it will be found that at a certain distance it becomes impossible to retain the lines in focus. If the card be now turned through a right angle, so that the lines become vertical, and the distance found at which they again become indistinct (the eye throughout being kept close to the lens), this distance will often be found slightly different from the previous one. Also, if the farthest distances be determined at which they can be clearly focussed, the distances will usually be different in the case of the horizontal and vertical lines respectively.

Take the reciprocals of these distances, and knowing the power (see p. 57) of the lens, determine the curvature of the light entering the eye in each of these four cases.

The difference between the curvatures when the horizontal lines are nearest and when they are farthest measures the accommodation of the eye.

Similarly, the difference of the curvatures when the vertical lines are nearest and farthest, measures the accommodation of the eye. These numbers should agree.

The difference between the curvatures of the horizontal and vertical waves measures the astigmatism in one direction, but is not necessarily the true astigmatism of the eye, because the greatest and least powers of the lens may not be horizontal and vertical, but may be oblique.

346. **Measurement of Astigmatism.**—Placing the card at the greatest distance at which it is clear, rotate it and see if in any position it can be seen at a still greater distance; if so take this distance. Then turn it through a right angle and again find the greatest distance at which it is visible. The difference between the curvatures in these two positions will more approximately measure the astigmatism of the eye, but the variation in the accommodation will generally upset the measurements. It is further supposed by many that the eye possesses in its ciliary muscles, a power of adjusting the curvature not only as a whole but locally, and thus by these muscles it is able to slightly correct the natural astigmatism of the lens.

To find the astigmatism it is necessary, therefore, to avoid th confusion caused by accommodation, and it is usual to fix attention upon a fan of radiating lines, and with a series of cylindrical

lenses discover which will cause all the lines to be equally black and clear. Special instruments have been devised to measure the variation in curvature of the cornea in different meridians (which is the chief cause of astigmatism) called "Ophthalmometers" or "Keratometers." The astigmatism can also be found by retinoscopy (see § 170).

Having found the difference between the curvatures, a lens the power of which in one direction is that amount, that is a cylindrical lens, will have to be added to the eye to increase its power in the required direction.

The correction of slight astigmatism by the ciliary muscles, or the attempt to correct it, produces a strain upon those muscles which is one of the most frequent and troublesome causes of headache.

Colour Vision.

347. **Experiments illustrating Absorption of Light.**—(*a*) Make a solution of alkaline litmus and another of chromate of potash. These will be found with a spectroscope to remove the yellow and the blue, respectively, from white light; a mixture will therefore leave red and green. Or, the two solutions may be placed in flat vessels one behind the other. Pass white light through them, it will appear yellow. (A yellow disc can be projected on a screen by passing the light of a lantern through the solutions.) Examine a sodium flame through the solutions one behind the other; it is invisible.

(*b*) Another yellow liquid similar in appearance can be made with a mixture of bichromate of potash, permanganate of potash, and copper sulphate, or chloride; cutting out the green, blue, and red, respectively, and leaving a yellow. Look through this at a white light and at a sodium flame.

(*c*) Place this yellow liquid behind the other one,[1] and look through them. They are practically opaque.

(*d*) Dissolve cupric chloride in dilute hydrochloric acid; and also dye some amyl alcohol with an aniline red. Place these two liquids in a flat bottle and cork it up. The amyl alcohol will float on the top of the cupric chloride, and the two solutions

[1] Lord Rayleigh in *Nature*, January, 1871.

will appear red and green respectively. Shake them up, and they will become black.

(*e*) Dissolve two parts of copper sulphate in twenty of water, and add ammonia until the precipitate is just dissolved. This gives a good blue solution, cutting off the red end of the spectrum. Make a solution of potassium ferro-cyanide. This gives a good red solution. The two solutions, one in front of the other, produce black.

348. **Recomposition of Light.**—(*a*) In front of a gas flame, or a piece of ground glass lighted by a window, or other white light,

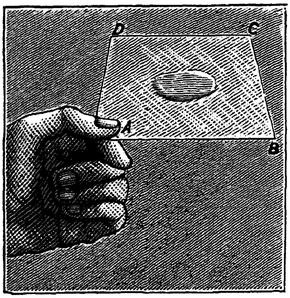


FIG. 362.

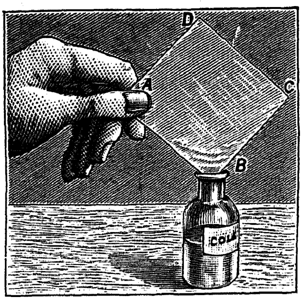


FIG. 363.

place a blue and yellow glass side by side, and examine them with a double image prism. This will cause part of the blue and yellow to overlap. Where they overlap white light will result. (If two lanterns are available, blue and yellow patches may be caused to overlap, one from each lantern, when again white will result.) If placed in front of one another, on a piece of ground glass, the blue and yellow frequently give green. A collodion film stained with methylene blue is better for this purpose than blue glass.

To produce this film, a piece of clean glass must be thoroughly dried; and a pool of pure collodion is poured on the middle of the glass, such as is sold for photographic purposes (the cheap collodion is opaque when dry). The glass must be held by one corner between the thumb and finger of the left hand, as in Fig. 362. By tilting the glass, the pool is caused to flow, firstly

to the corner opposite C, then counterclock-wise round the plate to the corner D, then to A, where the plate is held. Lastly, the excess is poured from the fourth and nearest corner B back into the bottle. The plate is slightly rocked while it is being poured off to avoid the collodion becoming streaked. In about a minute, before the collodion is thoroughly dry, it must be dipped in the staining solution (or the dyes may be mixed with the collodion).[1]

(*b*) Examine with the spectroscope the solutions above mentioned and explain the results. Also examine films dyed with

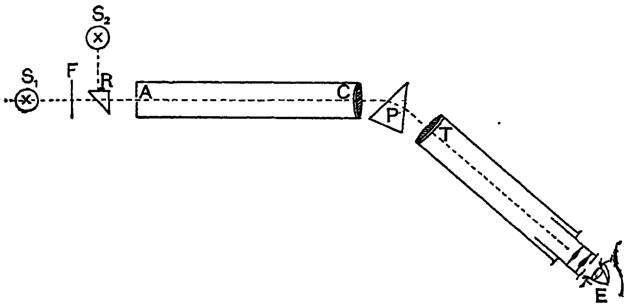


FIG. 364.—Examination of the Absorption produced by Coloured Films.

the following aniline colours: magenta, rosaniline, rhodamine, eosine—which leave the red and part of the blue end of the spectrum; chrysoidine, chrysophenine, cutting the blue end of the spectrum; picric acid, which cuts the ultra violet; and methylene blue, and malachite green, which cut the red end of the spectrum.[2]

The films, or solutions, may be placed half covering the slit of the spectrometer, or the small right-angled *comparison prism*

[1] Instead of collodion, an ordinary unexposed gelatine photographic plate, that has been cleaned in "hypo," may be stained by immersion for several hours in an aqueous solution of the dye.

[2] Small specimens in bottles of these dyes can be obtained of Messrs. Hartington Bros., 4 Oliver's Yard, City Road, at about 6d. each, containing enough dye for hundreds of films.

Stained gelatine films, such as are used on Christmas crackers, also make excellent colour absorption screens.

Messrs. Gallenkamp sell small bottles with worked flat sides, which are useful for introducing solutions in front of the slit of the spectrometer.

may be adjusted in front of the slit (as in Fig. 364), so that two spectra, one of which has suffered absorption by the film and the other of which is left unaffected, may be seen the one above the other, and a curve drawn showing approximately the absorption of each solution.

Knowing the colours left by each film independently, it is easy to predict what will occur when they are bound two or three together; and films may easily be arranged to give any colour effects that are desired.

The resolving power of the eye was discussed in § 268; the colour sensitiveness in § 325.

CHAPTER XXI

POLARISED LIGHT

Experiments on the Polarisation of Light.

349. *Apparatus.*—The polariscope used in the following experiments may be easily made of wood. To a base board about 6 inches square, attach two uprights, each about 12 inches high and $\frac{3}{8}$ inch by 1 inch section. On the top of these fasten a stage, EF, also 6 inches square, having a hole $\frac{1}{2}$ inch in diameter at its centre. About 6 inches from the top, fix a horizontal glass plate, D, to act as a platform, and at about 4 inches from the bottom hinge another glass plate, A, about 5 inches square. An ordinary silvered mirror, B, about 4 or 5 inches square is placed on the bottom; and a piece of zinc, Z, is bent round partly to inclose the space between the platform at the top and the glass stage. The zinc may be attached to the stand by a sort of bayonet joint, P, so that it can easily be removed. Cut a hole about 3 inches in diameter in the thin piece of wood 6 inches square, and fasten this to the top of the table, EF. The glass plate, A, can easily be hinged by inserting it in saw cuts made in small wooden blocks as indicated in Fig. 365. Cut a black card to lie on the glass

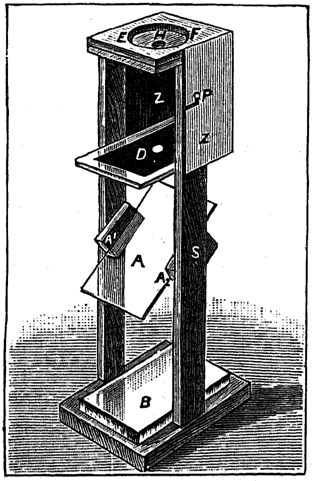


FIG. 365.—Norremberg Doubler.

stage, D, with a hole $\frac{1}{4}$ inch in diameter vertically under the hole in the top board. Draw lines on the top stage, EF, parallel to the sides, which would, if produced, pass through the centre of the circles, and also lines bisecting the angles between these, so as to mark every 45°.

Fitting easily in the 3 inch circle of the platform, EF, make three wooden discs each with a $\frac{1}{2}$ inch hole through its centre, of the same thickness as the board in which the 3 inch hole was made. To one of these, E_1 F_1 (Fig. 366), attach two uprights, and hinge between them in the same way as before, a small mirror, C (Fig. 366), about 1 inch by $1\frac{1}{2}$ inches, either of black glass, or of glass blackened at the back, with the reflecting surface downwards. On another disc, E_2 F_2, between two similar

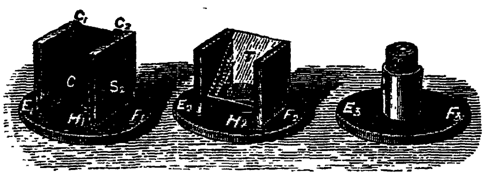


FIG. 366.—Fittings for Polariscope.

uprights, support a "pile of plates," T, of micro-cover glass at an angle of about 53°. The glasses used by botanists, about 2 inches by 1 inch, answer perfectly for this purpose. To the third E_3 F_3, attach a short tube which just fits a small nicol's N—one of those sold for attachment to the microscope is sufficiently large.

In addition to the polariscope, two rhombs of Iceland spar of about $\frac{1}{2}$ inch side, a second nicol, and a pair of tourmalines will be required; protractor, piece of spar cut perpendicular to the axis mounted in cork; tinfoil; two sewing needles stuck into a wood block with their points outwards and in the same vertical line.

350. **Use of Polariscope.**—(*a*) Set up the polariscope in front of window so that the light from the sky[1] shall be—

1. Reflected vertically down from the inclined glass plate, AA (Fig. 367).
2. Reflected vertically up from the silvered mirror, B.

[1] At night a sheet of white card may be set up in front of the instrument, with a good light illuminating it.

3. Partly transmitted by the glass plate, AA.

NOTE.—Nearly all the light is returned by the plate to the sky, but a small portion is transmitted.

4. Reflected by the black glass plate, C, into the eye.

(*b*) Without otherwise moving anything rotate the top black glass round a vertical axis, and observe the effect on the light.

In two positions the light will be nearly extinguished, and in two positions it will have maximum brightness.

Draw a diagram to indicate clearly the two positions in which it is dark, and the two in which it is bright.

Definition.—If plane polarised light be incident at the angle of polarisation on a glass surface (and the *plane* of incidence be varied by rotating the glass while the *angle* of incidence be kept the same), **the plane of incidence in which the light is best reflected is the plane of polarisation of the incident light.**

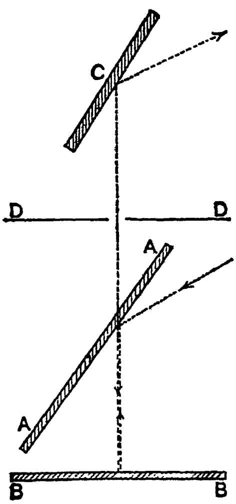


FIG. 367.—Path of the Rays through the Polariscope.

351. **Plane of Polarisation.**—(*a*) Examine the light transmitted by the plate A, and using the above definition, find its plane of polarisation. Also state, with reasons, whether the polarisation produced by the plate A was produced by (1) *the reflection* down to B, or (2) the *transmission* through A.

(*b*) Replace the black glass mirror, C, by the pile of plates, draw diagrams as in Experiment, § 350, and compare them with those obtained in that experiment.

Show that the planes of polarisation of reflected and transmitted light are at right angles.

352. **Brewster's Law.**—Replace the black glass mirror, and turn it to extinction. Now without rotating its plane alter its inclination, and that of the plate A, until the centre of the field is quite dark, and thus endeavour to find the angle of polarisation. Measure this angle with a protractor; and using the formula given by Brewster—

$$\tan i = \mu,$$

find the refractive index of the glass.

353. **Double Refraction.**—(*a*) Examine the small rhomb of Iceland spar—there are eight angular points; see that at six of these corners, there are two acute and one obtuse angle, but at the other two, which are diagonally opposite one another, there are *three* obtuse angles. *If the rhomb has all its edges equal* the line joining these two points is the *optic axis* of the crystal. Draw the crystal.

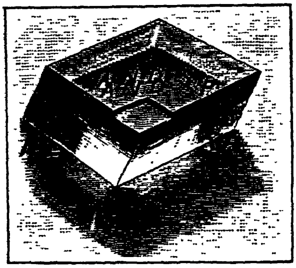


FIG. 368.—Iceland Spar.

(*b*) Make a small black dot on a piece of paper, and place one of the rhombs upon it. You will see two dots. Draw a diagram of the crystal as seen from above, inserting in the drawing, the optic axis, and the line joining the dots. Notice carefully the relation of these two lines.

(*c*) Rotate the crystal and draw a diagram of the appearance every 90°. Which dot moves?

(*d*) Raise the rhomb gradually from the paper, and notice carefully any change in position of the dots. Does their distance apart alter? Hence show by diagrams in a vertical plane the course of the rays to the eye.

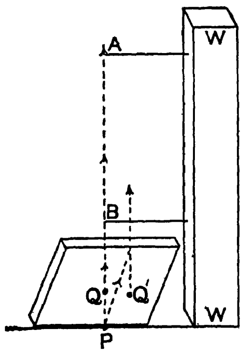


FIG. 369.—Discriminating between the Ordinary and Extraordinary Images.

(*e*) Set up the block with the two needles so that their points are vertically over a black dot on a piece of paper (Fig. 369). Place the rhomb over the dot, and using the needles as a guide, observe which of the two dots coincides with the original dot. The dot that coincides is called the one formed by the ordinary ray, and the other is formed by the extraordinary ray.

(*f*) See that it is the ordinary ray that *remains still* when the crystal is rotated.

(*g*) Look carefully at the apparent positions of the two dots in

the crystal. One will be seen to be higher than the other. Which is it? Hence which refractive index is the greater—that of the ordinary ray or that of the extraordinary ray?

354. i. **Polarisation by Double Refraction.**—(*a*) Make a pin-hole in a piece of tinfoil, and lay it on the stage of the polariscope (Fig. 365). Place a spar rhomb on it, and looking directly down through the hole in the top stage, observe the effect of rotating the rhomb. Hence show that the light of each dot is polarised. Which ray is polarised in the plane containing the two dots?

(*b*) Place the rhomb on a black dot on a piece of white paper, and examine with the "pile of plates" the two dots produced. Remembering the plane of polarisation of the transmitted light, see whether your results agree with those obtained by the last method.

(*c*) Place a second rhomb over the first, and draw diagrams of the appearance of the dots at every rotation of 45° of the upper rhomb. In particular explain the positions of the *two* dots in each of the four positions in which only two are left.

(*d*) Test your explanation by examining these two dots with the pile of plates.

ii. **The Optic Axis.**—Place the piece of spar cut at right angles to the optic axis, on a black dot or a pin-hole in tinfoil, and see that it does not divide the light into two. Test the light and see if it is polarised.

iii. **Nicol's Prism.**—(*a*) Place the *nicol's prism* on the upper stage of the polariscope, and find the plane of polarisation of the light it transmits. Is it along the line AB?

(*b*) See that two nicols produce more complete extinction than can be got with the reflecting polariscope.

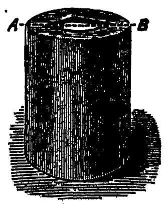


FIG. 370.—Nicol's Prism.

355. **Tourmalines.**—Examine the pair of *tourmalines*. Set them to allow the light to pass, and examine it with a nicol. See that it is plane polarised. Note that the light they transmit is coloured. (Hence for most purposes they are inferior to a nicol.) See that they produce very complete extinction.

Colours of Thin Plates.

356. *Apparatus.*—Polariscope; mica and selenite films; double-image prism; Fox's wedge.

i. **Mica and Selenite Plates.**—(*a*) Set up the polariscope with a nicol as analyser (*i.e.* in place of the pile of plates or the mirror C), and arrange it to extinguish the light. Also tilt the glass reflector, A, till the extinction is as complete as possible.

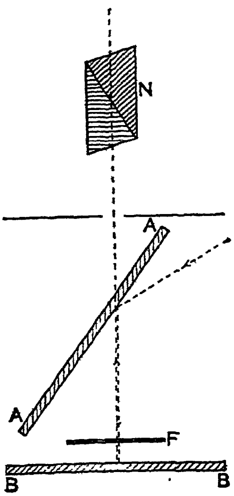


FIG. 371.—Polariscope arranged with a Nicol's Prism, N.

(*b*) Place a thin piece of *mica* or *selenite* on the stage, and note that colour is produced.

(*c*) Rotate the mica and see that in four positions at right angles it has no effect. In each of these four positions, rotate the nicol, and see that the effect is precisely the same as if the mica were not present.

(*d*) Place the nicol once more to extinction, and rotate the mica; note that the colour is brightest at angles half-way between the positions at which it has no effect.

(*e*) Leave the mica in the position in which it has the greatest effect and rotate the nicol. Notice that when it is turned 45° the mica once more has no effect; on rotating another 45° the colour is again bright, but is the complementary of what it was at first.

ii. **Double Image Prism.**—(*a*) Replace the nicol by the *double image prism* and see that the separate half images vary in the same way as those produced above, also that they are always complementary, and therefore where they overlap they form white. Note that the actual tint of the halves changes suddenly to its complementary, and that till that change they only vary by having more or less white mixed with them.

(*b*) Replace the mica by other pieces and note that the colour depends on the thickness.

iii. **Fox Wedge.**—Place the Fox wedge on the stage and note the colour changes. See that the colours produced proceed as in Newton's rings, and that in this wedge there are two orders of colour. (The wedges are made by cementing equal thicknesses of mica together so as to form a flight of steps each rising by an equal amount, see p. 461, Exercise 4, and § 395.)

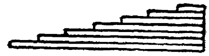


FIG. 372.—Method of arranging the Mica Films to form a Fox's Wedge.

357. In all the above cases plane polarised light falls on the crystal (mica) and is, in general, at once split into two beams, polarised in planes at right angles to one another, which may or may not be of equal intensity. These two rays travel through the crystal with different velocities, and, reaching the other side, have therefore gained or lost some fraction of a wave-length on one another. Here they recombine.

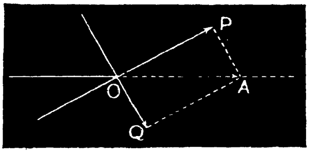


FIG. 373.—OA, amplitude and direction of a vibration incident on the crystal; OP, OQ, amplitudes and directions of vibration of the two components as they enter the crystal.

If either of the planes of polarisation of the light in the crystal happens to coincide with the plane of polarisation of the incident light, of course there will be only *one* ray in the crystal. In this case, there can be no interference, whatever the thickness of the crystal. Hence, in four positions at right angles, the crystal has no effect as in Experiment § 356, i. (*c*).

If the plane of the incident light *bisects* the angle between the planes of polarisation of the light in the crystal, as in Fig. 374, the two rays will be of *equal intensity*. In this case, the interference effects will be most marked; for should one ray of any colour gain half a wave-length, that colour will interfere completely.

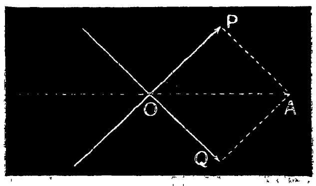


FIG. 374.—Component Vibrations with Equal Intensities.

If one ray gains any whole number of wave-lengths of any colour over the other ray, that colour will emerge in the same relative phase as it entered, and will combine therefore into a vibration in the same plane. So that, if the nicol is crossed, that is, if it is arranged for extinction, that colour will *remain* dark: or, if the field had been bright, that colour will remain bright.

If one ray gains an odd number of half wave-lengths over the other for any colour, then suppose OA (Fig. 375) to be the incident vibration; on entry it is resolved into two components at right angles, OP and OQ. On emergence, if we suppose one component vibration to be in the same phase as at entry, to be, for instance, at Q′ (Fig. 376), the other must be in the opposite phase to the one at entry; it will thus be at the other end of its swing at P′. The resultant of the component displacements OQ′ and OP′ will be the displacement OB. Also, as the component displacements at emergence will be along OQ′ and OP′, and will be equal in amount to those at incidence along OQ and OP, the resultant displacement will *always* be along OB. Therefore the resultant vibration will be along OB. That is, it is still plane polarised, but in a different plane. If, in addition, the angle AOP (Fig. 375) is 45°, it will be polarised in a plane at right angles, since the ∠A′OB is double the ∠A′OQ′. Therefore, in a dark field, this colour (being rotated 90°) will appear bright, and dark in a bright field. Thus, we see why the colour depends on the thickness and why it changes to its complementary on rotating the analyser through a right angle.

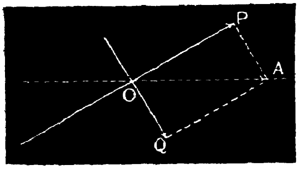


FIG. 375.—Decomposition of a ray at incidence on a crystal.

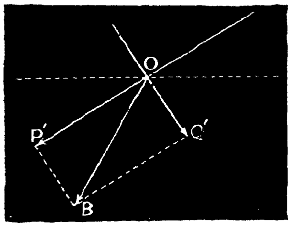


FIG. 376.—Recomposition of the light on emergence from a crystal when one component has gained half a wave-length on the other.

If one ray gains ¼ wave-length on the other for some colour, then Q will be in the middle of its swing when P is at the end of its swing, and *vice versa*. Hence, as they are at right angles and of equal amplitude, the resultant will be a circular vibration (Fig. 377). The colour will therefore appear equally bright with all positions of the analyser. This is called **circularly polarised light**. (This can only be true, of course, for one colour at a time, and should be therefore

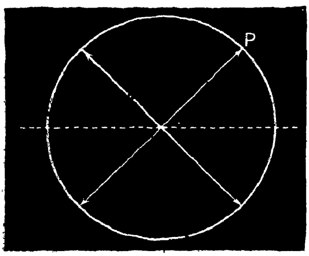


FIG. 377.—Recomposition of two components of equal amplitudes, on emergence from a crystal in which one has gained a quarter of a wave-length on the other.

observed with a sodium flame.) A film of mica of such a thickness that it produces this effect is called a *Quarter Wave Plate.*

If the gain is no exact wave-length, ½ wave-length, or ¼ wave-length, the emergent light will have an elliptic vibration, and be called **elliptically polarised light.** Such a colour will increase and fade as the nicol is rotated, but in no position can it be completely extinguished.

If the gain were ¼ wave-length, but the axes in the mica were not at 45° with the plane of the incident light, the amplitudes of the two rays OP, OQ, would be unequal (Fig. 378). The effect would again be that the vibrations of the emergent light would be elliptic and not circular.

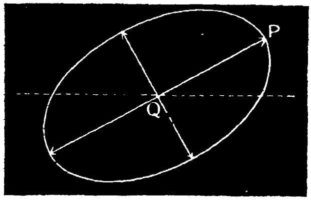


FIG. 378.—Elliptic vibration caused by a quarter wave plate, when the components have equal amplitudes.

358. i. **Norremberg Doubler.**—Place the mica film (§ 356, i.) on the mirror B. The colour will in general be quite different. The light now has to pass twice through the mica, and, therefore, as the ray which travelled most slowly on going *down* will again travel most slowly *after* reflection, it will lose *twice* as much as it would do were the mica on the stage D. The effect is thus the same as if a film of double thickness had been placed on D.

This form of polariscope is often called the Norremberg "doubler" in consequence of this action.

ii. **Quarter Wave Plate.**—(*a*) Examine a quarter wave plate of mica on the stage D (Fig. 365), using a sodium flame. On rotating the analyser see that the intensity remains unchanged. [The mica must have its axes at 45° with the plane of polarisation.]

(*b*) Place the quarter wave plate on B, and see that it is bright in a dark field, and dark in a bright one, *i.e.* it acts as a half wave plate.

iii. **Half Wave Plate.**—(*a*) Place a half wave plate (*i.e.* a film of mica of a thickness that delays one component of the light half a period) on D, and see that it behaves as ii. (*b*).

(*b*) Place it on B and note the effects.

iv. **Cause of Difference of Phase.**—(*a*) Take a small piece of mica of uniform thickness and scratch a line on it parallel to the direction in which it has no effect, cut it in two along some line

(such as the dotted one). Place the pieces on one another with the scratch on each parallel and place it at 45° as usual. Note the colour when on D (in white light). Now place a single piece on B and see that you get the same colour.

(*b*) Take the same two pieces, but this time place them so that the scratch on one is perpendicular to the scratch on the other. Place the pair on either stage and see that it has no effect on the light in any position. Explain this.

To produce the Polarisation of Light with a Plate of Spar cut normally to the Axis, using the difference between the Critical Angles of the Ordinary and Extraordinary Rays.

359. *Apparatus.*—A plate of Iceland spar cut perpendicular to the axis as large and as thin as can be obtained; a glass trough

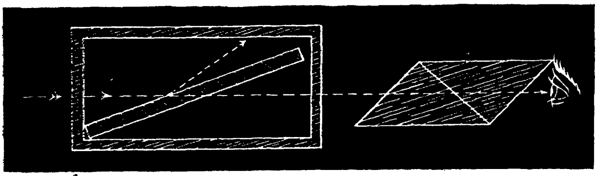


FIG. 379.—Polarisation by Critical Reflection.

with parallel sides at sufficient distance apart to allow the spar to be placed obliquely between them;[1] carbon bisulphide or some alpha-mono-bromo-naphthalene; a nicol or double-image prism.

Theory.—The refractive index of spar is given in the following table:

Rays.	Extraordinary Ray.	Ordinary Ray.
A.	1·4828	1·6501
C.	1·4847	1·6545
D.	1·4864	1·6585
F.	1·4908	1·6679
G.	1·4945	1·6762

[1] In place of the trough a glass beaker, or better, a tube with a plain glass bottom, may be set up with its axis vertical, and the spar placed in this at the angle of 20° with this vertical axis, and then the carbon bisulphide poured in until its surface is above the spar; it will be found that the light coming up vertically through the spar is polarised.

The refractive index of alpha-mono-bromo-naphthalene at 20° C. for the D line is 1·6582, which is almost identical with the refractive index of the ordinary ray in spar. Thus the critical angle for the ordinary ray is 90°, and for the extraordinary ray is about 64° (p. 28). The refractive index of carbon bisulphide is 1·67, and the critical angles are 83° and 62° respectively for yellow light.

Experiment.—If, therefore, the spar is placed in the cell with its plane vertical, and making an angle of about 70° with the face of the trough, and if the trough is filled with either of these liquids, the light from a distant source falling upon the surface of the

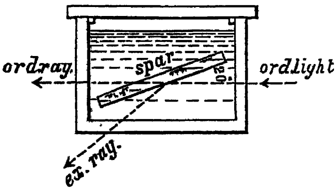


FIG. 380.—Polarisation by Critical Reflection.

glass normally will enter the liquid, and when it reaches the surface of the spar the extraordinary ray will strike the surface at an angle greater than the critical angle, and therefore will be entirely reflected. The ordinary ray will pass almost straight through it, as the refractive index of the ordinary ray in the spar is nearly the same as that in the liquid.

Examine each of the beams, the transmitted and reflected, with a nicol, and see that they are polarised in planes perpendicular to one another, the ordinary ray being polarised in the principal plane, which is the plane of incidence, since the optic axis coincides with the normal.

Jamin suggested this arrangement as a polarising apparatus (instead of the nicol's prism), but liquid prisms are awkward to use. Feussner made one of glass and spar (*Nature*, March 27, 1884), but as he was unable to find a cement of sufficiently high refractive index the prism was not very successful. The reflected beam will only be perfectly polarised if the refractive index is absolutely identical in the spar and the liquid.

Polarising Prisms.

360. The following table gives the angular field for a number of polarising prisms:

	Angular Aperture of Field.	Inclination of Section to Long Axis.	Ratio of Length to clear Aperture.
I. The Old Prisms.			
1. Nicol - - - - -	29°	22°	3·28
2. Shortened Nicol.			
(*a*) Cemented with Can. bal.	13	25	2·83
(*b*) ,, ,, copaiba ,,	24	25	2·83
3. Nicol with perp[l] ends.			
(*a*) With Canada balsam -	20	15	3·73
(*b*) With cement of index of refraction 1·523 - -	27	15	3·73
4. Foucault Prism - - -	8	40	1·528
5. Hartnach.			
(*a*) Original form - -	35	15·9	3·51
(*b*) With largest field - -	41·9	13·9	4·04
(*c*) Shorter piece of spar -	30	17·4	3·18
(*d*) ,, ,,	20	20·3	2·70
6. Glans Prism - - -	7·9	50·3	·831
II. New Feussner Prism.			
1. With calc. spar - - -	44	13·2	4·26
2. ,, ,, - -	30	17·4	3·19
3. ,, ,, - -	20	20·3	2·70
4. With nitrate of soda - -	54	16·7	3·53
5. ,, ,, - -	30	24	2·25
6. ,, ,, - -	20	27	1·96

The apertures in II. are calculated ones. It is impossible to obtain large slices of nitrate of soda, and only small prisms therefore are possible with this.

The commercial nicol's prisms have angular apertures of about 26° as a rule, as has also the ordinary commercial Hartnach.

ADDITIONAL EXERCISES ON CHAPTER XXI

1. Examine the light reflected from the surface of water with nicol's prism. It can be clearly distinguished at perpendicular incidence, but not so perfectly as that reflected from a glass plate at the angle of polarisation.

2. Stand a board vertically in the water, and insert pins in it to mark the direction in which the reflected ray is proceeding when the polarisation is most complete. The line, along which the water wets the board, will mark the surface of the water; the angle of polarisation can be measured, and hence the refractive index of the water determined by Brewster's law (§ 352).

3. Examine with a nicol the light reflected from a polished table, shiny book, leather, and any other polished surfaces.

4. Make a Fox's wedge. Split off as large a sheet of mica as can be obtained.[1] Its thickness must be uniform, and as nearly as possible about $\frac{1}{8}$ wave-length. This thickness will, when placed on the stage D, cause a dark field to become grey, but will give no colour. On the lower stage it should, of course, act as a $\frac{1}{4}$-wave plate. (It is not absolutely necessary that it should be $\frac{1}{8}$-wave.) Place it on the polariscope, and mark the direction in which it produces the most effect. Cut it into rectangles parallel to this direction, each rectangle being about $\frac{1}{2}$ inch in one direction, and in the other direction $\frac{1}{16}$, $\frac{1}{8}$, $\frac{3}{16}$, $\frac{1}{4}$ inch, and so on. These rectangles are now to be mounted upon a microscope slide, one upon another to form a flight of steps (Fig. 372), with Canada balsam dissolved in benzol or xylol. Leave them until the balsam is set; and then cover them with an ordinary thin glass, with more balsam. Be careful not to displace them in so doing.

[1] To split mica, insert a thin needle, perpendicular to an edge, about in the middle of one side, and then work it each way to split off the film so produced. By watching the *Newton's rings* which are seen where the films are separating, it is easy to see if the film is splitting properly. Any discontinuity forming in these rings, indicates a change in the thickness of the film. Before cutting the film, it must be examined with a polariscope, and if the thickness is not uniform all over, a scratch must be made surrounding the part which is of one thickness, and the rectangles must all be cut from this piece, so that each step in the wedge may increase by an equal amount. The wedge of Experiment § 356 (iii.) is supposed to have sixteen films. The scratches will all disappear when the films are mounted in Canada balsam.

CHAPTER XXII

SIMPLE SACCHARIMETERS

Saccharimetry.

361. *Apparatus.*—A tube to contain the substance under examination about 20 cms. long, $\frac{1}{2}$ inch in diameter, and closed with worked glass ends—patent plate will do—(it will probably be simplest to procure this tube ready made); pair of nicol's prisms, which may be those supplied for use in a microscope, having a clear aperture of about $\frac{1}{8}$ to $\frac{3}{16}$ of an inch; a biquartz; these are to be

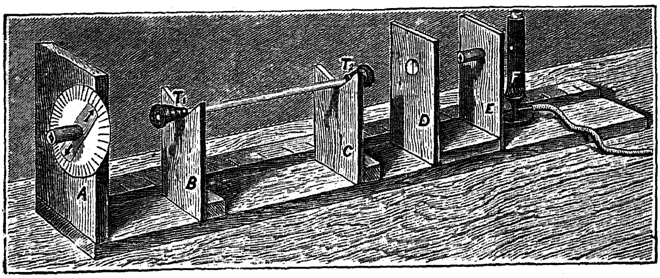


FIG. 381.—Saccharimeter.

mounted on a short optical bench similar to that in Fig. 51; beakers; cane sugar; a convex lens of about 4 inch focus; an incandescent gas flame.

The tube is supported on a pair of V's. The biquartz is fitted in a hole in the stand D, and against this is the convex lens. One of the nicol's prisms is supported in stand E behind the biquartz. At the end of this prism is a piece of card with a small aperture about $\frac{1}{2}$ cm. in diameter. A short brass tube, which will

just fit the cork in which the other nicol is mounted, is inserted in a hole in the stand A, in which it can be rotated smoothly. (If this hole is lined with cloth it will make the motion smoother.) A hole into which the brass tube will fit is cut in a piece of cardboard about 1 inch wide, and the tube pushed through it so that when the brass tube containing the nicol is rotated in the hole in the stand A, this piece of card will be carried with it, and act as a pointer.

A divided circle, such as that ordinarily used for magnetometers, is attached to the stand A, and the card is cut just to reach the graduations of this circle. A pair of arrows are drawn on the card at opposite ends of a diameter. By always taking the readings of both arrows, errors of centring will be eliminated.

Make a 10 per cent. sugar solution—that is a solution containing 10 per cent. of sugar by weight and 90 of water. The solution must be perfectly transparent, and, if necessary, be filtered and left to settle. It must not only be free from dust but optically transparent; that is to say, of uniform composition, and therefore also of uniform refractive index.

The incandescent gas flame is placed behind E to illuminate the aperture, and the distance of D from E adjusted until the image of this aperture is formed at A. The tube for the sugar solution being removed the nicol in A is rotated until the halves of the biquartz are of the same colour. Tint of passage. The readings of both pointers are then taken.

Fill the tube with the sugar solution. If the tube has been washed in ordinary water, a film of water will remain inside the tube, and on putting in the sugar solution would mix with it and alter the strength of the solution. Therefore a small quantity of sugar solution should be put in and well shaken up in the tube. A plug of cotton wool may be pushed backwards and forwards thoroughly to remove the film of water. Pour this away. Again partly fill the tube and notice if the solution remains transparent, or whether objects seen through it appear hazy. If so, the tube was not quite clean and must be further rinsed with sugar solution, this also being thrown away. Lastly, fill it right up, insert the tube between the stand A and the biquartz as in Fig. 381, and looking through the nicol A, the halves of the biquartz will no

longer be of the same tint. Rotate the nicol in A until the equality is restored and obtain the reading.

Both this reading and the reading with the tube removed should be repeated several times, the mean of each being taken. The difference between the reading with and without the solution, of course, measures the rotation caused by the solution. As the rotation is very small the readings will have to be taken very carefully to obtain any trustworthy results at all.

The rotation is proportional to the percentage of sugar and also to the length of the tube. Thus, with the tube 30 cms. long, the rotation produced will be three times as great as would be given by a tube 10 cms. long, filled with the same strength solution. A twenty per cent. solution will give a rotation twice as great as a ten per cent. solution. Dividing the rotation produced by the percentage strength of your solution and by the length of your tube in centimetres, will give the rotation per centimetre of a one per cent. solution.

362. With the same apparatus, by replacing the biquartz with a half-shadow analyser, the saccharimeter of **Laurent** can be imitated. One form of half-shadow polariser is formed by inserting in half the field a piece of quartz or spar, cut parallel to the axis and of such a thickness that it produces a retardation of half a wave-length between the ordinary and extraordinary rays. The other half of the field contains a glass plate of sufficient thickness to absorb the yellow light to the same extent as the quartz. A half-wave plate cut parallel to the axis decomposes a beam of polarised light into two components, one polarised in a plane parallel to the axis and the other perpendicular to it, and delays one of these half a wave-length. Thus, if OA (Fig. 375) is the plane of polarisation of the light which enters the plate, it is decomposed at once into the two components OQ and OP. When it reaches the other side one of these, OP, for instance, has been delayed half a wave-length relatively to the other, and it is relatively in the opposite phase. If, therefore, one component is represented by OQ′ (Fig. 376), on reaching the other side the other component will be represented by OP′. These will recombine into OB′ on emergence. The angle between OA, the plane of polarisation of the incident light, and OB′, the plane of polarisation of the emergent light, is double the angle between the plane of OA and the plane OP containing the optic axis. Thus, if OA in Fig. 375 is the

plane of polarisation of the incident light, the light emergent from the quartz will be polarised in the plane OB′, whilst that emergent from the glass which has no effect upon the plane will still be in the plane OA. Thus the plane of polarisation of the light in the halves of the polariscope will make an angle with one another, which is twice that between the plane of polarisation of the nicol E and the axis of the quartz in the analyser.

Fix this analyser in D in place of the biquartz (Fig. 381), and point it at a sodium flame. On looking through the nicol A (the tubes being removed), the two halves of the analyser will in general appear unequally bright. As the plate is a half-wave plate only for yellow light, it will be necessary to use a cell containing bichromate of potash, or a strong yellow solution, if a sodium flame is not used for the illuminant. Rotate the nicol in A until its plane of polarisation is either parallel or perpendicular to the bisector of the angle BOA′. The two halves of the analyser will now appear equally bright or dark. Keeping the nicol in A fixed, rotating E will vary the angle AOB′, and therefore vary the intensity of the light transmitted by A, and therefore the sensitiveness of the instrument. It is most sensitive when the field is nearly but not quite dark.

On introducing the sugar solution the light is equally rotated by the solution in both halves of the field, and therefore rotation of the nicol in A in the same direction will be necessary to restore the equality; the amount of this rotation is, of course, equal to that produced by the solution which we wish to determine.

363. A half-wave plate may be made by splitting a film of mica of the right thickness, and in this case, a piece of cover glass will be sufficient to produce the same absorption as the mica, or a second piece of mica cut from the same sheet with the optic axis at a slightly different inclination.

The lens attached to D and the stop at E are more necessary for the Laurent polariscope than the Soleil in order to make sure that the field is evenly illuminated, and indeed, without them a much larger nicol at E will be necessary, and a flame giving a uniform illumination over an area of ½″ would be required. The actual Laurent instrument is sufficiently like the one described to

need no further comment. The formula given by Tollens for the percentage of sugar for the length of tube supplied with this instrument is

$$P = \cdot 7475\theta - \cdot 001723\theta^2,$$

where θ is the angle of rotation of the plane of polarisation in degrees.

The weight c of 100 c.c. of sugar is given by

$$c = \cdot 75063\theta + \cdot 000766\theta^2.$$

364. In another form of half-shadow apparatus due to **Lippich**, there is a second small nicol, N_2, oriented so that the direction of vibration of the light coming through it makes a small angle with that emerging from the polariser. When the analyser is set so that its plane of polarisation is perpendicular to the bisector of the angle between the planes of the two nicols, the observer sees both halves of the diaphragm equally illuminated. This is the zero of the apparatus.

365. In still another form due to **Landolt** there are two small nicols behind the polariser dividing the field into three. This is to avoid errors caused by the variable sensitiveness of the eye of the observer. In looking at a field, one half of which is bright and the other half dark, the part of the retina upon which the image of the bright half of the field falls becomes fatigued; and if the two halves could now be suddenly made equally bright, that half which was originally brighter would appear darker than the other. In other words, the balance will seem to be obtained when this half is really brighter than the other. But if the field is divided into three, of which the two outer strips are equal and either both darker or both brighter than the central strip, by fixing attention alternately on the one or the other dividing line, the fatigue effort actually helps to increase the sensitiveness, for the part of the retina which was fatigued by the brighter strip when one dividing line is observed, is the part upon which the duller light falls when the eye is directed to the other dividing line.

CHAPTER XXIII

FURTHER EXPERIMENTS ON POLARISED LIGHT

The Refractive Indices of Iceland Spar or Quartz, by the Microscope Method.

366. **Being given a section perpendicular to the optic axis.**

Apparatus.—Microscope; tinfoil; millimetre scale; section of Spar or Quartz cut perpendicular to the axis.

Put a piece of tinfoil with a small cross cut in it, against the lower surface of the spar, and place them upon the stage of the microscope. Focus the cross seen through the spar with a $\frac{1}{2}''$ or $1''$ objective. It will be found that two images are formed vertically above one another.

367. This seems strange at first sight, as one would expect that along the axis of the crystal there would be no double refraction; but it must be remembered that the objective embraces a *cone* of rays, and that the light starting from the pin-hole will form two wave surfaces which touch one another on the axis, but have different radii of curvature there. When a spherical wave reaches the boundary between two media, its curvature is altered. If that boundary be a plane surface, the ratio of the radii of curvature is the ratio of the refractive indices of the two media. Consider first the ordinary wave. The radius of curvature when it reaches the upper surface of the crystal will be the real thickness of the crystal b. Its radius of curvature in the air will be the apparent thickness of the crystal d, and the ratio of one to the other will be the refractive index. Thus we obtain at once the refractive index of the ordinary wave:

$$\mu_0 = \frac{b}{d_0}.$$

Consider next the extraordinary wave surface. It is a spheroid of revolution having the cross as the centre. Its surface near the axis is therefore approximately a part of a sphere of radius $\frac{a^2}{b}$, where a and b are the major and minor semi-axis in the case of spar, or are the semi-axes respectively perpendicular and parallel to the optic axis in the case of any uni-axial crystal, as its section in any plane containing the axis is an ellipse, of which the optic axis is one of the axes. If, therefore, this wave could pass into air without any change of shape, the image of the cross would appear at a depth of $\frac{a^2}{b}$.

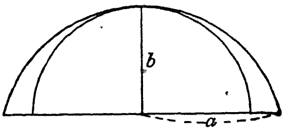


FIG. 382.

But it is obvious, since the velocity in the direction of the axis is increased in the ratio of the velocity in air to the axial velocity in the crystal—that is in the ratio of the refractive index of the ordinary wave to 1—that the curvature in air is increased also in this proportion. That is to say, the radius of curvature is diminished to $\frac{1}{\mu_0}$ of its value, and, therefore, becomes

$$\frac{a^2}{b}\cdot\frac{1}{\mu_0}=b\cdot\frac{a^2}{b^2}\cdot\frac{1}{\mu_0}$$

$$=b\cdot\frac{\mu_0^{\,2}}{\mu_e^{\,2}\mu_0}\quad\left(\text{since } a:b::\frac{1}{\mu_e}:\frac{1}{\mu_0}\right)$$

$$=b\cdot\frac{\mu_0}{\mu_e^{\,2}},$$

which is therefore the apparent depth of the extraordinary image.

Calling this d_e we have,

$$d_e=b\cdot\frac{\mu_0}{\mu_e^{\,2}}$$

or

$$\mu_e=\sqrt{\frac{b\cdot\mu_0}{d_e}}=\sqrt{\frac{b^2}{d_0d_e}}=\frac{b}{\sqrt{d_0d_e}},$$

which gives μ_e when μ_0 has been found.

As the images of the cross will tend to confuse one another, it is difficult to obtain the correct focus. But if a nicol prism is placed above the objective (or above the eye piece) with its plane of polarisation parallel to an arm of the cross, it will be found that that arm of the cross can only be seen in the ordinary image,

and the other arm of the cross only in the extraordinary image, that is, one arm of the cross will appear raised above the other arm. It will thus be easy to determine the apparent depths, d_o and d_e, of these images.

368. If the above theory is not quite clear, the following will perhaps help:

Let O be a point on the lower surface of the crystal and AMB be the upper surface of the crystal.

Imagine for a moment that the crystal extends above AMB, and let FADBH be the extraordinary surface for a wave spreading from O

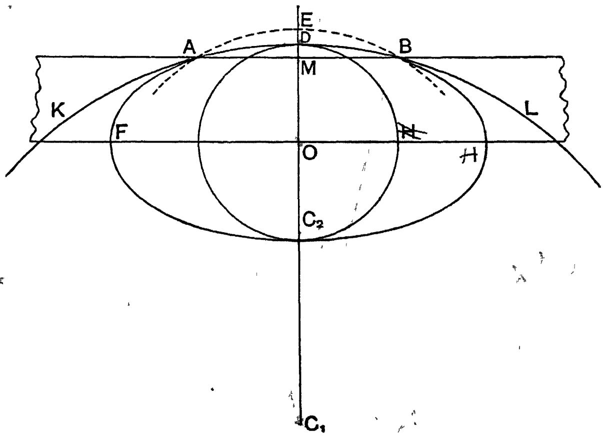


FIG. 383.

just after the wave has passed M. The part ADB may then be considered to be a small portion of a sphere, of which the centre is at C_1 and of which the radius is given by $\frac{a^2}{b}$, where a and b are the major and minor axes respectively of the ellipse FADBH, *i.e.* OH and OD.

$$C_1D = \frac{OH^2}{OD}. \qquad \text{(i)}$$

Then AMB being a chord of this sphere,

$$AM^2 = DM.(2C_1D - DM) = 2.DM.C_1D \qquad \text{(ii)}$$

when DM is small.

On emerging into the air, however, at AMB, the light begins to travel with a greater velocity, and the wave will therefore spread to AEB

instead of to ADB, where ME is to MD as the velocity of light in air is to the velocity in the spar along OD (*i.e.* to the velocity of the ordinary ray in spar), or inversely as the refractive index of air is to that of the ordinary ray.

Thus $$\frac{ME}{MD}=\mu_0. \qquad \text{(iii)}$$

But AEB will be approximately a sphere, of which the centre is at C_2, where as before

$$AM^2 = 2 . EM . C_2E,$$

or

$$\begin{aligned} C_2E &= \frac{AM^2}{2 . EM} \\ &= \frac{AM^2}{2 . \mu_0 . MD}, \quad \text{by (iii)}, \\ &= \frac{2 . DM . C_1D}{2\mu_0 MD}, \quad \text{by (ii)}, \\ &= \frac{1}{\mu_0} . \frac{OH^2}{OD}, \quad \text{by (i)}, \\ &= \frac{1}{\mu_0} . \frac{OH^2}{OD^2} . OD \\ &= \frac{1}{\mu_0} . \frac{\mu_0^2}{\mu_e^2} . b, \end{aligned}$$

when MD is infinitely small.

Since OH and OD are respectively proportional to the velocities of light along OH and OD, and therefore *inversely* proportional to the refractive indices along these lines.

Thus, as C_2E is ultimately the *apparent depth* d_e of the image of O formed by the extraordinary ray below the surface of the crystal, if this is determined with a microscope, as in § 25, page 34, we have

$$\frac{\mu_e^2}{\mu_0} = \frac{b}{d_e}.$$

369. **Section parallel to the Optic Axis.**—In this case a small cross must be cut in the tinfoil, and the crystal laid upon it so that its axis may be parallel to one of the arms of the cross. Place this on the stage of the microscope and focus it as before. The extraordinary wave surface will this time have different curvatures along and perpendicular to the axis, and thus the image of the cross formed by the extraordinary ray will be astigmatic. The image formed by the ordinary ray will be formed by an ordinary

spherical wave. These images will partly confuse one another, and they will more easily be observed if first one and then the other is cut off, by placing over the eye-piece of the microscope a small nicol prism. When its plane is parallel to the axis of the crystal, it will transmit only the ordinary wave. In this case, the image of the cross will appear as a cross with the two arms equally distinct, and the apparent depth can be determined as usual. This will give the ordinary refractive index.

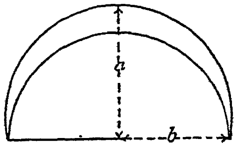


FIG. 384.

$$\mu_0 = \frac{\text{real depth}}{\text{apparent depth}} = \frac{a}{d_0}.$$

(This time calling the thickness of the crystal a, as it is the major axis of the ellipse.)

Turning the nicol now, so that its plane is perpendicular to the axes of the crystal, the extraordinary image will be obtained. This, as above remarked, will be astigmatic, the section of the extraordinary wave surface being a circle in a plane perpendicular to the optic axes and an ellipse in the plane containing the optic axis, and thus its radii of curvature in these two planes will be a and $\frac{b^2}{a}$ respectively, when a and b are, as before, the semi-axes of the spheroid perpendicular to and along its optic axis respectively. Thus the two arms of the cross will not both be in focus at the same depth.

The position of the arm parallel to the optic axis will be determined by the refraction in the plane perpendicular to this. Consider, therefore, the circular section of the wave whose radius was a. The velocity of this wave normal to the surface, which is that of the extraordinary wave perpendicular to the optic axis, is increased in the ratio μ_e to 1. Thus the curvature of this wave is increased also in that ratio, and the apparent distance of the image will therefore be $\frac{a}{\mu_e}$, or, calling this distance d_1,

$$\mu_e = \frac{a}{d_1}.$$

Now consider the arm of the cross perpendicular to the axis of the crystal. The image of this arm will be determined by the refraction

in the plane containing the optic axis; for the refraction perpendicular to this plane will only have the effect of elongating the image of each point on it in the direction of its length; it, therefore, will not introduce any haziness, but merely slightly increase the apparent length of the line. In the plane of the optic axis the radius of curvature is $\frac{b^2}{a}$. As the normal velocity on passing into the air is increased in the ratio μ_e to 1, the apparent depth will become

$$\frac{b^2}{a\mu_e}=\frac{a}{\mu_e}\cdot\frac{b^2}{a^2}=\frac{a}{\mu_e}\cdot\frac{\mu_e^2}{\mu_0^2}=\frac{a\mu_e}{\mu_0^2}.$$

Calling the apparent depth in this case d_2, the thickness being a, we have

$$\mu_e=\frac{d_2}{a}\cdot\mu_0^2;\ \text{ and } \mu_0=\sqrt{\frac{a\mu_e}{d_2}}=\frac{a}{\sqrt{d_1 d_2}}.$$

If the cross had been observed without the nicol prism, two images of each line would have been formed, and the experiment would be impossible, as, being equally bright and at different depths, in no position would they appear sharply defined.

Convergent Polarised Light.

370. *Apparatus.*—Tourmaline pincet; a piece of spar cut perpendicular to the axis, a piece of quartz cut perpendicular to the axis (the biquartz of the saccharimeter will do); any other crystal that can be obtained may, of course, be examined.[1]

The simplest apparatus is an ordinary tourmaline pincet. This is usually made by mounting a pair of ordinary tourmalines in brass rings, which are then supported facing one another in a bent wire spring. They are sold for examining the "pebble" (quartz) lenses sold by opticians, to see if they are cut perpendicularly to the axis.

If the crystal to be examined be placed between the two tourmalines and held close to the eye facing the light, the rings and brushes can be observed in most cases.

Two or three crystals are quite easily worked, and it is interesting to mount them for oneself. The simplest of these is the sugar crystal, grown on a string, and sold by confectioners as sugar candy. If the shopkeeper can be persuaded to allow his stock to be looked through, there will be no difficulty in finding several suitable crystals. These crystals are about the shape shown, and if mounted with Canada balsam, with a piece of cover glass on each of the large faces, the

[1] Crystals for this purpose can be obtained from Messrs. Steeg and Rutter of Homberg Vor der Höhe, or from Newton of Fleet Street, E.C.

rings and brushes can be easily seen. Generally the surface is rather opaque owing to loss of moisture, and has to be removed. This can be done by grinding it on a piece of very fine ground glass. Ordinary ground glass is useless for this purpose, but if two pieces of plate glass are ground together with coarse emery until the surface becomes matt, it will do very well. The sugar crystals are to be rubbed on this, using a small quantity of water. It must be remembered that when mounted with the balsam between glass plates, any slight opacity in the surface will disappear if it is due merely to scratches. This crystal is biaxial, and one optic axis will be found to be perpendicular to the surface.

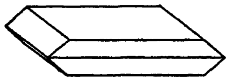


FIG. 385.

Another crystal which can be easily obtained and worked and gives beautiful rings is ordinary nitre. Specimens of these crystals may be obtained from an oilman. He will generally allow you to look over his barrel of nitre and select any crystals you wish. The crystals are long pointed ones, and are generally filled with striae running lengthwise. A crystal which is free from these striae should be found, and then a slice will have to be sawn off it, perpendicular to the long axis of the crystal. This can be done with an iron wire stretched on a fret-saw frame worked with emery and water. If too much water is used and there are any striae, the crystal will generally break up. A slice of about one-eighth or three-sixteenths of an inch thick should be cut off. This may then be rubbed on a piece of flat lead with coarse emery and water, to rather more than $\frac{1}{8}''$ thick, then rubbed on the plate glass with a little water until the coarse scratches are removed. To see if it is really cut perpendicular to the long axis, it must be examined in the pincet. It is a biaxial crystal, and if cut as directed the section will be normal to the bisector of the optic axis. It will probably be too opaque to see any trace of rings, but if a handkerchief be stretched over the plate glass, the centre of the handkerchief wetted and then the crystal rubbed to and fro across this part, it will be found easy to obtain a sort of polish, sufficient to make the rings fairly clearly seen. Generally it will be found that the crystal is not quite correctly cut; if the rings are visible at all, they appear to be in one corner of the field, and it is necessary to tilt the crystal in order to bring them to the centre of the field. Note how the crystal is tilted, and grind it accordingly until the rings become central. A slight touch should then be given once more on the glass plate, and the crystal mounted between two pieces of micro-cover glass with Canada balsam, without any polishing on the handkerchief. The

scratches left by the glass will be filled in by the balsam, and will not be visible, while the polish produced by the wet handkerchief through it removes the scratches and leaves the surface undulating.

Another crystal which can easily be obtained is the yellow prussiate of potash, potassium ferro-cyanide. But it s necessary to use a very large crystal, as the double refraction is very slight, and unless the crystal is large the rings will have very great diameter. It should be from three-sixteenths to a quarter of an inch thick, and, of course, must be perfectly transparent. The crystals split easily into flat flakes, and the natural surface may be left undisturbed, mounting a thin cover glass on each face to assist the transparency.

A good crystal of aragonite, about an eighth to three-sixteenths of an inch thick, should be procured, cut perpendicular to one of the optic axes, for the examination of internal and external conical refraction.

Harder crystals can be cut with an iron wire and emery, ground with a series of emeries on a flat piece of brass with water, alcohol, or petrol, and polished with rouge on a stretched cloth with water, or on paper dry with tripoli.

371. *Experiments.*—Begin with the spar crystal cut perpendicular to the axis. Hold the tourmalines up to the eye and rotate one of them. As it is turned round the field will become alternately bright and dark. Place it in the position in which it is as dark as possible. Now introduce the spar between the two tourmalines and look through. A series of concentric circular rings will appear somewhat similar to Newton's system of rings. These will be crossed by two black brushes at right angles to one another. Rotate the crystal without interfering with the tourmalines. No movement of the rings or brushes will take place. (If the crystal is not cut quite perpendicular to the axis the rings will not be quite in the centre of the field, and as the crystal is rotated they will move about slightly, but the brush will always pass through the centre of the rings and will not alter its direction.)

If the tourmaline be revolved as a whole, the crystal either being held still or being allowed to revolve, the brushes will move with the pincet, showing that the direction of the brushes depends upon the position of the tourmalines, and not upon that of the crystal.

Spar is a crystal which is symmetrical about its optic axis, so that rotating it about this axis can have no effect on the rings and brushes produced.

372. To explain this appearance, let us suppose a vibration to start from a point O on the upper surface of the crystal, spreading in all directions through the crystal, and let OA be the direction of the optic axis. In any given direction such as OP other than the optic axis, this vibration from O will travel with two different velocities, one of which is the same whatever the direction of OP may be. The other varies with this direction, being generally greater; but in the direction of the axis this latter velocity becomes equal to the former. So that the vibration from O spreads in a given time on to two wave surfaces, one of which is a sphere having O as centre and OA as radius, the other a spheroid, of which OA is the minor axis. In the direction OP at this moment, one vibration takes place at P′ and the other at P″. The vibration at P″ would be to and fro along the line in the plane of the paper perpendicular to OP′ indicated by the

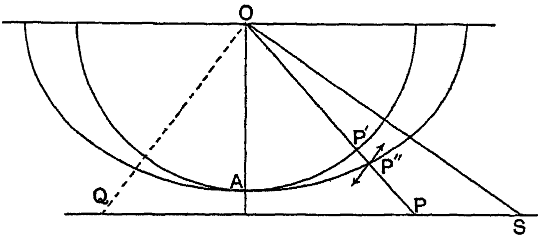


FIG. 386.

arrows. The vibration at P′ will be to and fro along a line perpendicular to the plane of the paper. So that the original vibration at O, which is to proceed in the direction of OP, is to be decomposed into two, one in the plane of the paper and the other perpendicular to it. The relative size of these two components will depend upon the direction of the vibration at O. If the point O vibrated in the plane of the paper there could be no component perpendicular to the plane of the paper, and the amplitude of the vibration at P′ would be zero and that at P″ a maximum. On the other hand, if the vibration at O had been perpendicular to the plane of the paper, the amplitude of the vibration at P″ would be zero and that at P′ a maximum. It is only when the vibration at O is neither in nor perpendicular to the plane OP that both vibrations P′ and P″ will exist. If, therefore, light polarised by passing through a tourmaline or nicol prism enters at O, and that prism is slowly rotated, when the plane of polarisation coincides with the paper—that is, when the light vibrates perpendicular to the plane of the paper—only a vibration at P′ will be produced.

As the nicol is rotated, P′ will gradually diminish and P″ increase until, when the nicol has been rotated 45°, the two amplitudes will become equal. On rotating it still further P″ will reach its maximum and P′ disappear. Rotate it another 90° and P′ will once more reach its maximum. In two positions, 180° apart, P′ will reach its maximum and P″ be zero, whilst in two positions bisecting these the opposite will be the case.

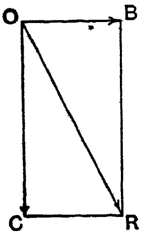


FIG. 387.

Let OB be a line in the plane of the paper, and OC a line perpendicular to it, OA standing up at the right angles to the plane of the paper, and let OR be the direction of the incident vibration.

Drop perpendiculars from R to OB and OC. Then the magnitudes of the vibrations at P″ and P′ will be represented by the lines OB and OC respectively. Suppose now the thickness of the crystal such that, by the time it reaches the other surface at P, the distance P′P″ has become half a wave-length, then the vibration OC will arrive at the surface half a wave-length behind OB. It will, that is to say, be at the opposite end of its swing. So if O′B′ is the vibration at P at any instant, the corresponding vibration in the plane at right angles to the paper will be O′C′, and on passing into the air these two vibrations will combine into vibration along O′R′; thus the light which emerges from P will be vibrating along a line O′R′. That which entered at O was vibrating along OR. The effect of the crystal has been to turn the plane of polarisation through twice the angle ROB. This angle will depend upon the relative lengths of OB and OC. It will be a right angle when OB and OC are equal; that is, when the plane of polarisation of the instant light makes an angle of 45° with the plane AOP. In this case the effect will be the greatest; for if the nicol or tourmaline used as an analyser were so placed that the light from the polariser if no crystal were interposed would be cut off, the revolution of the plane of polarisation produced by the crystal through a right angle, would just allow it to come through, and thus change dark into light. In the same way it would change light into dark.

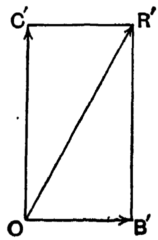


FIG. 388

If the plane of polarisation is rotated by the crystal through 180° or 0° the effect of the crystal would be nothing. This will occur if the incident vibration is either along OB or OC; that is, when the plane of polarisation of the incident light is in, or perpendicular to, the plane

AOP. In two planes at right angles, therefore, the crystal will have no effect, and if the two tourmalines or nicols are so placed that without the crystal the light is cut off, when the crystal is inserted it will still remain dark in these two planes, and there will appear a dark "brush" forming a cross.

As the wave surface is symmetrical about the axis OA, the direction of OP, for which the distance P′P″ is half a wave-length, will make a constant angle AOP with the axis OA. If the plane AOP is imagined to be rotated about OA, all the points P on the face of the crystal will lie on a circle, of which the centre is on the axis OA, and there will be the same difference of half a wave-length between the two waves emerging there.

At a greater distance from the centre another circle can be drawn such that the difference of path is a whole wave; still further from the centre one at which the difference of path will be a wave and a half, and so on. At the centre the difference of path is, of course, zero. The crystal should have no effect on the light at the centre, and again on a circle for which the difference of path is a wave-length, or any whole number of wave-lengths. On circles for which the difference of path is $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ wave-lengths, the effect of the crystal will be a maximum.

So that with monochromatic light, there will be, firstly, a dark cross (if the two nicols or tourmalines are set for extinction) extending right across the field, and between the arms of this cross will be pieces of circles, which are most clearly seen on the lines bisecting the arms of the cross, where the dark will be changed into light.

If the analysing tourmaline be turned through 90° (which would make the field bright if there were no crystal) the cross will become bright and the pieces of circles on the lines bisecting the arms of the cross will change from light into dark.

When using white light, the circles for which the difference of path will be any particular number of half wave-lengths, will be different in size for each colour, and the rings seen will be coloured, the colours being similar to those seen with Newton's rings. A dark or white cross will be visible as before.

373. The rings and brushes in crystals can also be seen with an ordinary microscope.

The sub-stage condenser will give the necessary convergence. The crystal is placed on the stage and the light is received by a lens of about $\frac{2}{5}$ or $\frac{1}{2}$ inch focus, which will produce the interference system of rings in its upper focal plane. To observe these a low

power objective is to be screwed in the lower end of the draw tube, and the draw tube adjusted until the rings are sharply focussed, or an additional eye-piece may be mounted above the ordinary one.

For the polariser a single black mirror mounted at an angle of 53° at the end of a rod takes the place of the ordinary silvered mirror, the rod being mounted in a tube so that its axis coincides with that of the microscope body.

The analyser may be placed either above the objective, or above the eye-piece, as preferred.

Using this arrangement, very much smaller crystals will be sufficient.

Some beautiful effects can be produced by growing the crystals on the stage of the microscope. The crystals should be mounted on an ordinary micro-slide, melting them as a rule in their own water of crystallisation, and then dropping the cover glass on them. Warm them, put them under the objective in polarised light, and allow them to cool.

Suitable crystals are: Benzoic acid, phenol-salicylate (rub a crystal on the edge to start the growth), paratoluidine, nitro-toluene, meta-dinitrobenzol, ortho- and para-nitrophenol, naphthalene.

To test whether a Disc or Lens has been properly annealed.

374. This can be done by an arrangement somewhat similar to that employed to test the aberrations of a mirror (see § 169). The

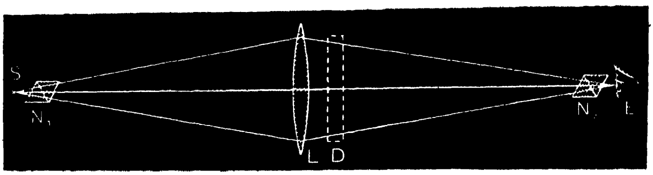


Fig. 389.—Test of Strain in a Lens.

light from a large pinhole at S is focussed by the lens to be tested at E. Thus an eye at E would see the lens full of light. Two nicols N_1 and N_2 are placed close to S and E respectively,

and turned to extinguish the light. If the lens is not properly annealed a bright cross will appear, due to double refraction produced in the glass by strain. A plain disc of glass or a weak concave lens may be tested by placing it in front of a lens L

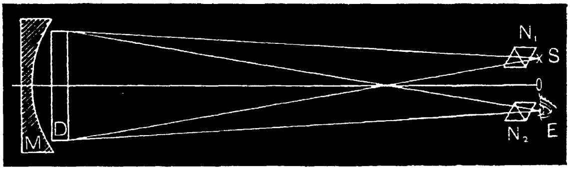


FIG. 390.—Test of Strain in a Plate.

which has been found to be free from strain, or by placing it in front of a convex mirror M, the light and eye being placed close to the centre of curvature O of the mirror. The light is cut off as before by two crossed nicols N_1 and N_2 placed close to S and E respectively, as shown in Fig. 390.

Internal and External Conical Refraction.

375. *Discussion.*—It was shown by Hamilton that, owing to the shape of the wave surface in the biaxial crystal, a ray of light incident normally on such a crystal cut perpendicular to one of the optic axe will on entering the crystal divide not into two rays as usual, but into a hollow cone of rays to emerge from the other surface as a hollow cylinder. This phenomenon is known as *internal* conical refraction.

Also if a ray of light be sent through such a crystal in the direction of single-ray velocity, it will emerge not as usual as two rays only, but as a hollow cone of rays. As the cone this time is formed in the air, this phenomenon is known as *external* conical refraction.

In the figure, let the plane of the paper be the section of wave surface in a biaxial crystal containing the optic axis. A plane has double contact with the wave surface at M and N in this plane, and does not—like an ordinary tangent plane—touch it only at these two points, for the point P is a "conical point" on the surface, and the tangent plane covers it up and touches the surface all round the perimeter of a circle of contact, of which MN is the diameter; OM is perpendicular to this plane, and is the optic axis of the crystal. Thus a plane wave of light travelling through the crystal with this tangent plane as its wave surface may have rays in any or all directions fro

O to the circumference of this circle of contact; that is, a hollow cone of rays, of which the vertex is O. This cone of rays will have only a single wave front. On reaching the surface of the crystal, this will give rise to a wave in the air having only a single wave front; so that we should expect that a ray incident on the crystal in such a direction that the refracted wave in the crystal is parallel to the plane MN, will divide in the crystal into a cone of rays, and these will emerge from the crystal as a hollow cylinder of rays. For as the cylinder has only one wave front, the rays in air must be parallel to one another.

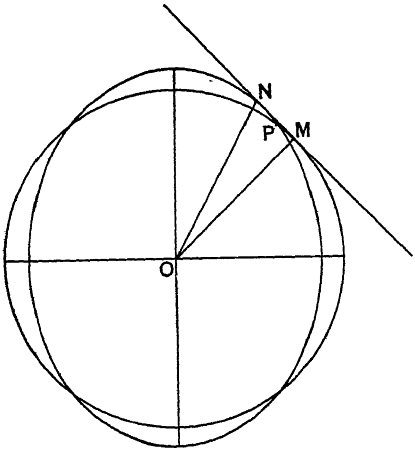


FIG. 391.

Again, a ray which travels through the crystal in the direction OP may correspond to an infinite number of waves, for an infinite number of waves can be drawn to touch the conical point of the surface at P. Thus a ray of light parallel to the line OP when it reaches the surface of the crystal will pursue a direction determined by any one of these tangent planes. The emergent beams from any point will form a hollow cone. The direction OP from the property just mentioned is known as the axis of single-ray velocity.

The experimental verification of the theory was first accomplished by Lloyd.

376. *Experiment.*—**Internal Conical Refraction.**—The experiment (not at all an easy one) may be performed upon an ordinary microscope, using a low power to observe the emergent cylinder. A biaxial crystal cut perpendicular to one of the optic axes is most convenient, *e.g.* a sugar crystal; but it is possible to manage with a crystal cut as usual perpendicular to the bisector of the optic axis if the angle between the axes is not too great. The thicker the crystal is the better. A piece of tinfoil P, with a pin-hole in it (or preferably a very thin sheet of zinc or brass, with a pin-hole drilled in it) is placed against the lower face of the

crystal C, which is then laid upon the stage of the microscope so that the pin-hole is visible in the field of the objective. It may be mounted on some pellets of plasticene, F, F, and so can be tilted to get the optic axis about parallel to the axis of the microscope. A small cone of rays is used, which may be produced by placing a diaphragm L with a small hole, about $\frac{1}{2}$ mm. diameter, upon the sub-stage R, and reflecting the light of a sodium flame through it. The objective is focussed upon either the upper surface of the crystal or a plane slightly above. Then two dots will be visible. On moving the diaphragm on the sub-stage about in its own plane, these two dots will alter their positions.

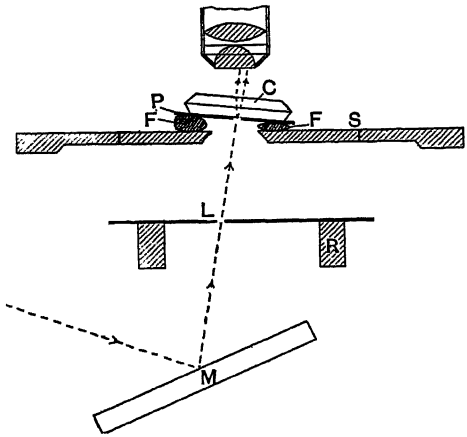


FIG. 392.

By considering the points of contact of a tangent plane to the wave surface when near the conical point, it will be obvious that the optic axis is to one side of the line joining the points of contact. On moving the diaphragm about, the direction of the line joining the dots will remain constant if the light does not enter the crystal in the neighbourhood of the optic axis. But when this neighbourhood is reached this line will alter its direction rapidly (in somewhat the same way that, when mapping the lines of force of a magnet, a compass needle alters its direction in the neighbourhood of the "neutral point," where the earth's field just neutralises that of the magnet). When the diaphragm, is very nearly in the correct position the two dots rotate rapidly round one another with the slightest movement of the diaphragm, and presently a position will be found in which they suddenly enlarge into a continuous circle of light. That the ring in the

air is a section of a cylinder can be shown by raising the body of the microscope so as to focus sections of the cylinder at different distances from the crystal; it will be seen that the diameter of the circle remains constant in size. With a micrometer eye-piece observe the diameter of this ring. By placing a scale on the stage of the instrument, the value of the divisions of the eye-piece can be found, and thus the actual diameter of the ring will be known. Measure the thickness of the crystal, and so knowing MN, the diameter of the ring, and OM, the thickness of the crystal, the angle MON can be found. (It should be 1° 55′ in the case of arragonite.)

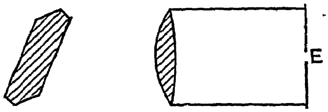


FIG. 393.

377. The internal conical refraction can also be observed by mounting a convex lens of not more than 1″ focus at the end of a short tube, closed at the other end by a disc with a pin-hole E in its centre, the lens being adjusted so that this pin-hole is at its principal focus. Then if the pin-hole is held up to the eye, the light which is to enter the eye from the pin-hole must have been parallel when it entered the lens. If, therefore, the crystal in which the internal conical refraction is to be observed has a pin-hole on its further surface, and is held in front of the lens facing a sodium flame, and tilted, there will be no necessity to use a second pin-hole. When the inclination is correct the two spots into which the light normally divides will brighten out into a circular ring, which can be seen as a bright circle by the eye placed at the pin-hole E.

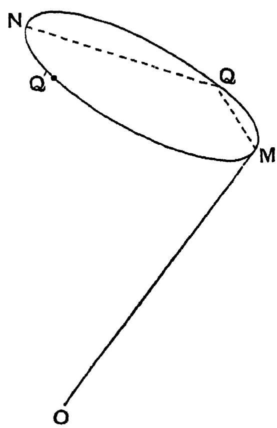


FIG. 394.

378. Observe the ring through a nicol prism, or use polarised light to form it.

Let MQN be the ring, and OM the optic axis. The direction of vibration at any point Q on the ring will be parallel to the lines QM, QN, M being the extremity of the optic axis and N the

opposite end of the diameter. So that at a point Q′ diametrically opposite to Q, the directions of vibration will be at right angles to those at Q; thus when the nicol prism is turned to extinguish the light at Q, that at Q′ will be the maximum, and the ring will fade away from Q′ round to Q.

379. **External Conical Refraction.**—Apparatus as before.

It is advisable to attach the plate of crystal this time to a glass plate on the stage of the microscope, having first pasted a pin-hole diaphragm on the lower surface of the crystal. A cone of rays must be concentrated upon this pin-hole either by the concave mirror or by a low-power sub-stage condenser. A second pin-hole diaphragm must then be placed upon its upper surface and be adjusted until the line joining the two pin-holes is parallel to the direction of single-ray velocity in the crystal. The objective must be focussed a short distance above the upper surface. On looking through the microscope two bright points will be visible, caused by the ordinary double refraction; a slight movement of the upper pin-hole will displace these, and if it is nearly in the right position will cause them to move round one another. The movement is very sensitive, and, as before, when the pin-hole is placed correctly, the two dots will open out into a circle. This time, on raising or lowering the objective, the circle will alter its size, showing it to be a section of a cone and not of a cylinder. The angle of the cone can be determined approximately by first focussing the objective on the upper pin-hole and then withdrawing it a measured distance, and observing the diameter of the ring at this distance. (The angular aperture should be $3^\circ\ 0'\ 58''$ for arragonite.)

If a crystal cut perpendicular to the bisector of the optic axis is used for this experiment, it will have to be tilted until the optic axis is approximately parallel to the axis of the instrument.

Thickness of a Film of Mica or other similar Crystalline Bi-refracting Plate.

380. Arrange the apparatus as in § 373 to obtain the circular rings and brushes with a spar crystal in convergent polarised light. Place an eye-piece with a micrometer scale in the observing

microscope. Introduce the film, of which the optical thickness is required, above the spar. It will deform the rings from circles into ellipses. Rotate it to produce its maximum effect—*i.e.* until the plane containing the optic axis is either parallel to or perpendicular to the plane of polarisation of the light entering it. Take the readings on the scale in the eye-piece of two or three of the rings—say the 8th, 9th and 10th. Now rotate the plate 90° and again read the positions of the same rings. The ratio of the displacement of a ring to the distance apart of two consecutive rings is double the fraction of the plate. For instance, a quarter wave-plate under such conditions will displace a ring by an amount equal to half the distance from that ring to the next one.

To find the Elements of the Ellipse of Elliptically Polarised Light by Stokes' Method.[1]

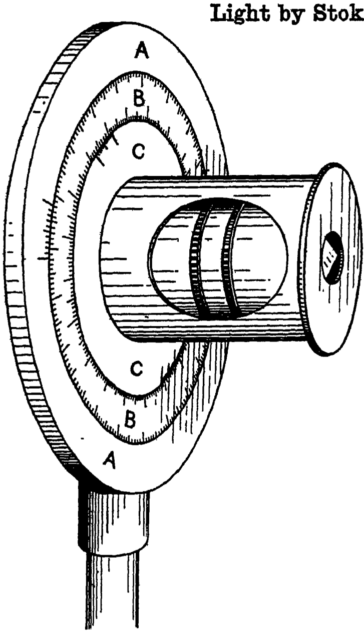


FIG. 395.—Stokes' Apparatus.

381. *Apparatus.*—A nicol; a thin crystalline plate of unknown thickness (for instance, a quarter-wave plate), to produce the elliptically polarised light; a sodium flame; and Stokes' apparatus for measuring the elements of the ellipse. This latter consists of a fixed ring A about 5″ diameter that can be attached to the optical bench; rotating in this is a divided circle B, which carries a plate of crystal which is approximately a quarter-wave plate; its exact thickness does not matter. Rotating on this circle again is another circle C, which carries one nicol. The circle carrying the quarter-wave plate is divided on both edges into degrees, which are numbered from 0° to 180°; there are pairs of verniers on

[1] *Phil. Mag.*, Nov. 1851.

the fixed ring 180° apart, and also on the circle C; the one divided circle does for both.

The divided circle B has a tube about 1″ in diameter and 2″ long attached to its face, with the plate at the end, and about half-way there is a milled ridge by which it may be rotated. The circle C has a tube which fits over the one attached to B, and it can be slipped on and off. These tubes form its only bearings. The nicol prism is fixed in the end of this tube. The tube is partly cut away on each side to allow the inner tube to be rotated. Thus, by holding the inner tube through this cut part, the two will be rotated together, and by holding the outer tube it can be rotated independently.

Experiment.—Set up in a line the polarising nicol, the thin plate to produce the elliptically polarised light and Stokes' apparatus,

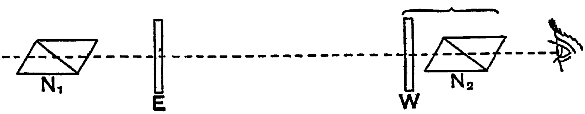


FIG. 396.--Optical Parts of Stokes' Apparatus.

and illuminate it with a sodium flame, so that the light may pass in succession through the polarising nicol N_1, the crystalline plate E, the quarter-wave plate W, nicol N_2, and then reach the eye.

The light which passed through the nicol N_1 will emerge plane polarised. On reaching E, this will be divided generally into two components at right angles, one of these being delayed an amount dependent upon the thickness of this plate. They will combine on emergence into an elliptically polarised beam, of which the axes will be parallel to the planes of polarisation in the crystal (which are supposed unknown), and of which the magnitude will depend upon its position relatively to the nicol N_1. On reaching W this elliptically polarised light will once more be decomposed into two components, of which the magnitude and the phase will depend upon the position of the axes of W relative to those of the ellipse, and which may be varied therefore by rotating W. One, or both, of these components will be delayed an amount depending upon the thickness of W. The light will emerge generally as elliptically polarised light; but as the relative phase of the two components into which the

elliptically polarised light from E is first decomposed by W may be varied by rotating W, it is possible by placing W in certain positions to make this difference of phase either equal and opposite to that which E itself introduces, and W will then bring them into the same phase; or to make it such that when added to the difference of phase that E introduces, it shall bring them into the same phase. There are thus two positions of W which will produce plane polarised light. Of course, if W be turned through 180° from either of these positions, the effect will be the same, so that in all there are four positions of W in which plane polarised light will result, and for these extinction can be produced by the nicol N_2.

Place W in any position. The light will probably emerge elliptically polarised. Rotate N_2, and the light will increase or diminish according as its plane is parallel to the major or to minor axis of the ellipse. Place it in the minimum position. W may be making the axes of the ellipse more nearly equal or more unequal. Rotate W very slightly in one direction, say clockwise, and again set N_2 so that the light may be at a minimum. Notice whether this minimum is greater or less than the preceding. If the light is now fainter than before, it shows that rotating W clockwise was making the ellipse flatter; if so, continue to rotate it in this direction, adjusting N_2 each time, and in this way it is quite easy to determine the position of both W and N_2 for which extinction is produced.

To obtain accurate readings of this position the sodium light must be very bright, so that until the exact position is found there shall still be some light visible.

Read the verniers on A and C. Rotate C through about 180° —there obviously should again be darkness. Adjust once more to extinction, and again read the verniers. The readings should be the same as before. Rotate B through 180°, carrying C with it. Adjust for extinction, and again read the verniers. Lastly, rotate C through 180°, adjust and read. There will be eight vernier readings for the position of A, which should all be the same (as B is numbered up to 180° twice), and eight similar readings for the position for C. All these readings of A should be the same, and also those of C.

Now turn W round, adjusting N_2 each time to keep the light at a minimum. It will grow to a certain point, and then it will be found that it will diminish again, and may be once more extinguished. The readings of A and C will be different from those previously obtained. Repeat all the readings for the new position as above. The readings may be entered as follows:

	Readings of the Verniers on A.		Readings of the Verniers on C.	
	1st position.	2nd position.	1st position.	2nd position.
Sum,				
Mean,	m_1	m_2	m_3	m_4

There will be eight readings in each column, and the mean of these readings is to be found. Call these means m_1, m_2, m_3, m_4 respectively. These readings are taken from an arbitrary zero.

Find angles θ_1 and θ_2, ϕ_1 and ϕ_2, such that

$$\left.\begin{aligned}\theta_1 - \theta_2 &= m_1 - m_2,\\ \theta_1 + \theta_2 &= 90^\circ.\end{aligned}\right\}$$

$$\left.\begin{aligned}\phi_1 - \phi_2 &= m_3 - m_4,\\ \phi_1 + \phi_2 &= 90^\circ.\end{aligned}\right\}$$

Then ρ, the difference of path introduced by the crystal W, and $\frac{b}{a}$, the ratio of the axes of the ellipse of the polarised light from E, are given by

$$\tan 2\phi_1 \cos \rho = -\tan 2\theta_1,$$
$$\cos 2\varpi \cdot \cos 2\theta = \cos 2\phi_1,$$

where $\tan \varpi = \frac{b}{a}$.

382. *Theory.*—Let OA and OB be the axes of the ellipse of the polarised light from E, a and b being the semi-major and -minor axes respectively. Let OX and OY be the directions of vibrations in the crystal W, and let the angle XOA be equal to θ. We may suppose the light from E incident on W to be equivalent to a vibration along OA, $a \cos pt$,

and one along OB, $b \sin pt$. On entering the crystal, these will be resolved into

$$\left.\begin{aligned} &a \cos pt \cos\theta + b \sin pt \sin\theta, \text{ along OX} \\ \text{and} \quad &-a \cos pt \sin\theta + b \sin pt \cos\theta, \text{ along OY.} \end{aligned}\right\} \quad \text{(i)}$$

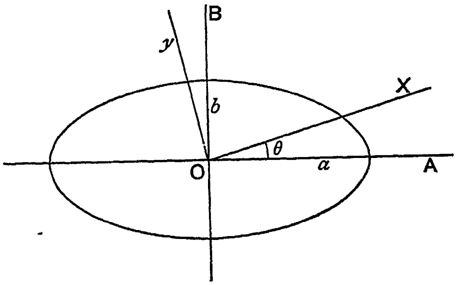


FIG. 397.

These are equivalent to

$$\left.\begin{aligned} &A\cos(pt+\epsilon_1), \quad \text{along OX.} \\ &B\cos(pt+\epsilon_2), \quad \text{along OY.} \end{aligned}\right\} \quad \text{(ii)}$$

If the plate W is to make this into plane polarised light by introducing a relative retardation ρ,

$$\rho = \epsilon_1 - \epsilon_2, \quad \text{(iii)}$$

and the plane of polarisation will make an angle ϕ with OX given by

$$\frac{B}{A} = \tan\phi. \quad \text{(iv)}$$

We have from (i) and (ii),

$$\left.\begin{aligned} A^2 &= a^2\cos^2\theta + b^2\sin^2\theta, \\ B^2 &= a^2\sin^2\theta + b^2\cos^2\theta. \end{aligned}\right\}$$

Hence
$$\cos 2\phi = \frac{1-\tan^2\phi}{1+\tan^2\phi} = \frac{A^2-B^2}{A^2+B^2} \quad \text{by (iv)}$$
$$= \frac{a^2-b^2}{a^2+b^2}(\cos^2\theta - \sin^2\theta)$$
$$= \cos 2\varpi \,.\, \cos 2\theta.$$

Again, from (i) and (ii),

$$AB(\cos\epsilon_1\cos\epsilon_2 + \sin\epsilon_1\sin\epsilon_2) = -(a^2-b^2)\sin\theta\cos\theta,$$

or [since $A^2+B^2 = a^2+b^2$],

$$\frac{2\,.\,AB}{A^2+B^2}\,.\,\cos(\epsilon_1-\epsilon_2) = -\cos 2\varpi\,.\,\sin 2\theta,$$

or
$$\sin 2\phi\,.\,\cos\rho = -\cos 2\varpi\,.\,\sin 2\theta,$$

or [using the above value for $\cos 2\phi$],

$$\tan 2\phi\,.\,\cos\rho = -\tan 2\theta.$$

APPENDIX I

The Rainbow.

383. The elementary theory of the rainbow will be found in *Preston's Light*, in which it is shown that the deviation produced by the reflection and refraction within a sphere is given by

$$\cos = \sqrt{\frac{\mu 2 - 1}{n^2 + 2n}},$$

where n is the number of internal reflections.

The more complete investigation by Airy is also given in the same book, showing that although the ray of minimum deviation is given by the above formula, the brightest part of the emergent light as seen at a distance will not be upon that line, but there will be a series of maxima, the first of which is the true position of the rainbow.

A non-mathematical student may substitute the following proof,[1] which, although not so accurate as Airy's investigation, gives almost the same results.

It is obvious that as the light emerging from the drop has a ray of minimum deviation, whilst those rays on either side are more deviated, the wave surface of the emergent light must be curved into a shape such as SCS′.

The equation to a curve of this character must be of the form $y = cx^3$ for the part of the curve near the origin, where c is some constant.

Consider the arc CS. The vibration due to it at a distant point P on a line making an angle θ with the axis will be dependent upon that of its pole A, with a loss of phase $\frac{\pi}{4}$, since AP is a minimum.

[1] Mascart, *Traite d'Optique*, vol. i. page 391.

In the same way the arc CS' will be considered equivalent to the effect of its pole A with a gain of $\frac{\pi}{4}$, since this arc is concave. Let $A'P - AP = \Delta = A'A'' = 2CB$.

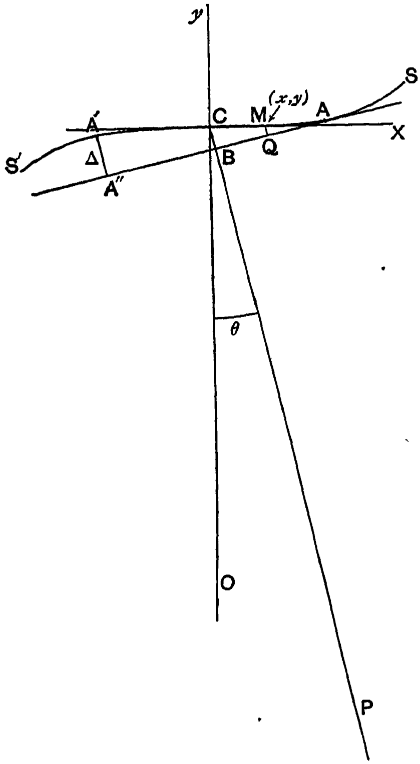


FIG. 398.

The difference of phase at P, due to the combined effect of this loss or gain of $\frac{\pi}{4}$ and of the extra length of path from A', will be

$$\delta = 2\pi\frac{\Delta}{\lambda} - \frac{\pi}{2}.$$

To find Δ, let M be any point on the curve whose coordinates are x and y.

$$CB - MQ = x\sin\theta - y\cos\theta = x\sin\theta - cx^3\cos\theta.$$

For a given value of θ, this will be a minimum when x is given by the equation $\tan\theta = 3cx^2$ (obtained by differentiating with respect to x). This gives the position of the pole A.

Substituting this value of x in

$$\Delta = 2(x\sin\theta - y\cos\theta),$$

or in

$$\Delta = 2x\cos\theta(\tan\theta - cx^2),$$

we get

$$\Delta = 2\cos\theta\sqrt{\frac{\tan\theta}{3c}}\left(\tan\theta - \frac{\tan\theta}{3}\right)$$

$$= \frac{4\sin\theta\sqrt{\tan\theta}}{3\sqrt{3c}}.$$

If A be supposed to be the amplitude due to either pole, the resulting amplitude in the direction θ is

$$2A\cos\frac{\delta}{2} = 2A\cos\pi\left(\frac{4\sin\theta\sqrt{\tan\theta}}{\lambda\,.\,3\sqrt{3c}} - \frac{1}{4}\right)$$

$$= 2A\cos\pi\left\{\frac{4}{\lambda\sqrt{a}}\left(\frac{\theta}{3}\right)^{\frac{3}{2}} - \frac{1}{4}\right\},$$

nearly, if θ is small.

It shows that the maximum value is not obtained when $\theta = 0$, but when this expression in the bracket vanishes, and that there will be a series of such maxima.

The value of the constant c depends upon the size of the drop and the order of the bow.

If we write $c = \frac{h}{3a^2}$, where a is the radius of the drop, the value of h for the first bow is 4·96, and the successive values of θ can easily be calculated.

If the radius of the jet is to be found by means of a microscope with a micrometer eye-piece, it is obvious from Fig. 399 that the

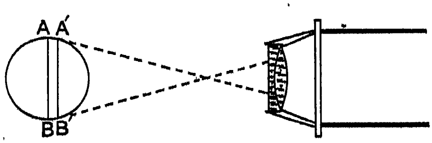


FIG. 399.

microscope will image the chord A′B′, and not the diameter AB, *i.e.* give x (Fig. 400) instead of a. [For clearness the front nodal point in

Fig. 399 is drawn at an exaggerated distance from the lens.] Calling d the distance from the nodal point to the axis of the jet, we have

$$a^2 = x^2 + \left(\frac{a}{d}\right)^2,$$

or

$$a^2\left(1 - \frac{}{d^2}\right) = x^2 \text{ nearly,}$$

or

$$a = x\left(1 + \frac{x^2}{2d^2}\right).$$

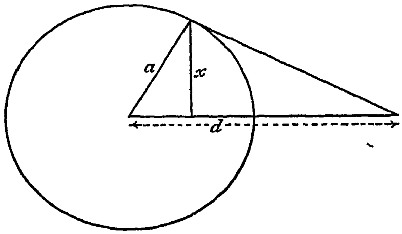


FIG. 400.

As θ will have to be positive, the deviation of the light forming the primary rainbow must be a little greater than that given by the elementary theory; that is to say, the radius of the primary bow is smaller than the ordinary theory would show.

The experimental verification of this theory was made by Miller, *Camb. Phil. Trans.*, vol. vii. page 277, and can be readily repeated (see p. 21).

APPENDIX II

Practical Hints.

384. **To cut Glass with a Glazier's Diamond.**—The diamond must be held between the first and second fingers of the right hand, the tips of these fingers pressing on the upper side of the wood on the two flats cut on it, and the tip of the thumb pressing upwards on the under flat.

The inclination of the stem requires generally to be about 60° to the glass; the angle must be found by trial. If it is held too upright, the diamond will scratch the surface of the glass only and flake off tiny splinters, the line will look rough; if held at too great a slant, it will not mark the glass except with considerable pressure. At the correct inclination it will make a clean scratch, and will also as a rule make a quiet hissing noise, which is quite easily recognised. The diamond should be drawn steadily along the glass, cutting not more than six inches a second. To guide it use a straight edge of sufficient thickness to come against the rectangular steel block. The diamond should not be drawn twice along the same line unless it failed to mark it the first time. If it has merely scratched the glass and not cut it, turn it over and try again on the other side. There is no great difficulty in the matter, and a few trials will soon teach anyone the knack.

385. **To grind Glass.**—The quickest grinding material is sharp silver sand and water on an iron plate, in the case of a surface of any size. The pressure of the glass imbeds the sand to some extent in the metal, and thus enables it to grind. For edges of plates, or the end of a tube, the most convenient thing to use is a sheet of coarse emery cloth, well moistened with turpentine. Do not press too hard or the emery will break off the cloth.

386. **To fine grind and polish Glass or Metal.**—The whole art of polishing depends upon the use of a series of grinding materials each finer than the last, and in perfect cleanliness, so that no trace of a coarser material shall get in the finer ones, and produce scratches. The first grinding having been completed with coarse emery—say 60 hole—a finer is used for perhaps a quarter of an hour or a little longer, until the scratches of the coarse emery have been removed. The polishing is commenced with a flour emery.

It is first necessary to "wash" the flour emery.[1] A couple of ordinary toilet basins are convenient for this. Fill one with water; stir up with the water about a quarter of a pound of flour emery, allow it to settle for one minute, and siphon off the liquid with a half-inch rubber tube into the second basin, being careful to leave the sediment behind with perhaps an inch of water. This sediment will contain grit, and must be rejected. Clean out the basin, stir up the liquid in the second basin thoroughly, let it settle for five minutes, and siphon the top liquid back into the first basin, again being careful to leave the sediment behind with say an inch of water. This sediment will be used for the first polishing. Leave it another few minutes to thoroughly settle, and then carefully pour away the inch of water that was left. The muddy sediment can then be collected and put in a saucer to dry. Wash out the basin. Stir up the water that was siphoned back into the first basin; let it settle for ten minutes, and siphon it off into the basin that has just been washed out. Proceed as before, putting the sediment in a second saucer to dry. Repeat these operations twice more, leaving the liquid to settle for twenty minutes and an hour respectively. The liquid siphoned off after standing for an hour may be thrown away. There will now be four saucers of material that have collected after standing five, ten, twenty, and sixty minutes respectively. There will not be much of each of the last ones, but on the other hand very little is required. The grinding with each grade of emery will take perhaps an hour; it must be continued until the scratches left by the preceding grade

[1] Fine grinding can be done with carborundum in about a quarter of the time that emery takes. The following grades ready prepared should be purchased, and about six 'wets' of each grade used, viz.: 80, 120, 150, 220, FFFF, 6 M, 15 M, 30 M. It should be complete in about six hours.

of emery have been removed. It may be done either on an iron plate or another piece of glass.

The final polish is given by rouge, which should be washed as before, that which settles in the first five minutes being rejected, and that which settles in the next half hour being used (only one grade is wanted). For most purposes a piece of broadcloth cemented on the tool used for the grinding may be used instead of the pitch. In the case of a large telescope mirror the rouge is applied by a *pitch* polisher. The pitch is melted and poured on a block of iron or glass, or other convenient support, and when partly cold is pressed on the glass that is to be polished so as to mould it to the shape of the glass. Then with a piece of wood a series of grooves from $\frac{1}{4}''$ to $\frac{3}{8}''$ apart (according to the size of the surface to be polished) are made in it parallel to one another, and a second series of similar grooves cutting the first at right angles; thus cutting up the surface of the pitch into a series of small square facets. The pitch is again pressed against the surface to be polished so as to remove the ridges which have been produced by the grooving, and left on the glass under pressure to cool. If it still is not quite right it will become so in a few days, or it may be softened by holding in front of a fire, and so moulded more rapidly.

The rouge is used with only a little water, rubbing hard and fairly quickly, and in about five minutes a polish will begin to come, and be completed in another five minutes. It is now that any scratches caused by want of care in removing all traces of the previous grade of emery in each stage of the grinding will show up. Except that they slightly scatter the light, scratches are not of serious importance optically. It is generally of much greater importance that the surface shall be truly flat or spherical, as the case may be.

387. Some very full directions for grinding and polishing and testing a telescope mirror were given in *Amateur Work*, 1886-7, by E. A. Francis, to which reference should be made by any one who wishes to attempt any grinding of this kind. In addition to the information there given, the following hints will be useful.

Fix the bench, tub, or other support firmly to the floor.

When cementing the glasses heat well in an oven, and have the ditch perfectly fluid. Avoid touching them while hot with any cold metal.

Make gauges very carefully out of zinc, grinding them together with emery.

Use silver sand and water for roughing out (not emery), then follow with 60 and 90 hole emery.[1] Get about a pound of each, and a pound of flour emery from Oakey, Wellington Mills, Westminster. (Penny packets of flour emery from an oil shop will do, however.)

After roughing out, true up with flour emery *completely* before proceeding with the washed emeries.

Mix the flour (and washed) emeries into a paste with water—not too much water—and use at least six 'wets' of each grade.

After trueing, the mirror should appear evenly grained all over, and have few, if any, pits. During fining it should appear, when held obliquely to the light, beautifully polished *all over*. Bring the fining to as high a state as possible. Examine the surface by transmitted light, with a microscope using a $1\frac{1}{2}''$ objective.

Try and keep a true spherical curve (not a parabolic one) during the fining. Use $\frac{1}{3}$ stroke with a slight side stroke.

In constructing the polisher, use best black pitch thoroughly liquid. Warm the tool in the oven, and pour the pitch on from the centre slowly in a spiral direction to the edge, continuously and evenly. The piece of wood to form the grooves should be cut carefully, and be curved to fit the tool.

In the first stage of polishing use the same stroke as above. Test it as soon as it is at all bright (in say half an hour). Test every ten minutes afterwards.

If the curve is an oblate spheroid use slightly longer strokes until it becomes spherical.

If the curve becomes hyperbolic you must return to the fine grinding and use a short stroke.

When the polish is complete, to get parabolic curve, use a long stroke for about ten minutes. Then silver it and test it in the telescope.

388. **To drill Glass.**—An ordinary drill left "glass-hard" (*i.e.* not tempered) will drill glass if used with turpentine.[2] Care is required just as the drill is coming through, or the glass may be split; otherwise there is no difficulty.

Large holes can be drilled with a copper tube of the diameter required, fed with emery, using either water or turpentine. The tube should be set up in a vertical position, and supported near

[1] Or carborundum, see footnote p. 494.

[2] Packets of suitable drills, known as watch drills, at 6d. a dozen, can be obtained from Grimshaw & Baxter, Goswell Road, Clerkenwell, the packets containing Nos. 12 to 24 are most useful.

the top and bottom by passing through holes in two pieces of wood. A lump of lead should be fixed to the top to press it on the glass, and then the tube may be rotated by passing a long piece of string round it once and pulling the ends alternately.

Glass may be turned with a steel tool moistened with turpentine, or with a diamond. The edges of lenses can be easily turned in this way.

389. **Cements.**—Glass is cemented to the chuck with shellac, or with a mixture made by melting 2 oz. orange shellac and adding a teaspoonful of oil of cassia.

A cement that will stand carbon-bisulphide can be made by dissolving eight parts of glue in one part of treacle. It is heated and applied hot to the glass, which must also be hot.

A cement that will join brass and glass is made by melting together five parts of resin, one part of red ochre, and one part of beeswax. It may be cast into sticks. It is sometimes called Electrical cement.

Canada balsam is mostly used for cementing glass together. It can be put in a glass syringe. It can then be warmed and a little forced out on to the glass. Keep the glass afterwards for at least an hour at a temperature of 200° F.

390. **Blacks.**—A dead black can be made by grinding up vegetable black dry in a mortar, or on a glass plate. Mix up with enough lacquer to form a paste. Put in a bottle and add methylated spirit. Filter through some very fine cotton gauze.

A dead black bronze for brass; mix 3 oz. Cu., ½ oz. silver nitrate, and 2 pints of nitric acid. Clean the brass, warm it, dip it until effervescence stops, burn it off, cool, and rub it with oil.

Photographic black is often useful.

391. **To silver Glass.**—Prepare the following two solutions in separate bottles:

Solution A.

Silver nitrate, 5 grams;
Distilled water, 40 cubic centimetres.

Dissolve the silver nitrate in the above quantity of water. Add ammonia slowly, a precipitate will be formed; continue to add ammonia until this precipitate is *nearly* dissolved. The solution should still be a little muddy.

Dilute to 500 c.c. with distilled water, and filter.

Solution B.

Silver nitrate, 1 gram;
Rochelle salt, ·83 gram;
Distilled water, 500 c.c.

Bring this to the boil, and filter it while hot. It must be allowed to cool before use.

The solutions can be kept for a long time in the dark in stoppered bottles.

In silvering use equal parts of A and B.

The glass to be silvered must be very thoroughly cleansed with nitric acid, water, strong sodium hydrate, and water alternately, until on wetting and allowing the water to drain off it does so without any sign of greasiness, leaving a uniform film of water on the glass. The vessel in which the silvering is to be done must also be thoroughly clean. The glass should be supported face downwards; it can be cemented to any convenient support with pitch, or it may be supported at the edges on two or three slanting glass rods. It must be kept with its face immersed in distilled water after being cleaned until the last moment. Then, having poured equal quantities of A and B into the silvering dish and mixed them with a clean glass rod, the glass is to be carefully transferred to the silvering bath, so that its face is just immersed. Care must be taken to avoid air bubbles by holding the face slightly inclined to the horizon while it is being lowered into the solution. If a thick deposit is required, it can be transferred at the end of about twenty minutes to a second silver bath mixed up in another dish.

392. **Alternative Method.**—Make up the following solutions which will be sufficient for a mirror about six inches in diameter:

C,	Nitrate of silver,	50 gr. in 2 oz. water.
	Potash, - -	50 gr. in 2 oz. ,,
D,	Glucose (pure), -	20 gr. in 2 oz. ,,

C. Add ammonia carefully to the silver solution until the precipitate is just dissolved. Add a few drops more silver nitrate solution to make a slight brownish precipitate.

Add about a third of the potash solution, and just clear with

ammonia. Add another third of the potash solution and again clear. Then add the last third and again clear. Add a few drops of silver nitrate until the liquid is a brownish yellow.

To use it put half the solution C, so made, into the silvering dish; put enough distilled water to make up the bath; add half the reducing solution D, stir it, and immerse the mirror in it. Then pour the other half of C into the dish of distilled water the mirror has been standing in, and when the silvering in the first bath is complete (in about 20 minutes), put the rest of the reducing solution in the second bath, stir it, and transfer the mirror to it. When the silvering in the second bath is complete (shown by the bath becoming brown and turbid), take the mirror out, rinse it gently under the tap for about a quarter of an hour, and stand it up to dry for about a day in a clean place.

393. **To silver Brass.**—Clean the brass thoroughly—it may be rubbed in parallel lines with very fine emery cloth, then cleaned with whiting or washed with salt brine. Dissolve some crystals of silver-nitrate in distilled water, and precipitate with an aqueous solution of common salt. Filter and wash the silver chloride precipitate so obtained with water. Mix the washed precipitate with an equal quantity of cream of tartar and enough water to form a paste. Rub the paste well on to the brass. Wash it and dry it gently with a soft cloth. To get a better coat it may be again rubbed with the paste and washed. Dry it carefully, warm it, and lacquer it with colourless lacquer. This treament is very suitable for the scale and vernier of a spectrometer, etc.

APPENDIX III

Practical Notes on Working Mica and Selenite.

394. A piece of mica should be selected as transparent as possible and free from striae. To split it, a long fine needle will be required to start the split. A very fine needle is necessary, which should be mounted in the end of a pen-holder (as the needles of botanists are mounted). It may be split under water. It is best to always split the piece of mica in equal thicknesses; that is to say, it is best to insert the needle in the middle of the thickness, so as, from the original, to produce two pieces each of half the thickness, then to halve the thickness of one of these pieces, and so on. When the thickness gets very small the finest needle will be necessary to start the split, and it should be inserted at the middle of one side. This needle should be pushed lengthwise right across the film. It may then be worked gradually to each extremity in turn.

The cleavage must be carefully watched, by observing the appearance of the Newton's rings at the line where the separation is occurring. As long as these rings remain unbroken, it is proceeding well, but if at any point the rings become broken or displaced it shows that the thickness is suddenly changing there, and if possible the cleavage should be worked round that point from the unbroken part of the ring. In this way, sometimes it may be possible to reach that point from another side, and avoid the change of thickness which would otherwise have occurred. When the film begins to get thin, it may be examined under the polariscope to see if the thickness is uniform.

After several trials, it may be possible to obtain pieces that are

approximately $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ wave thickness. Such pieces should be carefully laid aside.

Pieces of odd thicknesses may be used to form geometrical designs.

Before splitting into thin pieces, in fact as soon as colours can be seen, each piece should be placed under the polariscope and turned to the position in which it has no effect upon the light, which can be found fairly accurately; then make a scratch upon the film parallel to one of the sides of the instrument, so that the scratch is either parallel or perpendicular to the line joining the optic axes.

If convergent light is available, the position of the optic axes can be more accurately obtained with its assistance. Turn the film until the brush is rectangular and its arms parallel to the base of the instrument, and make the scratch as before. But the former method is sufficiently accurate for ordinary purposes.

Each piece as it is split off should then be laid on the scratched piece, so that the scratch is seen through, and the scratch repeated upon it, so that all the pieces shall be similarly scratched.

395. **Fox Wedge.**—If an eighth-wave plate can be split off, a "Fox wedge" can be made. It will be a very thin sheet, about ·02 mm. thick. The sheet must be cut into strips all of the same width (about $\frac{5}{8}''$), and of lengths varying from $\frac{1}{4}''$ to about $2''$, each length being $\frac{1}{8}''$ longer than the preceding one. These strips must be all cut from one film, with their lengths *parallel to one another*, and it is advisable to mark them before commencing to cut them. Their length should make an angle of 45° with the scratch.

These pieces are then to be mounted like a flight of steps, commencing at the largest one, on a piece of glass with Canada balsam, and a piece of micro-cover glass is to be put over the whole. If any difficulty is found in keeping them in place while they are being built up, they may all be put into position with a little gum on the edge A, but if so, a *very* small amount of gum must be used, as the part over which the gum spreads is almost opaque when finished.

When all are mounted, they may be warmed up and placed on a glass slip and a pool of balsam applied to one edge; it will be drawn in by capillary attraction. Any air bubbles that remain

must be squeezed out. Use plenty of balsam, as it can be easily removed afterwards. Make a big pool of balsam on the top, and place the cover glass on. A little pressure must be applied either by a weight or by a bent wire spring, and the whole left to dry for several days. A water bath will assist this. When thoroughly dry, the excess of balsam may be cleared off.

396. **Geometrical Designs.**—Simple geometrical designs of triangles, etc., may easily be constructed of pieces of mica of odd thicknesses.

In constructing these, care must be taken that all the pieces are put together with the scratches that were made upon them parallel to one another. The designs should not be too large, as if more than 1″ or $1\frac{1}{4}$″ diameter it will be difficult to project or examine them with any but very large nicols.

397. **Polariscope.**—A convenient instrument for the examination and construction of these designs is easily made by placing a few

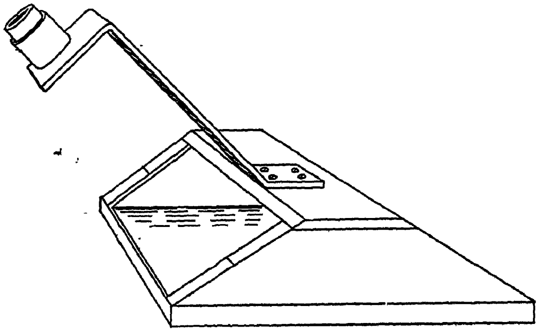


FIG. 401.

pieces of plane glass (such as old quarter-plate negatives from which the films have been removed) at the bottom of a box with a slanting front of clear glass, making an angle of 37° with the base, and a similar slanting back of ground glass.

There should be a ledge at the bottom in front on which the design to be examined may rest. An arm of bent wire or thin sheet brass should stand up from the top of the box, to which a short tube carrying a nicol prism is soldered, the prism being on a line perpendicular to the centre of the front glass plate. By placing this in front of a window or a lamp, the light reflected

from the "pile of plates" in the bottom of the box will be received by the nicol prism at the polarising angle, and the centre of the field can be extinguished by rotating the nicol. It will be observed that the extinction is only complete for a very short distance from the centre. The design to be examined, therefore, should be placed in this part of the field, and must not be very large. By attaching the arm which is to carry the nicol to the upper surface of the box free play is given for manipulating the films.

Two lines marked on the sides of the box parallel to the base, will be useful in making the scratch on the film to denote the position of its axis.

398. **Selenite or Mica Designs.**—A very simple and effective design consists of a triangle ABC, in which a second triangle of half the size, DEF, is fitted, followed by a third triangle GHK, and perhaps a fourth. Each triangle must have the scratch upon it in the same direction; if they are cut out of the one piece, and if the triangle DEF was originally in the dotted position E′F′, and GHK in the dotted position G′H′K′, this will necessarily be

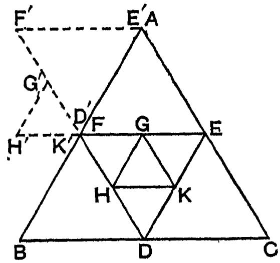


FIG. 402.

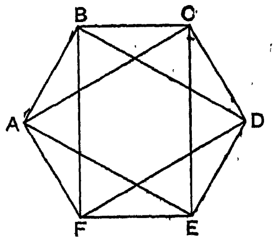


FIG. 403.

the case. The scratch upon the film indicates the position of the film when it has no effect upon the light (*i.e.* if it is either parallel or perpendicular to the line joining the optic axis); it should, therefore, in their designs make an angle of 45° with the base BC, so that when the triangle is viewed it may be vertical.

Another equally effective design is a hexagon ABCDEF formed by three rectangles BCEF, ABDE, CDFA, each rectangle being cut out in the proper direction, and mounted upon one another to construct the hexagon. They are most easily cut by drawing

the hexagon on paper, placing the film on the paper, and marking them out one by one.

This may be made more elaborate by adding films cut to the shape of the two triangles ACE and BDF. In the same way an octagon may be constructed of rectangles.

These designs may be complicated at will, and any geometrical patterns may be built up.

As a rule the designs when finished are too thin to show good colours. If a greater thickness of mica is used, its want of transparency spoils the effect, and it is better to increase the thickness by adding a film of selenite. This can be mounted separately, and one or two flat films will do for all the designs.

Special endeavours should be made to procure mica films of exactly $\frac{1}{4}$, $\frac{1}{2}$, and 1 wave thickness. The thickness may be found approximately by examining the films in the "doubler" both with the white light and the sodium flame, as already described on page 457, and may be further tested by the more elaborate methods given on pages 483 and 484.

In splitting the mica it will frequently happen that the thickness will vary from one part to another. If this is the case, the film should be laid on the instrument and examined by polarised light, then a scratch can be made round the parts which are the same thickness, so that in using the films for designs, parts where the thickness varies may be avoided.

A design like that of the hexagon described, which will not be symmetrical unless the three rectangles are of the same thickness, must have those rectangles cut all from one piece. The two triangles superimposed on them must be the same thickness as one another, but need not be the same thickness as the rectangles.

The triangular design first described may be constructed of films of unequal thickness.

The best results as regards colour will generally be produced if these films are all about one-eighth wave thickness, and an additional thickness of three waves is added by a selenite. If any difficulty is experienced in splitting a plate of selenite large enough, it may be mentioned that the selenite will be equally effective if placed against the lower surface of the nicol prism, and then it need not have a diameter greater than $\frac{1}{2}''$.

I prefer to keep the selenite separate from the design, as then by using a variety of selenites a number of colours can be produced from a single design.

399. **To split Selenite.**—To split a large film of selenite is not at all an easy task. It is necessary, in the first place, to obtain a very good specimen, for there is a great difference in the ease with which different specimens will split. To start the split a very sharp blade of steel, such as a doctor's lancet, is needed; a sewing needle is not fine enough, though perhaps if a needle were ground flat at opposite sides to make it into a thin pointed blade, it might do. This must be inserted a very short distance or the selenite will crack. The split so started will be nearly circular in shape, the outline being easily seen by observing the Newton's rings in reflected light. To continue it, use pieces of hard, good, thin notepaper, cut to the shape shown in Fig. 404. The pointed end of the paper is to be inserted in the split caused by the steel blade, which may often be removed before the paper is inserted, though sometimes it is necessary to put it in first. The paper must be pushed in as far as it will go, and then by rotating it about the point in a counter-clockwise direction the split will easily be continued.

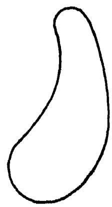


FIG. 404.

The appearance of the Newton's rings must be carefully watched and sudden breaks avoided, as explained in the case of the mica film.

The greatest difficulty usually occurs round the edges, as the cut edge of the selenite is nearly always blurred. When the paper gets crinkled it must be replaced by a new one. Several sizes of paper cut to the same shape will be useful.

Only experience can show the best method of using these, and many pieces will probably be spoilt before a good film is split. The surface will appear very scratched, as selenite is very soft, and even paper is hard enough to scratch it; but these scratches will quite disappear when the film is mounted with Canada balsam. By this method, I have succeeded in splitting films an inch and a half in diameter showing brilliant colour.

The film once split off may be cut to a convenient size—circular, I prefer—and mounted between two circular discs of glass with

Canada balsam; the glass must be optically worked, spectacle "flats" will do, and can be procured ground to a circular shape very cheaply.

The films should finally be mounted in brass rings, as they will then be much less likely to be broken. Or better still, they may be mounted between a circular piece of patent plate and a micro-cover glass. The brass rings will then not be required. All the designs of mica and selenite should be mounted on glass circles of uniform size (which will cost very little), and they will be much more convenient for use.

APPENDIX IV

Light Sources.

400. **Salt Flames.**—A very simple and effective way of producing flames of the ordinary salts (lithium, sodium, potassium, etc.) is to allow the spray produced in a rapid electrolysis to mingle with the flame. A glass tube about $\frac{1}{2}''$ diameter has two platinum wires sealed through one end which is closed. A side tube is sealed on of such a size that it fits into the mouth of a Bunsen

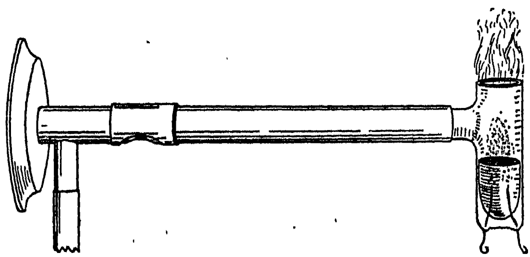


FIG. 405.—Electrolytic Salt Flamer.

burner, which is placed horizontally. A solution of the salt is put in the bottom of the tube, and decomposed rapidly with an electric current. The gas is turned on and lighted as it issues from the top of the glass vessel. A fairly strong, steady light can be produced in this way, and will last for a very long time.

See also § 197.

401. **Mercury Lamp.**—It is very little use to make a mercury lamp of glass, as it almost always cracks after a few hours' work, and although the first cost is large, one made of fused quartz should be got if possible.

A simple form of quartz lamp is shown in the figure. SS′ is the quartz tube with projections A and B, through which wires

pass into small quantities of mercury. Thin black copper vanes surround the tube at each end, by which it is air-cooled.

The water-cooled lamp shown is about the best glass one. To a tube SS with flat end are joined side tubes A, B, which are filled with mercury. A thick platinum wire passes through the bottom of each of these into a reservoir also containing mercury.

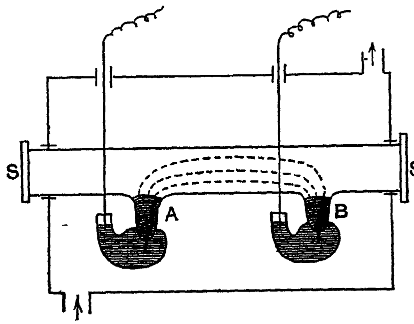


FIG. 406.—Water-cooled Mercury Lamp.

The terminals are passed into these reservoirs. The whole lamp is enclosed in a metal box through which water is kept circulating. The lamp takes a current of 5 to 10 amperes, and the arc is started by shaking a drop of mercury across from A to B. The light issues from the ends of the tube S. If the lamp is made of quartz, it may be made the same shape, but the reservoirs may be omitted, and some leaves of blackened copper put round the tube S to air-cool the lamp in place of the water circulation.

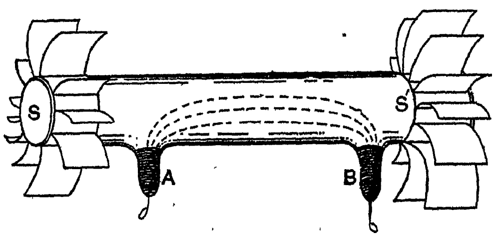


FIG. 407.—Quartz Mercury Lamp.

In using a Quartz Mercury Lamp it is most important to avoid looking at the Light directly.—Even a minute's exposure to the ultra-violet light given off would cause severe inflammation of the eye, and may permanently damage it. A glass screen cuts it off almost entirely,

but even with a glass screen it is not well to look at it for any length of time.

402. **Cadmium Lamp.**—Michelson's lamp was constructed of two glass globes A and B, connected by a horizontal tube C. The current from a 5″ coil with a mercury break entered and left by two aluminium rings D, E, 8 mm. diameter. The wires from the terminals to the rings were enclosed in two tubes G and H to prevent the vapour condensing in the cold part outside the box.

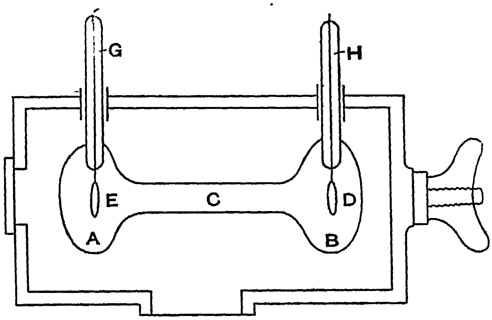


Fig. 408.—Cadmium Lamp.

Some cadmium filings were put in the bulb A. The lamp is enclosed in a metal box, and the temperature maintained between 200° and 300° C.

Instead of mercury, a cadmium amalgam containing 5 to 10 per cent. of cadmium may be used. The amalgam must be filtered hot through capillary tubes into the exhausted lamp to avoid the film of oxide that would prevent the formation of an arc. The arc can be produced between the two quantities of the amalgam, as in the ordinary mercury lamp. It must be heated to about 100° C., and fed by a current of ten amperes. It shows both the mercury and cadmium lines.

RECIPROCALS.

(**Differences** to be *subtracted.*)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	10000	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	27	36	45	54	63	72	81
11	9091	9009	8928	8849	8772	8696	8621	8547	8475	8403	8	16	23	30	38	46	53	60	68
12	8333	8264	8197	8130	8064	8000	7936	7874	7812	7752	6	13	19	26	32	38	45	51	57
13	7692	7633	7576	7519	7463	7407	7353	7299	7246	7194	6	11	16	21	27	32	38	43	49
14	7143	7092	7042	6993	6944	6897	6850	6803	6757	6711	5	10	15	19	24	29	34	38	43
15	6667	6623	6579	6536	6493	6452	6410	6369	6329	6289	5	9	13	17	21	25	29	34	38
16	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	8	11	15	19	22	26	30	34
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	19	23	26	29
18	5556	5525	5494	5464	5435	5405	5376	5347	5319	5291	3	6	9	12	15	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5050	5025	3	5	8	11	13	16	18	21	24
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	9	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	6	9	11	13	15	17	19
22	4545	4525	4504	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
23	4348	4329	4310	4292	4273	4255	4237	4219	4202	4184	2	4	6	7	9	11	13	15	16
24	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
25	4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	15
26	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	2	3	4	6	7	9	10	11	13
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
28	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
29	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	8	9	10
31	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
32	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	2941	2933	2924	2916	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
35	2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	1	2	3	4	4	5	6	7
37	2702	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	3	4	5	6	6
38	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
39	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	5	5	6
40	2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	6
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2386	1	1	2	2	3	3	4	5	5
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
43	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
45	2222	2217	2212	2207	2203	2198	2193	2188	2183	2179	0	1	2	2	2	3	3	4	4
46	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	2	3	3	4
49	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
51	1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0	1	1	2	2	2	3	3	4
52	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3	3
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3	3

RECIPROCALS.

(Differences to be *subtracted*.)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
56	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	2	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	1	1	1	1	2	2	2	3
59	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	2
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
64	1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
65	1538	1536	1533	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
66	1515	1513	1511	1508	1506	1504	1501	1499	1497	1495	0	0	1	1	1	1	2	2	2
67	1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
68	1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	1	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	1	2	2
70	1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
71	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
72	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
73	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	1	2
74	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
75	1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	1	1	1	1	1	1	1
78	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
81	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
82	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
85	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
88	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
89	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	0	1	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1102	1101	1100	0	0	0	0	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	0	0	1	1	1	1	1
92	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
94	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
95	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1041	1040	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
97	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	1	1	1	1	1

NATURAL SINES.

	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
0°	0000	0029	0058	0087	0116	0145	3	6	9	12	15	17	20	23	26
1	0175	0204	0233	0262	0291	0320	3	6	9	12	15	17	20	23	26
2	0349	0378	0407	0436	0465	0494	3	6	9	12	15	17	20	23	26
3	0523	0552	0581	0610	0640	0669	3	6	9	12	15	17	20	23	26
4	0698	0727	0756	0785	0814	0843	3	6	9	12	15	17	20	23	26
5	0872	0901	0929	0958	0987	1016	3	6	9	12	14	17	20	23	26
6	1045	1074	1103	1132	1161	1190	3	6	9	12	14	17	20	23	26
7	1219	1248	1276	1305	1334	1363	3	6	9	12	14	17	20	23	26
8	1392	1421	1449	1478	1507	1536	3	6	9	12	14	17	20	23	26
9	1564	1593	1622	1650	1679	1708	3	6	9	12	14	17	20	23	26
10	1736	1765	1794	1822	1851	1880	3	6	9	12	14	17	20	23	26
11	1908	1937	1965	1994	2022	2051	3	6	9	11	14	17	20	23	26
12	2079	2108	2136	2164	2193	2221	3	6	9	11	14	17	20	23	26
13	2250	2278	2306	2334	2363	2391	3	6	8	11	14	17	20	23	25
14	2419	2447	2476	2504	2532	2560	3	6	8	11	14	17	20	23	25
15	2588	2616	2644	2672	2700	2728	3	6	8	11	14	17	20	22	25
16	2756	2784	2812	2840	2868	2896	3	6	8	11	14	17	20	22	25
17	2924	2952	2979	3007	3035	3062	3	6	8	11	14	17	19	22	25
18	3090	3118	3145	3173	3201	3228	3	6	8	11	14	17	19	22	25
19	3256	3283	3311	3338	3365	3393	3	5	8	11	14	16	19	22	25
20	3420	3448	3475	3502	3529	3557	3	5	8	11	14	16	19	22	25
21	3584	3611	3638	3665	3692	3719	3	5	8	11	14	16	19	22	24
22	3746	3773	3800	3827	3854	3881	3	5	8	11	14	16	19	21	24
23	3907	3934	3961	3987	4014	4041	3	5	8	11	14	16	19	21	24
24	4067	4094	4120	4147	4173	4200	3	5	8	11	13	16	19	21	24
25	4226	4253	4279	4305	4331	4358	3	5	8	11	13	16	18	21	24
26	4384	4410	4436	4462	4488	4514	3	5	8	10	13	16	18	21	23
27	4540	4566	4592	4617	4643	4669	3	5	8	10	13	15	18	21	23
28	4695	4720	4746	4772	4797	4823	3	5	8	10	13	15	18	20	23
29	4848	4874	4899	4924	4950	4975	3	5	8	10	13	15	18	20	23
30	5000	5025	5050	5075	5100	5125	3	5	8	10	13	15	18	20	23
31	5150	5175	5200	5225	5250	5275	2	5	7	10	12	15	17	20	22
32	5299	5324	5348	5373	5398	5422	2	5	7	10	12	15	17	20	22
33	5446	5471	5495	5519	5544	5568	2	5	7	10	12	15	17	19	22
34	5592	5616	5640	5664	5688	5712	2	5	7	10	12	14	17	19	22
35	5736	5760	5783	5807	5831	5854	2	5	7	10	12	14	17	19	21
36	5878	5901	5925	5948	5972	5995	2	5	7	9	12	14	16	19	21
37	6018	6041	6065	6088	6111	6134	2	5	7	9	12	14	16	18	21
38	6157	6180	6202	6225	6248	6271	2	5	7	9	11	14	16	18	20
39	6293	6316	6338	6361	6383	6406	2	4	7	9	11	13	16	18	20
40	6428	6450	6472	6494	6517	6539	2	4	7	9	11	13	15	18	20
41	6561	6583	6604	6626	6648	6670	2	4	7	9	11	13	15	17	20
42	6691	6713	6734	6756	6777	6799	2	4	6	9	11	13	15	17	19
43	6820	6841	6862	6884	6905	6926	2	4	6	8	11	13	15	17	19
44	6947	6967	6988	7009	7030	7500	2	4	6	8	10	12	15	17	19

NATURAL SINES.

	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
45°	7071	7092	7112	7133	7153	7173	2	4	6	8	10	12	14	16	18
46	7193	7214	7234	7254	7274	7294	2	4	6	8	10	12	14	16	18
47	7314	7333	7353	7373	7392	7412	2	4	6	8	10	12	14	16	18
48	7431	7451	7470	7490	7509	7528	2	4	6	8	10	12	13	15	17
49	7547	7566	7585	7604	7623	7642	2	4	6	8	9	11	13	15	17
50	7660	7679	7698	7716	7735	7753	2	4	6	7	9	11	13	15	17
51	7771	7790	7808	7826	7844	7862	2	4	5	7	9	11	13	14	16
52	7880	7898	7916	7934	7951	7969	2	4	5	7	9	11	12	14	16
53	7986	8004	8021	8039	8056	8073	2	3	5	7	9	10	12	14	16
54	8090	8107	8124	8141	8158	8175	2	3	5	7	8	10	12	14	15
55	8192	8208	8225	8241	8258	8274	2	3	5	7	8	10	12	13	15
56	8290	8307	8323	8339	8355	8371	2	3	5	6	8	10	11	13	14
57	8387	8403	8418	8434	8450	8465	2	3	5	6	8	9	11	12	14
58	8480	8496	8511	8526	8542	8557	2	3	5	6	8	9	11	12	14
59	8572	8587	8601	8616	8631	8646	1	3	4	6	7	9	10	12	13
60	8660	8675	8689	8704	8718	8732	1	3	4	6	7	9	10	11	13
61	8746	8760	8774	8788	8802	8816	1	3	4	6	7	8	10	11	12
62	8829	8843	8857	8870	8884	8897	1	3	4	5	7	8	9	11	12
63	8910	8923	8936	8949	8962	8975	1	3	4	5	6	8	9	10	12
64	8988	9001	9013	9026	9038	9051	1	3	4	5	6	8	9	10	11
65	9063	9075	9088	9100	9112	9124	1	2	4	5	6	7	8	10	11
66	9135	9147	9159	9171	9182	9194	1	2	3	5	6	7	8	9	10
67	9205	9216	9228	9239	9250	9261	1	2	3	4	6	7	8	9	10
68	9272	9283	9293	9304	9315	9325	1	2	3	4	5	6	7	9	10
69	9336	9346	9356	9367	9377	9387	1	2	3	4	5	6	7	8	9
70	9397	9407	9417	9426	9436	9446	1	2	3	4	5	6	7	8	9
71	9455	9465	9474	9483	9492	9502	1	2	3	4	5	6	6	7	8
72	9511	9520	9528	9537	9546	9555	1	2	3	4	4	5	6	7	8
73	9563	9572	9580	9588	9596	9605	1	2	2	3	4	5	6	7	7
74	9613	9621	9628	9636	9644	9652	1	2	2	3	4	5	5	6	7
75	9659	9667	9674	9681	9689	9696	1	1	2	3	4	4	5	6	7
76	9703	9710	9717	9724	9730	9737	1	1	2	3	3	4	5	5	6
77	9744	9750	9757	9763	9769	9775	1	1	2	3	3	4	4	5	6
78	9781	9787	9793	9799	9805	9811	1	1	2	2	3	3	4	5	5
79	9816	9822	9827	9833	9838	9843	1	1	2	2	3	3	4	4	5
80	9848	9853	9858	9863	9868	9872	0	1	1	2	2	3	3	4	4
81	9877	9881	9886	9890	9894	9899	0	1	1	2	2	3	3	3	4
82	9903	9907	9911	9914	9918	9922	0	1	1	2	2	2	3	3	3
83	9925	9929	9932	9936	9939	9942	0	1	1	1	2	2	3	3	3
84	9945	9948	9951	9954	9957	9959	0	1	1	1	1	2	2	2	2
85	9962	9964	9967	9969	9971	9974	0	0	1	1	1	1	2	2	2
86	9976	9978	9980	9981	9983	9985	0	0	1	1	1	1	1	1	2
87	9986	9988	9989	9990	9992	9993	0	0	0	1	1	1	1	1	1
88	9994	9995	9996	9997	9997	9998	0	0	0	0	0	0	1	1	1
89	9998	9999	9999	1·0000	1·0000	1·0000	0	0	0	0	0	0	0	0	0

NATURAL TANGENTS.

(In most cases it is sufficiently accurate to use the values to four figures.)

Angle.	·0 =·0′.	·1 =6′.	·2 =12′.	·3 =18′.	·4 =24′.	·5 =30′.	·6 =36′.	·7 =42′.	·8 =48′.	·9 =54′.
0°	·00000	·00174	·00349	·00524	·00698	·00873	·01047	·01222	·01396	·01571
1	·01745	·01920	·02095	·02269	·02444	·02619	·02793	·02968	·03143	·03317
2	·03492	·03667	·03842	·04016	·04191	·04366	·04541	·04716	·04891	·05066
3	·05241	·05416	·05591	·05766	·05941	·06116	·06291	·06467	·06642	·06817
4	·06993	·07168	·07343	·07519	·07695	·07870	·08046	·08221	·08397	·08573
5	·08749	·08925	·09101	·09277	·09453	·09629	·09805	·09981	·10158	·10334
6	·10510	·10687	·10863	·11040	·11217	·11394	·11570	·11747	·11924	·12101
7	·12278	·12456	·12633	·12810	·12988	·13165	·13343	·13520	·13698	·13876
8	·14054	·14232	·14410	·14588	·14767	·14945	·15124	·15302	·15481	·15660
9	·15838	·16017	·16196	·16376	·16555	·16734	·16914	·17093	·17273	·17453
10	·17633	·17813	·17993	·18173	·18353	·18534	·18714	·18895	·19076	·19257
11	·19438	·19619	·19800	·19982	·20164	·20345	·20527	·20709	·20891	·21073
12	·21256	·21438	·21621	·21803	·21986	·22169	·22353	·22536	·22719	·22903
13	·23087	·23271	·33455	·23639	·23823	·24008	·24192	·24377	·24562	·24747
14	·24933	·25118	·25304	·25490	·25676	·25862	·26048	·26234	·26421	·26608
15	·26795	·26982	·27169	·27357	·27545	·27732	·27920	·28109	·28297	·28486
16	·28675	·28863	·29053	·29242	·29432	·29621	·29811	·30001	·30192	·30382
17	·30573	·30764	·30955	·31146	·31338	·31530	·31722	·31914	·32106	·32299
18	·32492	·32685	·32878	·33072	·33266	·33459	·33654	·33848	·34043	·34238
19	·34433	·34628	·34824	·35019	·35216	·35412	·35608	·35805	·36002	·36199
20	·36397	·36595	·36793	·36991	·37190	·37388	·37587	·37787	·37986	·38186
21	·38386	·38587	·38787	·38988	·39189	·39391	·39593	·39795	·39997	·40200
22	·40403	·40606	·40809	·41013	·41217	·41421	·41626	·41831	·42036	·42242
23	·42447	·42654	·42860	·43067	·43274	·43481	·43689	·43897	·44105	·44314
24	·44523	·44732	·44942	·45152	·45362	·45573	·45784	·45995	·46206	·46418
25	·46631	·46843	·47056	·47270	·47483	·47697	·47912	·48127	·48342	·48557
26	·48773	·48989	·49206	·49423	·49640	·49858	·50076	·50295	·50514	·50733
27	·50953	·51173	·51393	·51614	·51835	·52057	·52279	·52501	·52724	·52947
28	·53171	·53395	·53620	·53844	·54070	·54296	·54522	·54748	·54975	·55203
29	·55431	·55659	·55888	·56117	·56347	·56577	·56808	·57039	·57271	·57502
30	·57735	·57968	·58201	·58435	·58670	·58904	·59140	·59376	·59612	·59849
31	·60086	·60324	·60562	·60801	·61040	·61280	·61520	·61761	·62003	·62245
32	·62487	·62730	·62973	·63217	·63462	·63707	·63953	·64199	·64446	·64693
33	·64941	·65189	·65438	·65688	·65938	·66189	·66440	·66692	·66944	·67197
34	·67451	·67705	·67960	·68215	·68471	·68728	·68985	·69243	·69502	·69761
35	·70021	·70281	·70542	·70804	·71066	·71329	·71593	·71857	·72122	·72388
36	·72654	·72921	·73189	·73457	·73726	·73996	·74267	·74538	·74810	·75082
37	·75355	·75629	·75904	·76180	·76456	·76733	·77010	·77289	·77568	·77848
38	·78129	·78410	·78692	·78975	·79259	·79544	·79829	·80115	·80402	·80690
39	·80978	·81268	·81558	·81849	·82141	·82434	·82727	·83022	·83317	·83618
40	·83910	·84208	·84507	·84806	·85107	·85408	·85710	·86014	·86318	·86628
41	·86929	·87236	·87543	·87852	·88162	·88473	·88784	·89097	·89410	·89725
42	·90040	·90357	·90674	·90993	·91313	·91633	·91955	·92277	·92601	·92926
43	·93252	·93578	·93906	·94235	·94565	·94896	·95229	·95562	·95897	·96232
44	·96569	·96907	·97246	·97586	·97927	·98270	·98613	·98958	·99304	·99651

NATURAL TANGENTS.

(In most cases it is sufficiently accurate to use the values to four figures.)

Angle.	·0 = ·0′.	·1 = 6′.	·2 = 12′.	·3 = 18′.	·4 = 24′.	·5 = 30′.	·6 = 36′.	·7 = 42′.	·8 = 48′.	·9 = 54′.
45°	1·0000	1·0035	1·0070	1·0105	1·0141	1·0176	1·0212	1·0247	1·0283	1·0319
46	1·0355	1·0391	1·0428	1·0464	1·0501	1·0538	1·0575	1·0612	1·0649	1·0686
47	1·0724	1·0761	1·0799	1·0837	1·0875	1·0913	1·0951	1·0990	1·1028	1·1067
48	1·1106	1·1145	1·1184	1·1224	1·1263	1·1303	1·1343	1·1383	1·1423	1·1463
49	1·1504	1·1544	1·1585	1·1626	1·1667	1·1708	1·1750	1·1792	1·1833	1·1875
50	1·1918	1·1960	1·2002	1·2045	1·2088	1·2131	1·2174	1·2218	1·2261	1·2305
51	1·2349	1·2393	1·2437	1·2482	1·2527	1·2572	1·2617	1·2662	1·2708	1·2753
52	1·2799	1·2846	1·2892	1·2938	1·2985	1·3032	1·3079	1·3127	1·3174	1·3222
53	1·3270	1·3319	1·3367	1·3416	1·3465	1·3514	1·3564	1·3613	1·3663	1·3713
54	1·3764	1·3814	1·3865	1·3916	1·3968	1·4019	1·4071	1·4123	1·4176	1·4229
55	1·4281	1·4335	1·4388	1·4442	1·4496	1·4550	1·4605	1·4659	1·4715	1·4770
56	1·4826	1·4882	1·4938	1·4994	1·5051	1·5108	1·5166	1·5223	1·5282	1·5340
57	1·5399	1·5458	1·5517	1·5577	1·5637	1·5697	1·5757	1·5818	1·5880	1·5941
58	1·6003	1·6066	1·6128	1·6191	1·6255	1·6318	1·6383	1·6447	1·6512	1·6577
59	1·6643	1·6709	1·6775	1·6842	1·6909	1·6977	1·7045	1·7113	1·7182	1·7251
60	1·7320	1·7390	1·7461	1·7532	1·7603	1·7675	1·7747	1·7820	1·7893	1·7966
61	1·8040	1·8115	1·8190	1·8265	1·8341	1·8418	1·8495	1·8572	1·8650	1·8728
62	1·8807	1·8887	1·8967	1·9047	1·9128	1·9210	1·9292	1·9375	1·9458	1·9542
63	1·9626	1·9711	1·9797	1·9883	1·9969	2·0057	2·0145	2·0233	2·0323	2·0412
64	2·0503	2·0594	2·0686	2·0778	2·0872	2·0965	2·1060	2·1155	2·1251	2·1348
65	2·1445	2·1543	2·1642	2·1742	2·1842	2·1943	2·2045	2·2147	2·2251	2·2355
66	2·2460	2·2566	2·2673	2·2781	2·2889	2·2998	2·3109	2·3220	2·3332	2·3445
67	2·3558	2·3673	2·3789	2·3906	2·4023	2·4142	2·4262	2·4382	2·4504	2·4627
68	2·4751	2·4876	2·5002	2·5129	2·5257	2·5386	2·5517	2·5649	2·5781	2·5916
69	2·6051	2·6187	2·6325	2·6464	2·6605	2·6746	2·6889	2·7033	2·7179	2·7326
70	2·7475	2·7625	2·7776	2·7920	2·8083	2·8239	2·8396	2·8555	2·8716	2·8878
71	2·9042	2·9208	2·9375	2·9544	2·9714	2·9887	3·0061	3·0237	3·0415	3·0595
72	3·0777	3·0961	3·1146	3·1334	3·1524	3·1716	3·1910	3·2106	3·2305	3·2506
73	3·2709	3·2914	3·3122	3·3332	3·3544	3·3759	3·3977	3·4197	3·4420	3·4646
74	3·4874	3·5105	3·5339	3·5576	3·5816	3·6059	3·6305	3·6554	3·6806	3·7062
75	3·7321	3·7583	3·7848	3·8118	3·8391	3·8667	3·8947	3·9232	3·9520	3·9812
76	4·0108	4·0408	4·0713	4·1022	4·1335	4·1653	4·1976	4·2303	4·2635	4·2972
77	4·3315	4·3662	4·4015	4·4373	4·4737	4·5107	4·5483	4·5864	4·6252	4·6646
78	4·7046	4·7453	4·7867	4·8288	4·8716	4·9152	4·9594	5·0045	5·0504	5·0970
79	5·1446	5·1929	5·2422	5·2924	5·3435	5·3955	5·4486	5·5026	5·5578	5·6140
80	5·6713	5·7297	5·7894	5·8502	5·9124	5·9758	6·0405	6·1066	6·1742	6·2432
81	6·3138	6·3859	6·4596	6·5350	6·6122	6·6912	6·7720	6·8548	6·9395	7·0264
82	7·1154	7·2066	7·3002	7·3962	7·4947	7·5958	7·6996	7·8062	7·9158	8·0285
83	8·1443	8·2636	8·3863	8·5126	8·6427	8·7769	8·9152	9·0579	9·2052	9·3572
84	9·5144	9·6768	9·8448	10·019	10·199	10·385	10·579	10·780	10·988	11·205
85	11·430	11·664	11·909	12·163	12·429	12·706	12·996	13·300	13·617	13·951
86	14·301	14·669	15·056	15·464	15·895	16·350	16·832	17·343	17·886	18·464
87	19·081	19·740	20·446	21·205	22·022	22·904	23·859	24·898	26·031	27·271
88	28·636	30·145	31·821	33·694	35·801	38·188	40·917	44·066	47·740	52·081
89	57·290	63·657	71·615	81·847	95·489	114·59	143·24	190·98	286·48	572·96

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